Vibration response of sandwich plate under Low-velocity impact loading

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Abstract - The vibration response of sandwich panels, under low-velocity impact, is presented by using finite element method (FEM) and Hertz contact low. The vibrations on the sandwich plate under the impact are evaluated using the Wigner-Ville distribution. As a result, by formulating a simple model involving the motion of a rigid impactor combined with dynamic equation of sandwich plate, the effect of initial impact velocity and the geometric properties of plate are identified. This numerical simulation that determine the impact force and estimate the behaviour under impact would be very helpful during the sandwich structures design.

Keyword: - Low-velocity/ Impact/ Hertz contact law / Impact force/ Wigner-Ville distribution/ Indentation

1 Introduction

Impact is defined as the process involved in the collision of two or more objects. Now, elastic advanced plates have been widely used in aircraft, aerospace, automotive and marine.

During impact, kinetic energy is lost. The loss of kinetic energy is caused by wave propagation, plastic deformation or viscoelastic phenomena and depends on the shapes and material properties of the colliding bodies as well as on their relative velocities [1]. Some methods and results of the impacts on the flexible structures are recapitulated by Goldsmith [1], Johnson [2] and Stronge [3].

In low-velocity impact, Finite element analysis and a modified Hertz contact law were employed to relate the impactor movement with the displacement history of the sandwich panels, [4, 5]. An other approach based on a system having three-degree of freedom, consisting of spring-mass-damper-dashpot (SMDD) or spring-mass-damper (SMD), used by Malekzadeh et al. [6] to model the interaction between the impactor and the composite sandwich panel. Also, in the modelling of the composite structures impact, Abrate [7] used an energy-balance model and spring-mass models to analyse dynamic impacts.

In this paper, the low-velocity impacts between a rigid sphere and a clamped sandwich plate have been modelled. Impact response analysis is performed using FEM and Hertz contact law. The impact problem formulation is presented, the method used to calculate impact forces and then an algorithm for solving elastic impact problems is developed. Numerical results, including contact force, velocity of impactor, indentation and dynamic response of plate subjected to low-velocity impact is presented.

2 Formulation of the problem

In this section impact problem of a rigid sphere against a fully clamped sandwich plate is formulated. The sandwich plate is modelled by using the finite-element method (FEM).The Hertz contact law is used to simulate the pressure distribution in the contact area between the sphere and the sandwich plate.

2.1 Bi-dimensional sandwich plate model

Three perfectly glued layers form the studied sandwich plate. The two external layers, called skins, are supposed elastic, isotropic and homogeneous, and the core layer is assumed to be viscoelastic. A partial cross section of the plate in the xz-plane as shown in Fig. 1.

To model the sandwich plates a linear longitudinal displacement field hypothesis is adopted in each layer. The two sandwich plate skins and the core follow, respectively, the assumption of Love Kirchhoff and the assumption of Mindlin.

Taking into account the above-mentioned assumptions, the two skins and the core longitudinal displacement can be expressed in terms of 7 fundamental quantities, the longitudinal displacements u_m and v_m, the relative longitudinal displacements $[u]$ and $[v]$, the transversal displacement w_f and the skins rotations β_x and β_y, (Fig. 1) [8]:

Skins plates

$$\begin{cases} u_f = u_m + \dfrac{[u]}{2} + \left(z - z_f\right)\beta_x \\ v_f = v_m + \dfrac{[v]}{2} + \left(z - z_f\right)\beta_y \\ w_f = w \end{cases}$$

Viscoelastic core

$$\begin{cases} u_c = u_m - z_m\beta_x + z\varphi_x \\ v_c = v_m - z_m\beta_y + z\varphi_y \\ w_c = w \end{cases}$$

(f=1,2) (1)

where,

$$u_m = \frac{u_{10} + u_{20}}{2} \qquad v_m = \frac{v_{10} + v_{20}}{2} \qquad (2)$$

$$[u] = u_{10} - u_{20} \qquad [v] = v_{10} - v_{20} \qquad (3)$$

$$\varphi_x = \frac{[u]}{h_c} + \left(\frac{z_1 - z_2}{h_c} - 1\right)\beta_x \qquad \varphi_y = \frac{[v]}{h_c} + \left(\frac{z_1 - z_2}{h_c} - 1\right)\beta_y \qquad (4)$$

$$z_m = \frac{z_1 + z_2}{2} \qquad (5)$$

From displacement expression, the stress-strain relationship at each point for fth layer of the skin plate is:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}_f = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix}^f \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}_f \quad (f=1,2) \qquad (6)$$

where,

$$D_{11} = D_{22} = \frac{E_f}{1 - \nu_f^2}\ ; \quad D_{12} = \frac{E\,\nu_f}{1 - \nu_f^2}\ ;$$

$$D_{33} = \frac{E_f}{2\left(1 + \nu_f\right)} \qquad (7)$$

In the linear viscoelastic core, the stress-strain relation is obtained by considering a relaxation model, [9, 10]:

$$\sigma_{ij}\left(t\right) = \int_0^t C_{ijkl}(t - \tau)\frac{d\varepsilon_{kl}}{d\tau}\, d\tau \qquad (8)$$

This representation, used to model viscoelastic material behaviour, leads to complex relationship between the stress and strain in the core, [10].

