## Introduction

Frequency analysis implies the study of a signal's distribution along the frequency axis. Such analysis has traditionally been carried out mathematically or by means of analog electrical *filters* constructed of conventional electrical components. With today's digital technology, there are two methods that are primarily used: the *Fast Fourier Transform* (*FFT*), and *digital filtering*. Both make use of digitized measurement values. Each type of filter is named after its affect on the signal's frequency spectrum (see Figure 9.13):

(*i*) *Low pass filter.*

(*ii*) *High pass filter*

(*iii*) *Band pass filter*

(*iv*) *Band stop filter*

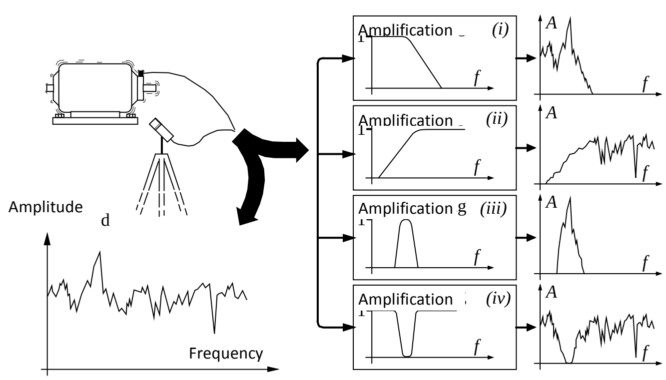


**Figure 9.13** Different filters influence on a noise signal's frequency spectrum when the signal passes through them:
(*i*) Low pass filter (*ii*) High pass filter (*iii*) Band pass filter (*iv*) Band stop filter (Source: Brüel & Kjær, course material.) [1]

The filter type that is most common is the low pass filter. Such filters are often used at the input to a measurement system to filter away frequency components higher than those to be analyzed. These removed components would otherwise introduce errors during the digitization process by contaminating the low frequency components ("aliasing"). A filter

### Band pass filters

An ideal *band pass filter*, such as the one in Figure 9.14a, suppresses components at all frequencies except those that lie within the *bandwidth B* (i.e., "passes" those in *B*). In practice, however, the edges of the band have a certain slope, as shown in Figure 9.14b, which implies that the frequency components immediately outside of the pass band are not completely eliminated. A common way to define the upper $f_u$ and lower $f_l$ frequency limits of the band is to indicate the frequencies at which the signal is reduced by 3 dB.

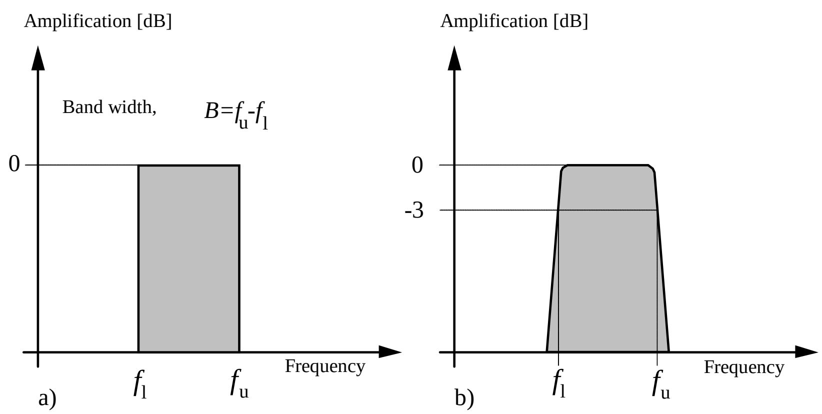


**Figure 9.14** a) Ideal band pass filter with infinitely steep cutoffs. b) Real filters have imperfect cutoffs. The upper and lower bounding frequencies are then defined by the frequencies at which the filter reduces the signal by 3 dB.

Band pass filters are named according to how the bandwidth varies along the frequency axis. Filters with bandwidths that do not vary along the frequency axis are called *constant absolute bandwidth* (*CAB*) *filters*; see Figure 9.15 a. A filter with a bandwidth proportional to its center frequency, $f_C$, is called a *constant relative bandwidth* (*CRB*) *filter*; see Figure 9.15 b.

