Use of Flexure Formula:

Illustrative Problems:

An I - section girder, 200mm wide by 300 mm depth flange and web of thickness is 20 mm is used as simply supported beam for a span of 7 m. The girder carries a distributed load of 5 KN /m and a concentrated load of 20 KN at mid-span.

Determine the

(i). The second moment of area of the cross-section of the girder

(ii). The maximum stress set up.

Solution:

The second moment of area of the cross-section can be determained as follows :

For sections with symmetry about the neutral axis, use can be made of standard I value for a rectangle about an axis through centroid i.e. (bd 3)/12. The section can thus be divided into convenient rectangles for each of which the neutral axis passes through the centroid. Example in the case enclosing the girder by a rectangle

$$I_{girder} = I_{rectangle} - I_{shaded\ portion}$$

$$= \left[\frac{200 \times 300^3}{12}\right] 10^{-12} - 2\left[\frac{90 \times 260^3}{12}\right] 10^{-12}$$

$$= (4.5 - 2.64)10^{-4}$$

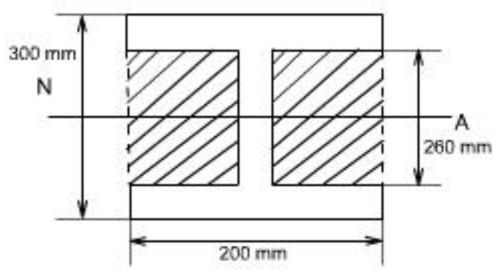
$$= 1.86 \times 10^{-4}\ m^4$$

The maximum stress may be found from the simple bending theory by equation

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

i.e.

$$\sigma_{max^m} = \frac{M_{max^m}}{I}\ y_{max^m}$$



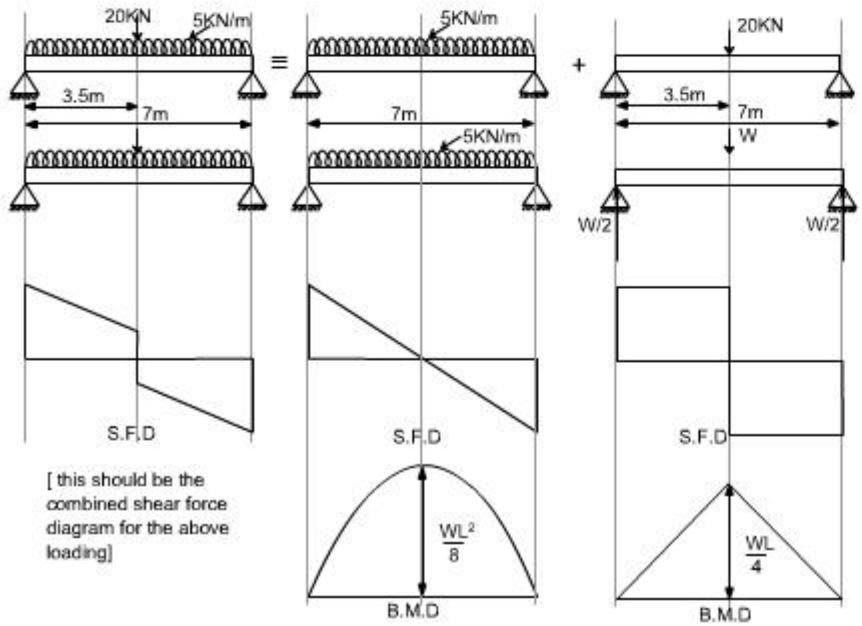
<u>Computation of Bending Moment:</u>

In this case the loading of the beam is of two types

(a) Uniformly distributed load

(b) Concentrated Load

In order to obtain the maximum bending moment the technique will be to consider each loading on the beam separately and get the bending moment due to it as if no other forces acting on the structure and then superimpose the two results.