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix}_c = \begin{bmatrix} C_{11}^* & C_{12}^* & 0 & 0 & 0 \\ C_{12}^* & C_{22}^* & 0 & 0 & 0 \\ 0 & 0 & C_{33}^* & 0 & 0 \\ 0 & 0 & 0 & C_{44}^* & 0 \\ 0 & 0 & 0 & 0 & C_{55}^* \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}_c \qquad (9)$$

where,

$$C_{11}^* = C_{22}^* = \frac{E_c^*}{1 - \nu_c^2}\ ; \ C_{12}^* = \frac{E_c^*\ \nu_c}{1 - \nu_c^2}\ ;$$

$$C_{33}^* = C_{44}^* = C_{55}^* = G_c^* \qquad (10)$$

The dynamic equation is obtained by the use of FEM. So a quadrilateral homogenized sandwich finite element is used which has four nodes and seven degrees of freedom per node (u_m, v_m ,$[u]$, $[v]$,β_x, β_y, w).

The equation of the sandwich plate motion can be written as:

$$M\ddot{W}\left(t\right) + K^* W\left(t\right) = F\left(t\right) \qquad (11)$$

In case of proportional damping, there is relationship between hysteretic and viscous damping models [11], leading to equivalent dynamic system of the sandwich plate which is gives (*Appendix B*):

$$M\ddot{W}\left(t\right) + C\dot{W}\left(t\right) + KW\left(t\right) = F\left(t\right) \qquad (12)$$

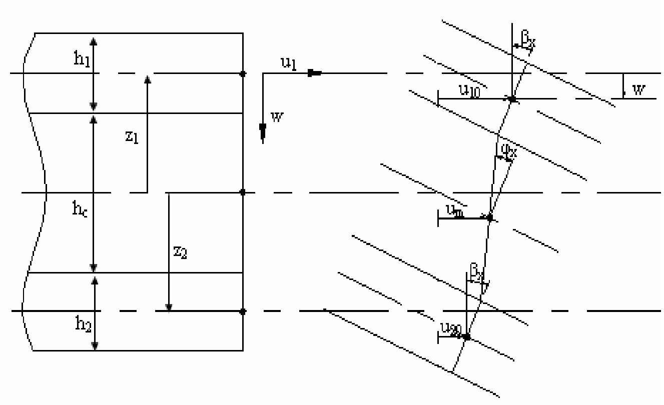

Fig. 1. Sandwich plate displacements in the x-z-plane.

2.2 Application of Hertz law in ball-plate contact

If the indentation due to the impact load is much smaller than the plate thickness, the contact force between the impactor and the sandwich plate during the impact is assumed to be governed by the nonlinear Hertzian contact law of the form:

$$F(t) = k_c \left[\delta(t) \right]^{3/2} \tag{13}$$

where δ is the indentation of the ball-plate bodies at the contact points, defined by:

$$\delta = Z_I - W\left(\frac{b}{2}, \frac{b}{2}, \frac{h_I}{2} \right) \tag{14}$$

where Z_I denotes the impactor displacement and W is the transverse top face plate displacement at the impact location.

And k_c is the Hertzian contact stiffness, defined in reference [4, 12] by:

$$k_c = \frac{4\sqrt{R_I}}{3\left(\frac{1-\nu_I^2}{E_I} + \frac{1-\nu_P^2}{E_P} \right)} \tag{15}$$

Therefore, for applying equation (15) in sandwich plates impact analysis, E_P and ν_P can be estimated using the following relations, [13]:

$$E_P = \frac{h}{h_1/E_1 + h_2/E_2 + h_c/E_c};$$

$$\nu_P = \nu_1\left(\frac{h_1}{h} \right) + \nu_2\left(\frac{h_2}{h} \right) + \nu_c\left(\frac{h_c}{h} \right) \tag{16}$$

2.3 Numerical models for the impact process

In this section we study the problem of the frictionless normal low velocity impact of a rigid sphere against an elastic plate.The dynamic equations for both the sandwich plate and impactor (rigid sphere) can be written as:

$$\begin{cases} M\,\ddot{W}(t) + C\,\dot{W}(t) + K\,W(t) = F(t) & (17) \\ \text{where, } W(0) = \dot{W}(0) = \ddot{W}(0) & \\ \qquad \text{and} & \\ M_I\,\ddot{Z}_I + F(t) = 0 & (18) \\ \text{where, } Z_I(0) = \ddot{Z}_I(0) = 0\ ;\ \dot{Z}_I(0) = \mathrm{V}_0 & \end{cases}$$

For the resolution of the two dynamic equations (17) and (18) simultaneously, an iterative numerical method according to the procedure described below was used.