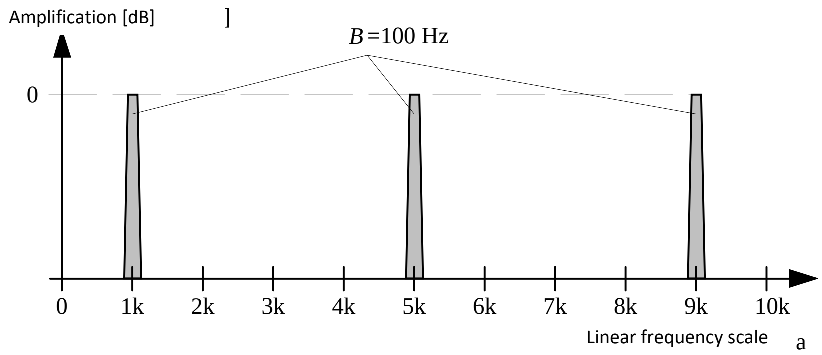


**Figure 9.15 a** CAB filter, with a bandwidth that does not vary along the frequency axis; it is typically presented with a linear frequency axis.

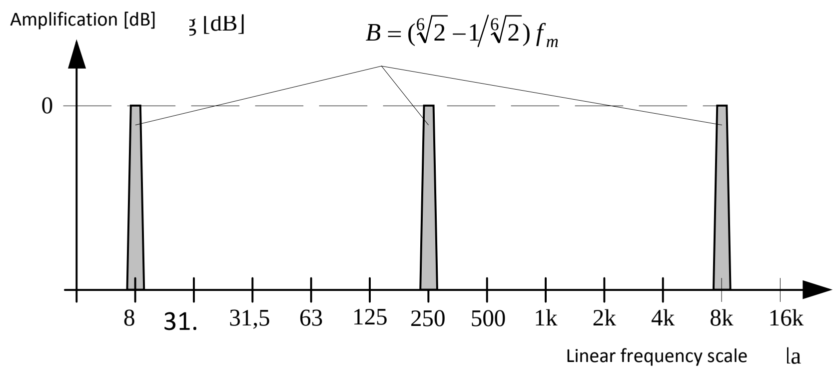


**Figure 9.15 b** CRB filter, with a bandwidth that is a certain percentage of the center frequency $f_c$; it is typically presented with a logarithmic scale. Because of the logarithmic scale, the stacks in the figure do not get wider, moving to the right along the axis. The example in the figure is called a third octave band filter and has a band width that is about 23% of the center frequency.

### Third-octave and octave band filters

*Third-octave* and *octave band filters* are CRB filters very widely used in the field of sound and vibrations. *Center frequencies* are standardized, and listed in table 9.1. Both types of filters are named with band numbers, as in table 9.2, or more often by their center frequencies, $f_c$. As is evident from table 9.2, each octave band spans three third-octave bands, which explains the name of this category of filters.

**Table 9-1** Definition of third-octave and octave band filters.

| | Octave band filter | Third-octave band filter |
|---|---|---|
| **Lower frequency limit** | $f_l = f_c / \sqrt{2}$ | $f_l = f_c / \sqrt[6]{2}$ |
| **Upper frequency limit** | $f_u = \sqrt{2} f_c$ | $f_u = \sqrt[6]{2} f_c$ |
| **Bandwidth** | $B = f_u - f_l = \left(\sqrt{2} - 1/\sqrt{2}\right) f_c$ | $B = f_u - f_l = \left(\sqrt[6]{2} - 1/\sqrt[6]{2}\right) f_c$ |
| **Center frequency** | $f_c = \sqrt{f_l f_u}$ | $f_c = \sqrt{f_l f_u}$ |

**Table 9-2** Standardized center frequencies and upper and lower frequency limits of third-octave and octave band filters. Shading indicates octave bands.