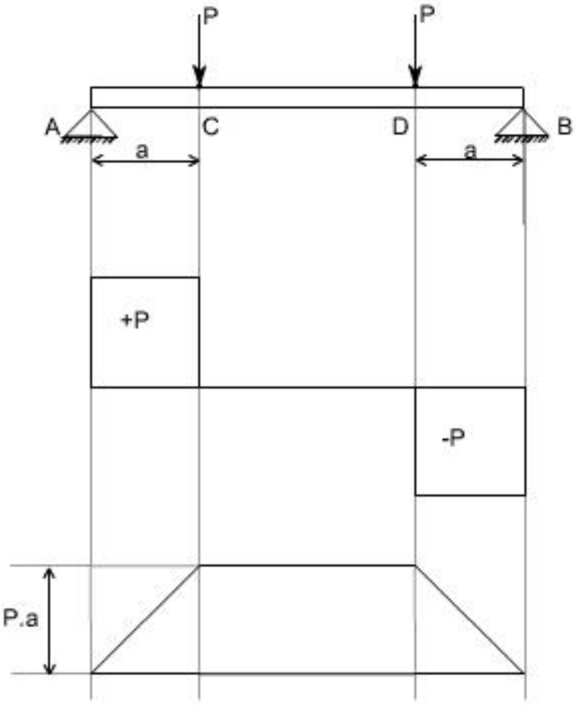
Hence

$$
\begin{aligned}
M_{max^m} &= \frac{wL}{4} + \frac{wL^2}{8} \\
&= \frac{20 \times 10^3 \times 7}{4} + \frac{5 \times 10^3 \times 7^2}{8} \\
&= (35.0 + 30.63)\,10^3 \\
&= 65.63 \text{ k Nm}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{max^m} &= \frac{M_{max^m}}{I}\, y_{max^m} \\
&= \frac{65.63 \times 10^3 \times 150 \times 10^3}{1.06 \times 10^{-4}} \\
\sigma_{max^m} &= 51.8 \text{ MN/m}^2
\end{aligned}
$$

Shearing Stresses in Beams

All the theory which has been discussed earlier, while we discussed the bending stresses in beams was for the case of pure bending i.e. constant bending moment acts along the entire length of the beam.



Let us consider the beam AB transversely loaded as shown in the figure above. Together with shear force and bending moment diagrams we note that the middle potion CD of the beam is free from shear force and that its bending moment. M = P.a is uniform between the portion C and D. This condition is called the pure bending condition.

Since shear force and bending moment are related to each other F= dM/dX (eq) therefore if the shear force changes than there will be a change in the bending moment also, and then this won't be the pure bending.

Conclusions :

Hence one can conclude from the pure bending theory was that the shear force at each X-section is zero and the normal stresses due to bending are the only ones produced.

In the case of non-uniform bending of a beam where the bending moment varies from one X-section to another, there is a shearing force on each X-section and shearing stresses are also induced in the material. The deformation associated with those shearing stresses causes " warping " of the x-section so that the assumption which we assummed while deriving the relation $\frac{\sigma}{y}=\frac{M}{I}=\frac{E}{R}$ that the plane cross-section after bending remains plane is violated. Now due to warping the plane cross=section before bending do not remain plane after bending. This complicates the problem but more elaborate analysis shows that the normal stresses due to bending, as calculated from the equation $\frac{\sigma}{y}=\frac{M}{I}=\frac{E}{R}$.

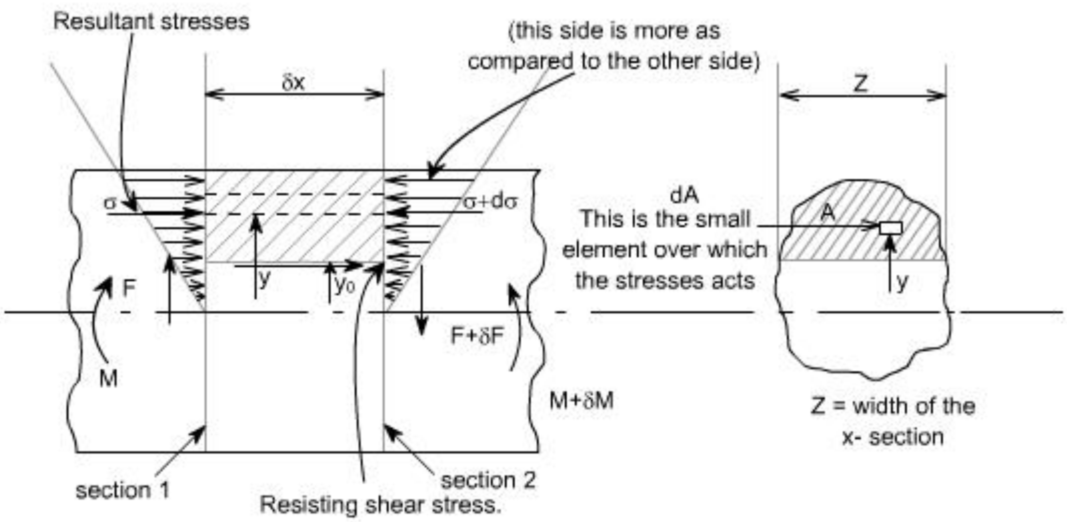
The above equation gives the distribution of stresses which are normal to the cross-section that is in x-direction or along the span of the beam are not greatly altered by the presence of these shearing stresses. Thus, it is justifiable to use the theory of pure bending in the case of non uniform bending and it is accepted practice to do so.