At the initial moment $t_0 = 0$ (beginning of the shock), the sphere punches the plate with an initial velocity V_0. We assume an arbitrary contact force value $F(t) = F_0$.

The dynamic equations (17) and (18) are solved using the Newmark integration scheme. That leaded to calculate the indentation δ. The force $F(t)$ is corrected then by using the Hertzian contact law expressed by the relation (13) after having determined the penetration δ of the impactor in the plate, given by the expression (14). The value of $F(t)$, thus obtained, is reinjected in the equations (17) and (18), and to take again the iterative process until convergence of $F(t)$. Two cases are possible:

- If $\delta > 0$, the contact between the sphere and the plate exist; the impact force is defined by the Hertzian contact law and $F(t)$ will be recalculated using a new value of δ, and is then injected into the dynamic equations to be solved again.
- If $\delta \le 0$, the contact is lost and the dynamic responses are determinate.

3 Numerical results and discussion

3.1 Validation

In order to ensure the accuracy of the present model and the computer program developed, impacted elastic plate system is validated by Liu et al. [14].
Liu et al considered a clamped 100 x 100 x 0.2 mm^3 steel plate impacted by a steel sphere at its centre. The initial condition of zero displacement and zero velocity are assumed for the plate.

The mechanical properties of the plate are given in Table1.

Table 1. Mechanical features of elastic plate

Young modulus (GPa)	Density (kg/m^3)	Poisson coefficient
210	7800	0.3

The radius of the sphere is 5 mm and the velocity just before impact is V_0=10 m/s.
Fig. 2 illustrated the comparison between the results of the centre plate deflection obtained by the present study and by Liu et al. [14], who used the Lagrangian approach.
The numerical results showed conform reproducibility with Liu's results at the impact moment ([0, 0.2] period time). After 0.2 ms there was no contact between the impactor and the plate, and these curves represented the plate free vibration after impact.

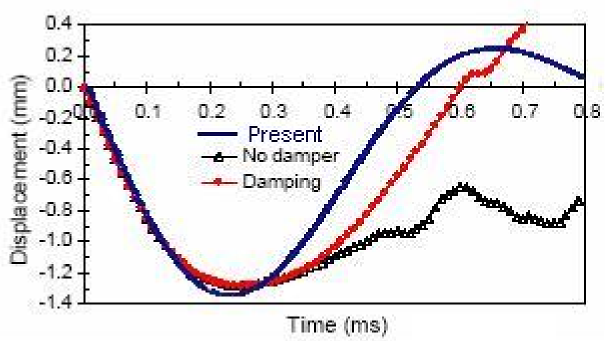

Fig. 2. Comparison of displacement history for a steel plate.

3.2 Study of impacted sandwich plate

In this section, a parametric study of a sandwich plate subjected to impact loading is conducted, and the effects of varying the different parameters, such as initial velocity and thickness of the skins, on the impact process are investigated using proposed model.
The impactor is considered rigid, with a mass 32.8 g and radius R_I=10 mm, and it is assumed that this spherical impactor hits the plate at its center with an initial velocity V_0
The geometrical and mechanical characteristics of ıe studied plates are mentioned on Tables 2 and 3.

Table 2. Geometrical sandwich plate characteristics

Sandwich plate	Side b (mm)	Thickness (mm)
P1	100	h_1=0.5;h_c=1; h_2=0.5
P2	100	h_1=1 ; h_c=1 ; h_2=1

Table 3. Mechanical sandwich plate characteristics

	Young Modulus E (GPa)			Poisson Coefficient ν			Density ρ	Material properties
Skins	E =69 GPa			0.33			2700 kg/m^3	Aluminium
	E'(MPa)	E"(MPa)	η_E (loss factor)	G'(MPa)	G"(MPa)	η_G (loss factor	Density ρ	Material properties
Core	113.5	3.27	0.0288	18.86	1.26	0.067	130 kg/m^3	HEREX C70.130 [10]

3.2.1 Influence of impactor initial velocity

In this part the effect of the initial velocity on the dynamic behaviour of sandwich plate P1 is studied.
Figs. 3 and 4 illustrate the impact force and the impactor velocity, according to the time axies, for the sandwich plate P1 and with different initial impactor velocity. These figures showed that the increase in impactor velocity led to an increase in impact force but involved a decrease in the loading time.
Fig. 5 illustrates the frequency responses of the sandwich plate P1 for various incidental velocities

of the impactor, whereas Fig. 6 illustrates the temporal responses of the sandwich plate P1 for various incidental velocity of the impactor.

