Stress Concentration

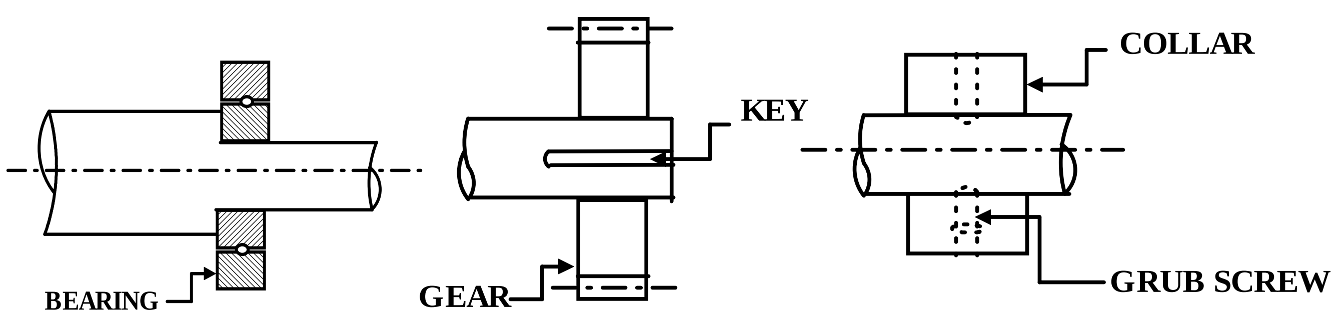
Instructional Objectives

At the end of this lesson, the students should be able to understand

- Stress concentration and the factors responsible.
- Determination of stress concentration factor; experimental and theoretical methods.
- Fatigue strength reduction factor and notch sensitivity factor.
- Methods of reducing stress concentration.

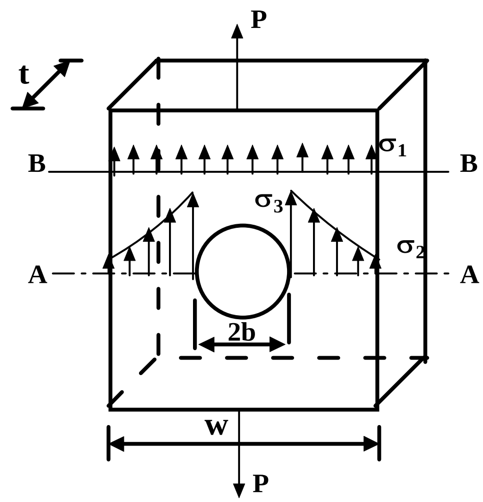
3.2.1 Introduction

In developing a machine it is impossible to avoid changes in cross-section, holes, notches, shoulders etc. Some examples are shown in **figure- 3.2.1.1**.



3.2.1.1F- *Some typical illustrations leading to stress concentration.*

Any such discontinuity in a member affects the stress distribution in the neighbourhood and the discontinuity acts as a stress raiser. Consider a plate with a centrally located hole and the plate is subjected to uniform tensile load at the ends. Stress distribution at a section A-A passing through the hole and another section BB away from the hole are shown in **figure- 3.2.1.2**. Stress distribution away from the hole is uniform but at AA there is a sharp rise in stress in the vicinity of the hole. Stress concentration factor k_t is defined as $k_t = \sigma_3 / \sigma_{av}$, where σ_{av} at section AA is simply $P/t(w-2b)$ and $\sigma_1 = P/tw$. This is the theoretical or geometric stress concentration factor and the factor is not affected by the material properties.

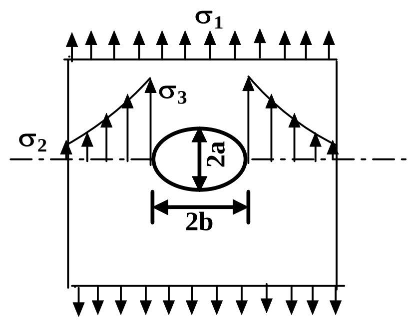


3.2.1.2F- Stress concentration due to a central hole in a plate subjected to an uni-axial loading.

It is possible to predict the stress concentration factors for certain geometric shapes using theory of elasticity approach. For example, for an elliptical hole in an infinite plate, subjected to a uniform tensile stress σ_1 (**figure- 3.2.1.3**), stress distribution around the discontinuity is disturbed and at points remote from the discontinuity the effect is insignificant. According to such an analysis

$$\sigma_3 = \sigma_1 \left(1 + \frac{2b}{a}\right)$$

If a=b the hole reduces to a circular one and therefore $\sigma_3 = 3\sigma_1$ which gives k_t=3. If, however 'b' is large compared to 'a' then the stress at the edge of transverse crack is very large and consequently k is also very large. If ‘b’ is small compared to a then the stress at the edge of a longitudinal crack does not rise and k_t =1.