Let us study the shear stresses in the beams.

Concept of Shear Stresses in Beams :

By the earlier discussion we have seen that the bending moment represents the resultant of certain linear distribution of normal stresses σ_x over the cross-section. Similarly, the shear force F_x over any cross-section must be the resultant of a certain distribution of shear stresses.

Derivation of equation for shearing stress :



Assumptions :

1. Stress is uniform across the width (i.e. parallel to the neutral axis)

2. The presence of the shear stress does not affect the distribution of normal bending stresses.

It may be noted that the assumption no.2 cannot be rigidly true as the existence of shear stress will cause a distortion of transverse planes, which will no longer remain plane.

In the above figure let us consider the two transverse sections which are at a distance **' δx'** apart. The shearing forces and bending moments being **F, F + δF** and **M, M + δM** respectively. Now due to the shear stress on transverse planes there will be a complementary shear stress on longitudinal planes parallel to the neutral axis.

Let **τ**□be the value of the complementary shear stress (and hence the transverse shear stress) at a distance '$\mathbf{Y'_0}$ from the neutral axis. **Z** is the width of the x-section at this position

A is area of cross-section cut-off by a line parallel to the neutral axis.

$\bar{y}$ = distance of the centroid of Area from the neutral axis.

Let **σ , σ + dσ** are the normal stresses on an element of area **δA** at the two transverse sections, then there is a difference of longitudinal forces equal to (**dσ . δA)** , and this quantity summed over the area A is in equilibrium with the transverse shear stress **τ** on the longitudinal plane of area z δ**x** .

i.e $\tau . z \delta x = \int d\sigma . dA$

from the bending theory equation

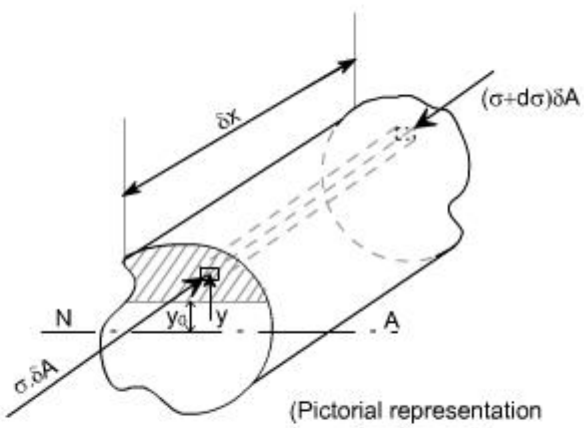
$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\sigma = \frac{M . y}{I}$$

$$\sigma + d\sigma = \frac{(M + \delta M) . y}{I}$$

Thus $$d\sigma = \frac{\delta M . y}{I}$$

The figure shown below indicates the pictorial representation of the part.



(Pictorial representation of entire part)

$$d\sigma = \frac{\delta M.y}{I}$$

$$\tau.z\delta x = \int d\sigma.dA$$

$$= \int \frac{\delta M\ .\ y.\delta A}{I}$$

$$\tau.z\delta x = \frac{\delta M}{I} \int y.\delta A$$

But $F = \frac{\delta M}{\delta x}$

i.e. $\tau = \frac{F}{I.z}\int y.\delta A$

But from definition, $\int y.dA = A\bar{y}$

$\int y.dA$ is the first moment of area of the shaded portion

and $\bar{y}$ = centroid of the area 'A'

Hence

So substituting $$\tau = \frac{F.A.\bar{y}}{I.z}$$

Where 'z' is the actual width of the section at the position where 'τ ' is being calculated and I is the total moment of inertia about the neutral axis.

Source: http://nptel.ac.in/courses/Webcourse-contents/IIT-ROORKEE/strength%20of%20materials/homepage.htm