In the case of an elasto-dynamic impact problem, the shock, on the sandwich plate P1, using a steel sphere (impactor) excites the same frequencies even if we varied the initial velocity of the impactor. we can also notice that only the amplitude of acceleration increases with the increase of the initial impact velocity.

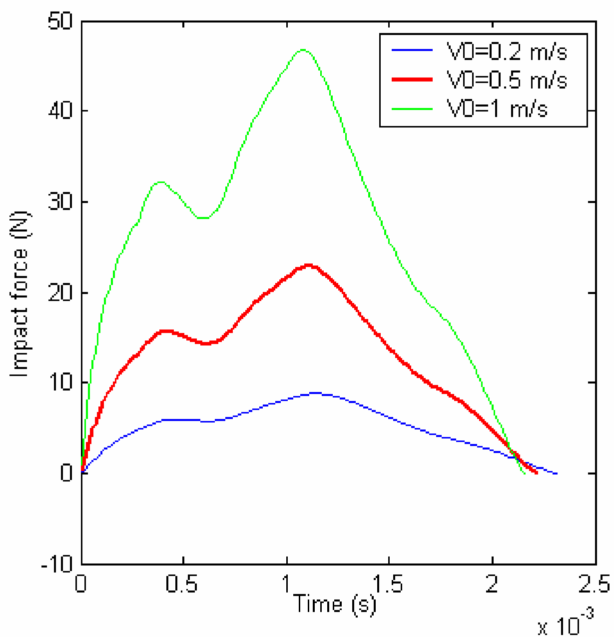

Fig. 3. impact force on the sandwich plate P1

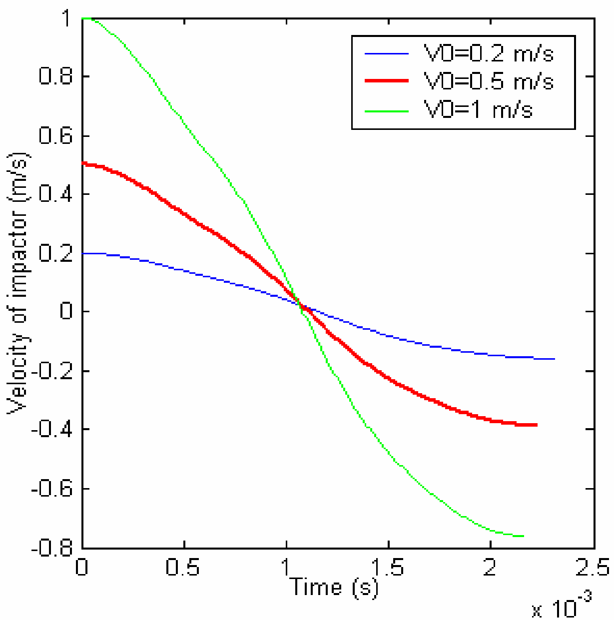

Fig. 4. Velocity of the impactor during the impact process.

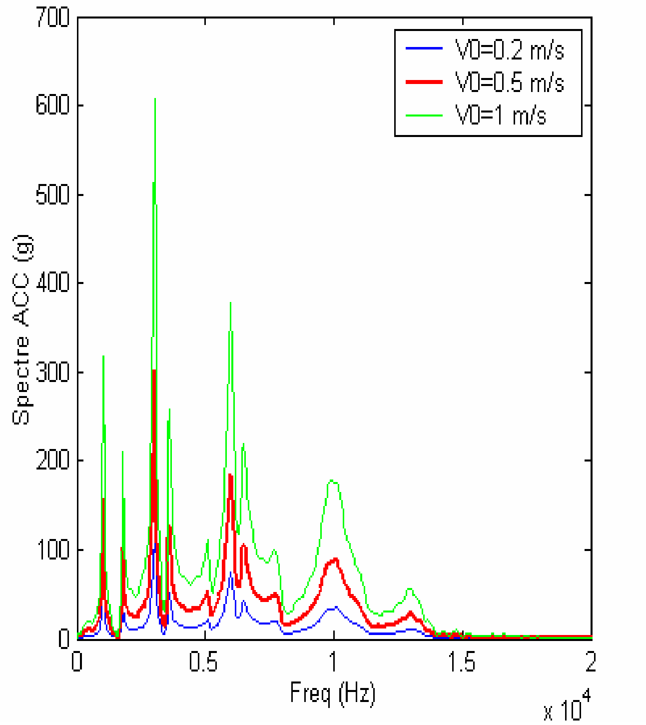

Fig. 5. Frequency responses of impacted sandwich plate for different initial velocity of impactor.