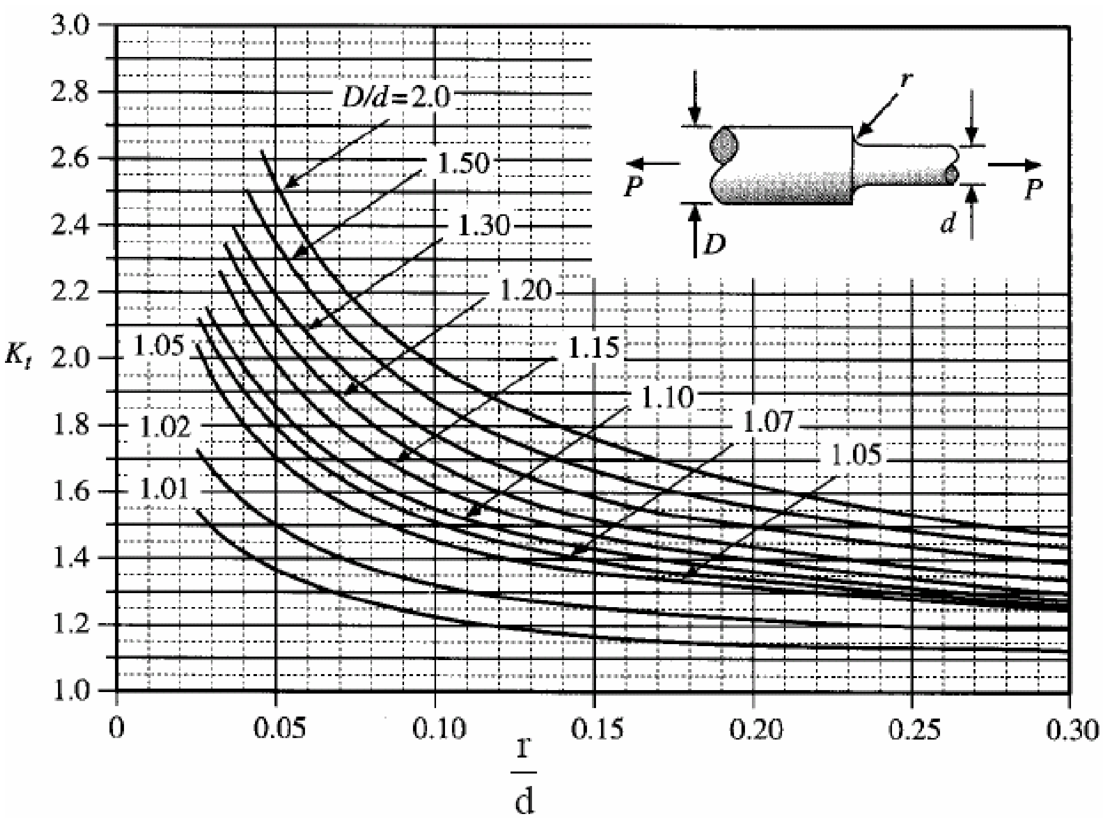
3.2.1.3F- Stress concentration due to a central elliptical hole in a plate subjected to a uni-axial loading.

Stress concentration factors may also be obtained using any one of the following experimental techniques:

1. Strain gage method
2. Photoelasticity method
3. Brittle coating technique
4. Grid method

For more accurate estimation numerical methods like Finite element analysis may be employed.

Theoretical stress concentration factors for different configurations are available in handbooks. Some typical plots of theoretical stress concentration factors and r/d ratio for a stepped shaft are shown in **figure-3.2.1.4.**



***3.2.1.4F**- Variation of theoretical stress concentration factor with r/d of a stepped shaft for different values of D/d subjected to uni-axial loading (Ref.[2]).*

In design under fatigue loading, stress concentration factor is used in modifying the values of endurance limit while in design under static loading it simply acts as stress modifier. This means Actual stress= $k_t \times$ calculated stress.

For ductile materials under static loading effect of stress concentration is not very serious but for brittle materials even for static loading it is important.