| Band no. | Center frequency $f_c$ [Hz] | 3rd-octave band filter $f_l - f_u$ [Hz] | Octave band filter $f_l - f_u$ [Hz] | Band no. | Center frequency $f_c$ [Hz] | 3rd-octave band filter $f_l - f_u$ [Hz] | Octave band filter $f_l - f_u$ [Hz] |
|---|---|---|---|---|---|---|---|
| 1 | 1.25 | 1.12 - 1.41 | | 23 | 200 | 178 - 224 | |
| 2 | 1.6 | 1.41 - 1.78 | | 24 | **250** | 224 - 282 | **178 - 355** |
| 3 | **2** | 1.78 - 2.24 | **1.41 - 2.82** | 25 | 315 | 282 - 355 | |
| 4 | 2.5 | 2.24 - 2.82 | | 26 | 400 | 355 - 447 | |
| 5 | 3.15 | 2.82 - 3.55 | | 27 | **500** | 447 - 562 | **355 - 708** |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 6 | **4** | 3.55 - 4.47 | **2.82 - 5.62** | 28 | 630 | 562 - 708 | |
| 7 | 5 | 4.47 - 5.62 | | 29 | 800 | 708 - 891 | |
| 8 | 6.3 | 5.62 - 7.08 | | 30 | **1000** | 891 - 1120 | **708 - 1410** |
| 9 | **8** | 7.08 - 8.91 | **5.62 - 11.2** | 31 | 1250 | 1120 - 1410 | |
| 10 | 10 | 8.91 - 11.2 | | 32 | 1600 | 1410 - 1780 | |
| 11 | 12.5 | 11.2 - 14.1 | | 33 | **2000** | 1780 - 2240 | **1410 - 2820** |
| 12 | **16** | 14.1 - 17.8 | **11.2 - 22.4** | 34 | 2500 | 2240 - 2820 | |
| 13 | 20 | 17.8 - 22.4 | | 35 | 3150 | 2820 - 3550 | |
| 14 | 25 | 22.4 - 28.2 | | 36 | **4000** | 3550 - 4470 | **2820 - 5620** |
| 15 | **31.5** | 28.2 - 35.5 | **22.4 - 44.7** | 37 | 5000 | 4470 - 5620 | |
| 16 | 40 | 35.5 - 44.7 | | 38 | 6300 | 5620 - 7080 | |
| 17 | 50 | 44.7 - 56.2 | | 39 | **8000** | 7080 - 8910 | **5620 - 11200** |
| 18 | **63** | 56.2 - 70.8 | **44.7 - 89.1** | 40 | 10000 | 8910 - 11200 | |
| 19 | 80 | 70.8 - 89.1 | | 41 | 12500 | 11200 - 14100 | |
| 20 | 100 | 89.1 - 112 | | 42 | **16000** | 14100 - 17800 | **11200 - 22400** |
| 21 | **125** | 112 - 141 | **89.1 - 178** | 43 | 20000 | 17800 - 22400 | |

| | | | | |
|---|---|---|---|---|
| 22 | 160 | 141 - 178 | | |

**ADDITION OF FREQUENCY COMPONENTS**

There are different ways to describe a signal's distribution in the frequency dimension or in frequency bands. Narrow bands give detailed information on the distribution of energy, with relatively low amplitudes in each band. Figure 9.16 shows the same spectrum presented with different bandwidths.

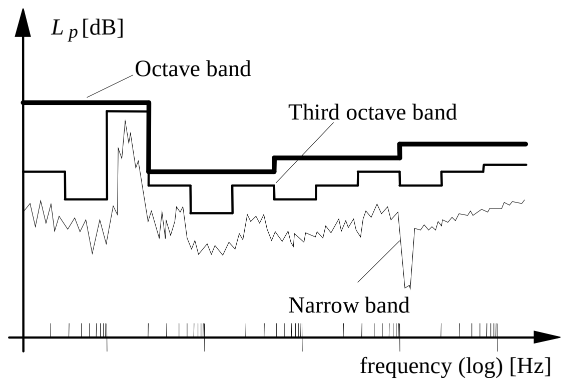


**Figure 9.16** The same sound spectrum presented in narrow band, third octave bands, and octave bands. The bigger the bandwidth, the more frequency components that contribute to any band, giving higher levels. The logarithmic frequency axis causes the CRB filters to have the same apparent width per band over the entire spectrum, while CAB filters, with their fixed band width over the entire spectrum, appear to grow more dense as frequency increases.

Summation of the sound pressures of individual frequency components is carried out in the same way as for the summation of sound pressures from multiple sources,

$$\boxed{\tilde{p}_{tot}^2 = \sum_{n=1}^{N} \tilde{p}_n^2} \qquad (9.33)$$

and

$$\boxed{L_{p_{tot}} = 10 \cdot \log \sum_{n=1}^{N} 10^{L_{pn}/10}} \qquad (9.34)$$

where the index $n$ stands for individual frequencies or frequency bands, instead of distinct sources. The proof for each of these formulas is from *Parseval's relations* for periodic and non-periodic functions, which in turn comes from Fourier analysis.

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**Example 09-1**

The 1000 Hz octave band includes the 800, 1000, and 1250 Hz third-octave bands. Determine the octave band level, if the third-octave band levels are 79, 86 and 84 dB, respectively.

*Solution*

Formula (2-36) gives $L_p = 10 \cdot \log(10^{7.9} + 10^{8.6} + 10^{8.4}) = 88.6 \approx 89$ dB.

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Source:
http://nptel.ac.in/courses/112107088/40