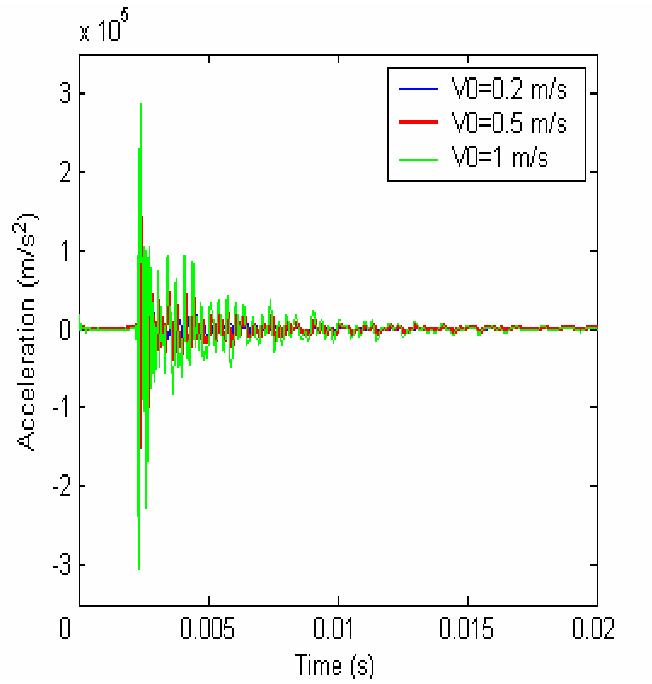

Fig. 6. Temporal responses of impacted sandwich plate for different initial velocity of impactor.

3.2.2 The skins thickness Influence

In order to analyze the influence of the skins thickness on the dynamic behaviour of the sandwich plate subjected to impact, two examples of sandwich plate are taken.

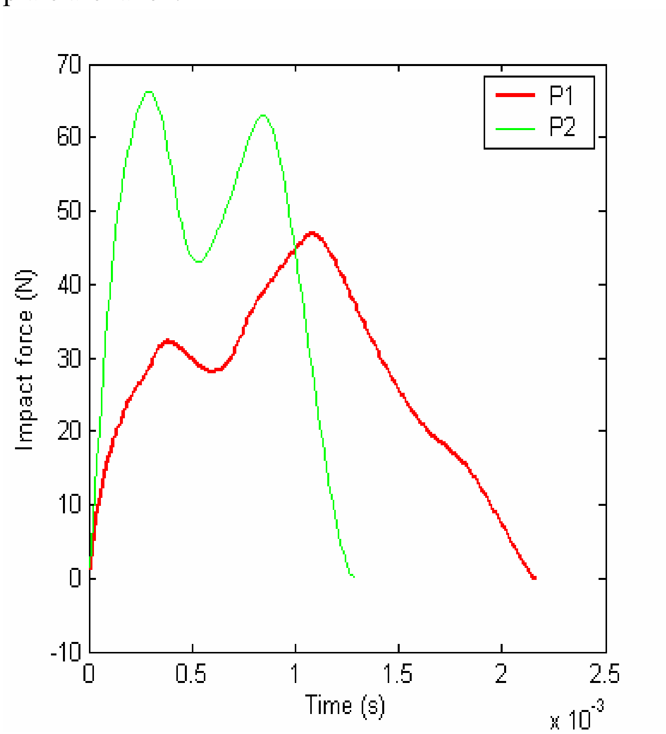

Fig. 7. Impact force of sandwich plate P1 and P2 for an initial impact velocity $V_0 = 1$m/s.