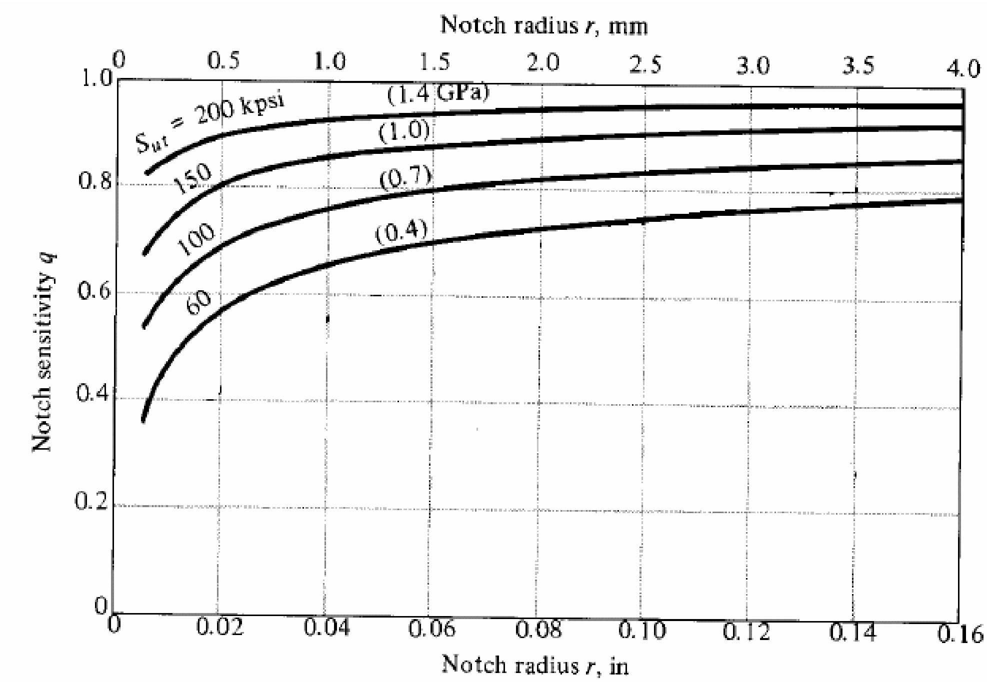
It is found that some materials are not very sensitive to the existence of notches or discontinuity. In such cases it is not necessary to use the full value of k_t and instead a reduced value is needed. This is given by a factor known as fatigue strength reduction factor k_f and this is defined as

$$k_f = \frac{\text{Endurance limit of notch free specimens}}{\text{Endurance limit of notched specimens}}$$

Another term called Notch sensitivity factor, q is often used in design and this is defined as

$$q = \frac{k_f - 1}{k_t - 1}$$

The value of 'q' usually lies between 0 and 1. If q=0, k_f =1 and this indicates no notch sensitivity. If however q=1, then $k_f = k_t$ and this indicates full notch sensitivity. Design charts for 'q' can be found in design hand-books and knowing k_t, k_f may be obtained. A typical set of notch sensitivity curves for steel is



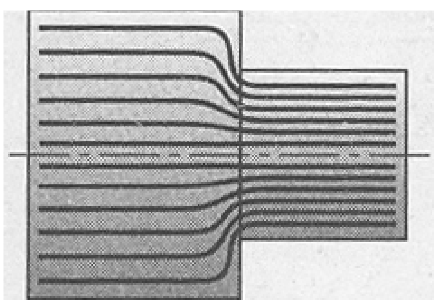
3.2.1.5F- Variation of notch sensitivity with notch radius for steels of different ultimate tensile strength (Ref.[2]).

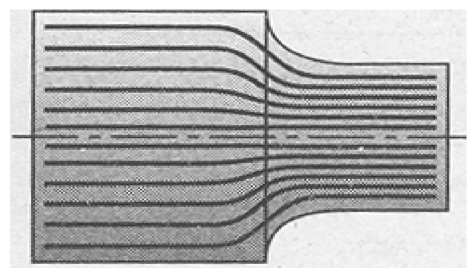
3.2.2 Methods of reducing stress concentration

A number of methods are available to reduce stress concentration in machine parts. Some of them are as follows:

1. Provide a fillet radius so that the cross-section may change gradually.
2. Sometimes an elliptical fillet is also used.
3. If a notch is unavoidable it is better to provide a number of small notches rather than a long one. This reduces the stress concentration to a large extent.
4. If a projection is unavoidable from design considerations it is preferable to provide a narrow notch than a wide notch.
5. Stress relieving groove are sometimes provided.