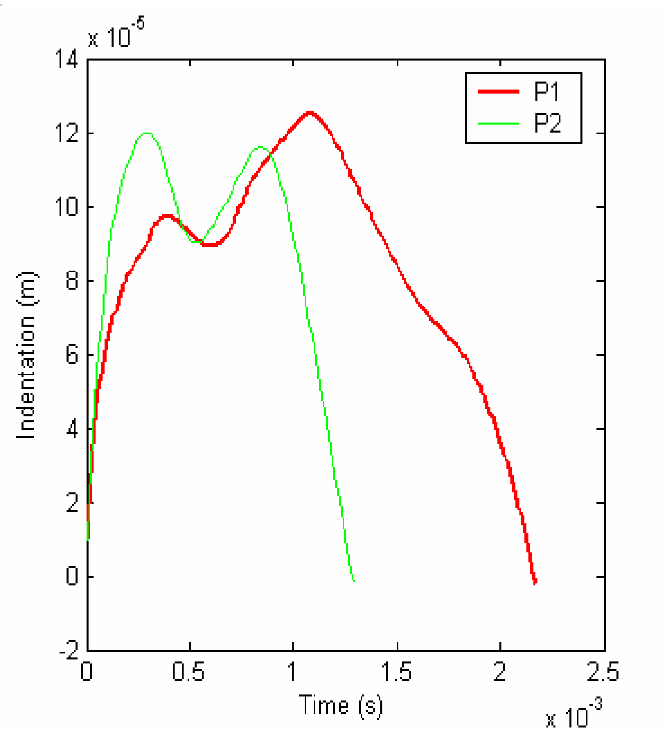

Fig. 8. Indentation of sandwich plate P1 and P2 for an initial impact velocity $V_0 = 1$m/s.

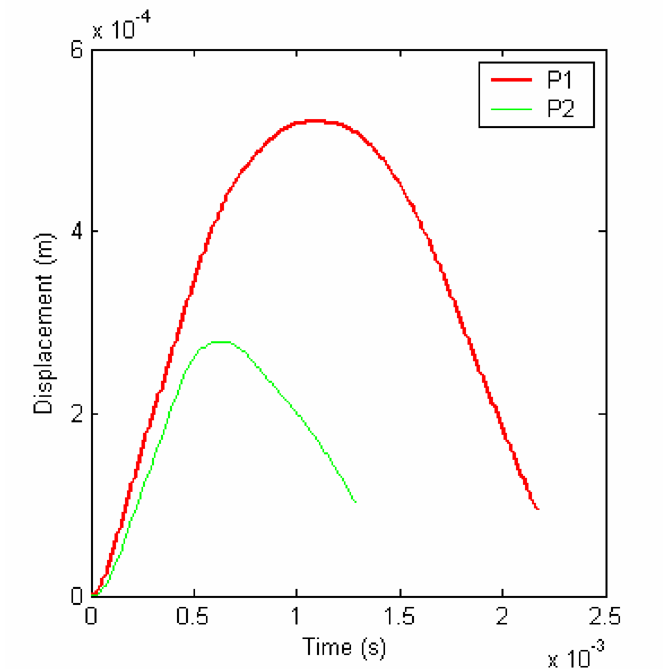

Fig. 9. Displacements of the center plates for an initial impact velocity $V_0 = 1$m/s.

Fig. 7 represents the evolution according to the time of the impact force on the plates P1 and P2 for an initial impact velocity $V_0 = 1$m/s.

This figure shows that the increase of the skins thickness of the plates generates an increase in the impact force and a reduction in the shock duration. This is due to the increase in the rigidity of the plate with the increase of the skins thickness. Also the effect of viscoelastic core during the impact process decreases with the increase of the layers thickness.

Fig. 8 illustrates the indentation, according to the time axes, for the tow plates.

Fig. 9 illustrates the comparison between the results of the centre plate deflection of sandwich plates P1 and P2

Comparison between two plates P1 and P2 showed that: The increasing of skins thickness caused a decrease in indentation and in displacement.

During collisions, energy is both absorbed by the bending of the plate and the absorbing character of the viscoelastic core. By observing Figs 8 and 9, we note that the energy absorbed by flexing of the plate is larger in the case of plate P1 (displacement of plate P1 is large than plate P2), and the energy absorbed by transverse shear of the core is larger for the plate P1 (indentation in the plate P1 is large than in the plate P2).

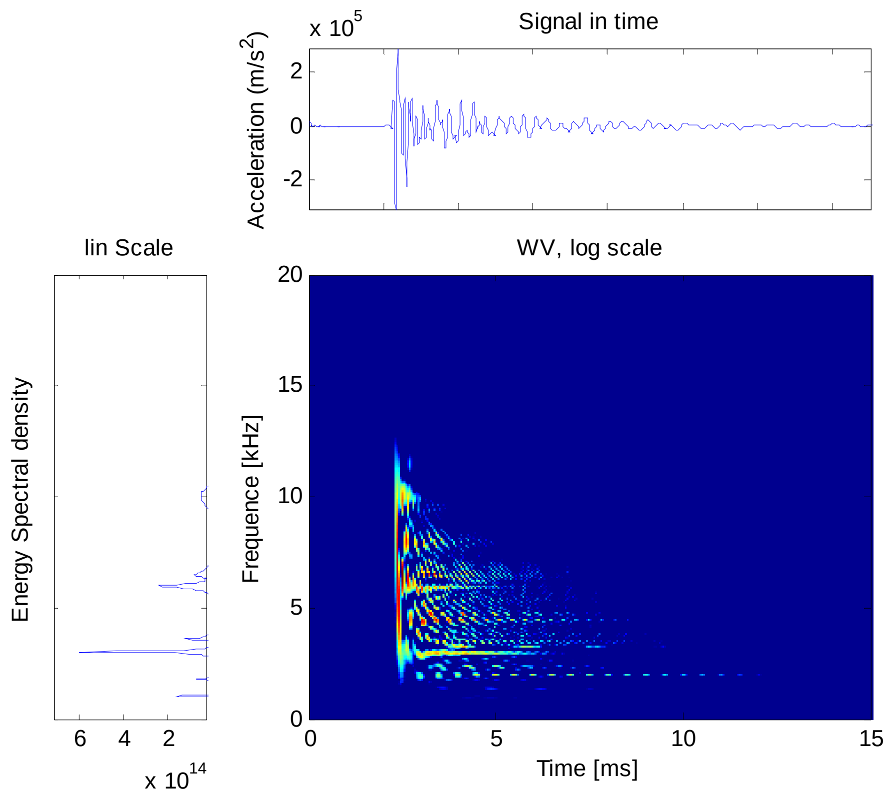

Fig. 10. Wigner-Ville Distribution, frequency response and temporal response of the Sandwich plate P1 under impact of a sphere with an initial velocity V_0 =1m/s.

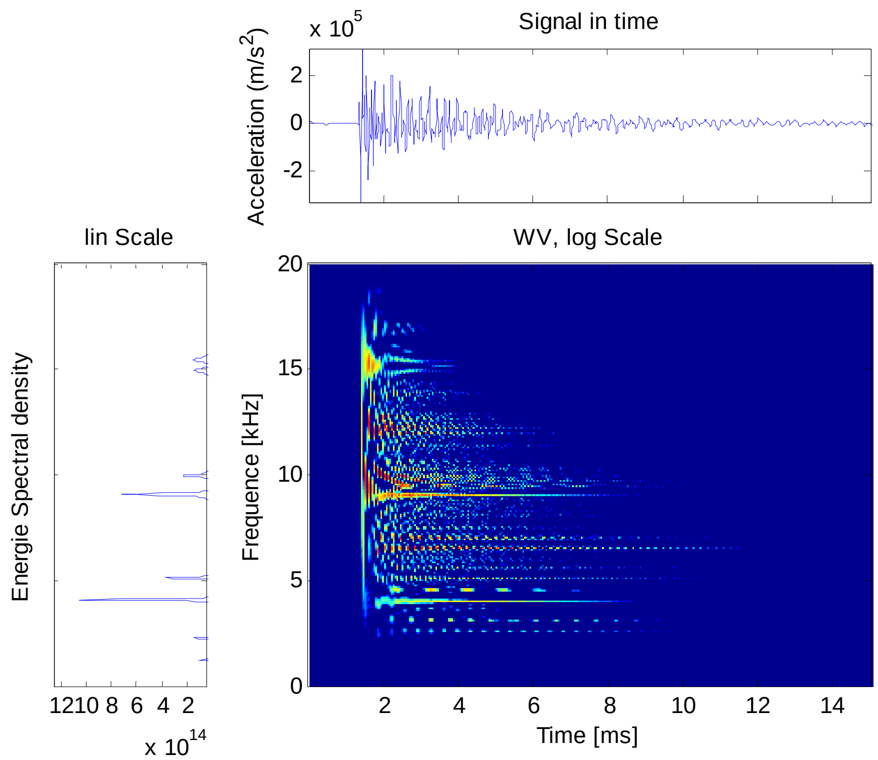

Fig. 11. Wigner-Ville Distribution, frequency response and temporal response of the Sandwich plate P2 under impact of a sphere with an initial velocity V_0 =1m/s.

Figs. 10 and 11 illustrate a succession of local analysis of the signal observed through a window using the energy distribution of the signal in frequency-time plan. For plates P1 and P2 there are three main components are separated by their amplitude and duration. It then distinguishes other secondary components for the two plates P1 and P2.
From Figs. 10 and 11, we notice that after impact the oscillations of sandwich plates P1 and P2 are therefore non-sustained (not maintained). We note also that the frequencies begin with a delay that depends on the duration of shock, these delayed mounted, visible on the Wigner-Ville distribution differs from one structure to another, represent the natural frequencies of sandwich plates P1 and P2.
During free oscillations, the vibration of the sandwich plate P1 dissipates very quickly. This faster dissipation, in the plate P1 is due in our parametric study to the percentage of the viscoelastic core participation in the general behaviour of the plate.

We also note that the spectral density of energy (vibration level) is higher during an impact for the plate P2. By comparing the two distributions of Wigner-Ville, the frequency components of vibration in the plate P2 (less damped) are maintained at relatively high levels compared to the frequency components of the vibration of the plate P1 (quickly dissipated).

4. Conclusion

This work deals the dynamic analysis of impacted sandwich plate. For this purpose, a numerical method is developed in order to estimate the impact force and determine the dynamic behaviour of the structure under impact. In low velocity impact problem, Hertzian contact law is appropriate to study the impact on sandwich plate at small indentation.
The parametric study of the sandwich plate shows the influence of the layers thickness on dynamic behaviour. In fact the numerical results of the dynamic analysis confirmed the capacity of attenuation of the vibrations of a sandwich plate.
Finally, it was shown that the present approach could be used to study the low-velocity impact response analysis problem.

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Appendix A.: **Nomenclature**

b	side of the square plate
C	viscous damping matrix of sandwich plate
$C_{ijkl}(t-\tau)$	relaxation modulus
D	hysteretic damping matrix
E_1, E_2, E_I	Young's modulus
E_P	effective equivalent transverse Young's modulus of the impacted plate
E_c^*	complex Young's modulus
F	nodal impact force vector
G_1, G_2	shear modulus
G_c^*	complex shear modulus
h	thickness of plate
h_1, h_2	thickness of two faces layers
h_c	thickness of the core layer
k_c	stiffness contact
K	stiffness matrix of sandwich plate
K^*	complex stiffness matrix
M	mass matrix of the plate
M_I	mass of the sphere
R_I	radius of the sphere
t	time
u_m, v_m	longitudinal displacements of a point located on the axis of the plate
u_1, v_1, u_2, v_2	longitudinal displacements of the two skins plates
u_{f0}, v_{f0}	displacement of a point located on the *f*th layer axis
u_c, v_c	longitudinal displacements of the core
$[u]$, $[v]$	relative longitudinal displacements of the two skins
V_0	initial impact velocity
w_f	transversal displacement
$W(t)$	displacement vector
$\dot{W}(t)$	velocity vector
$\ddot{W}(t)$	acceleration vector
W_r, W_c	mechanical work
Z_I	displacement of the impactor
z	normal coordinate
z_f	distance between the *f*th skin layer axis and plate axis
z_m	z coordinate of the middle plan of the plate
β_x, β_y	skins rotations
φ_x, φ_y	core rotations
δ	indentation
ν_1, ν_2, ν_c, ν_I	Poisson's ratio
ν_P	effective transverse Poisson's ratio of the impacted plate
ε_{yy}, ε_{xx}	strains
γ_{xy}, γ_{xz}, γ_{yz}	shear strains
σ_{xx}, σ_{yy}	normal stresses
σ_{xy}, σ_{xz}, σ_{yz}	shear stresses
τ	relaxation time
η_i	*i*th modal damping loss factor
ϕ_i	eigenvectors
Φ	mass-normalizied eignevoctor matrix

Appendix B.:

The equation of motion for sandwich plate can be written as:

$$M\ddot{W}(t) + K^* W(t) = F(t)$$

where, $K^* = K + j\,D$

The eigenvalue problem can be expressed as:

$$\left(-\lambda_i^2 M + K + j\,D\right)\phi_i = 0 \quad i = 1, n$$

Solving the above eigenproblem leads to the solution containing complex eigenvalues λ_i^2 and eigenvectors ϕ_i

The eigensolution processes the orthogonality properties, which are defined by the equations

$$\Phi^{\mathbf{t}} M \Phi = \mathbf{I} \quad \text{and} \quad \Phi^t\left(K + jD\right)\Phi = \left[\lambda_i^2\right]$$

Where Φ is the mass-normalizied eignevoctor matrix and $\left[\lambda_i^2\right]$ is diagonal matrix of complex eigenvalues wich can be expressed as:

$$\lambda_i^2 = \omega_i^2\left(1 + j\eta_i\right)$$

Where η_i is *i*th modal damping loss factor

The realsionship between viscous and hysteretic damping models is modelled by [11]:

$$2\xi_i = \eta_i$$

Then The dynamic equation may be written as :

$$M\ddot{W}\left(t\right) + C\dot{W}\left(t\right) + KW\left(t\right) = F\left(t\right)$$