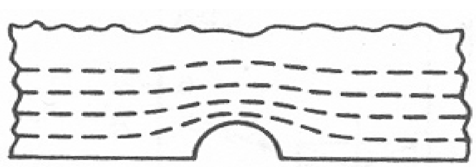
These are demonstrated in **figure- 3.2.2.1.**

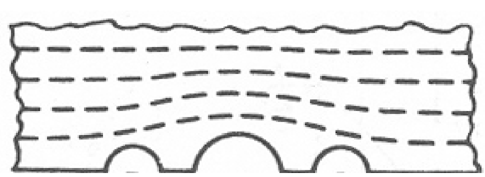




(a) Force flow around a sharp corner

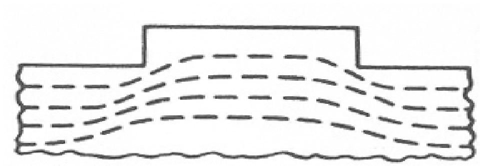
Force flow around a corner with fillet: Low stress concentration.

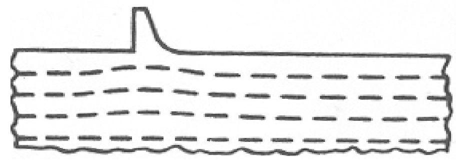




(b) Force flow around a large notch

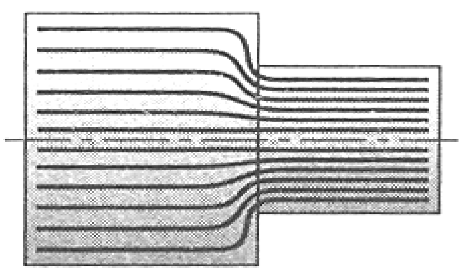
Force flow around a number of small notches: Low stress concentration.

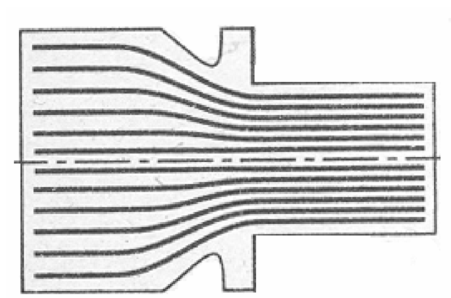




(c) Force flow around a wide projection

Force flow around a narrow projection: Low stress concentration.





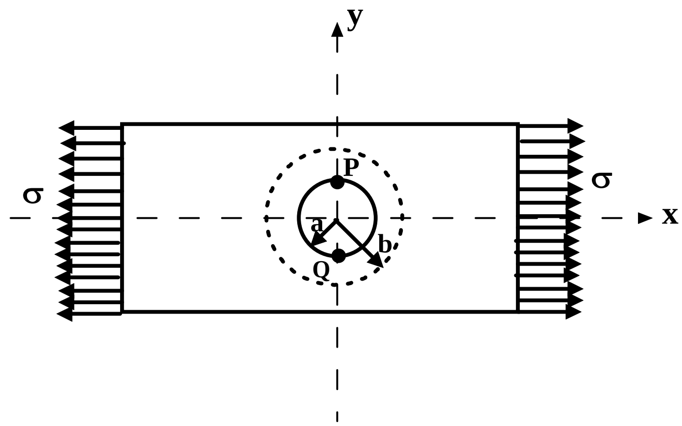
(d) Force flow around a sudden change in diameter in a shaft

Force flow around a stress relieving groove.

***3.2.2.1F**- Illustrations of different methods to reduce stress concentration (Ref.[1]).*

3.2.3 Theoretical basis of stress concentration

Consider a plate with a hole acted upon by a stress σ. St. Verant's principle states that if a system of forces is replaced by another statically equivalent system of forces then the stresses and displacements at points remote from the region concerned are unaffected. In **figure-3.2.3.1** 'a' is the radius of the hole and at r=b, b>>a the stresses are not affected by the presence of the hole.



3.2.3.1F- *A plate with a central hole subjected to a uni-axial stress*

Here, $\sigma_x = \sigma$, $\sigma_y = 0$, $\tau_{xy} = 0$

For plane stress conditions:

$$\sigma_r = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\cos\theta\sin\theta$$

$$\sigma_\theta = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\cos\theta\sin\theta$$

$$\tau_{r\theta} = \left(\sigma_x - \sigma_y\right)\sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

This reduces to

$$\sigma_r = \sigma\cos^2\theta = \frac{\sigma}{2}\left(\cos 2\theta + 1\right) = \frac{\sigma}{2} + \frac{\sigma}{2}\cos 2\theta$$

$$\sigma_\theta = \sigma\sin^2\theta = \frac{\sigma}{2}\left(1 - \cos 2\theta\right) = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta$$

$$\tau_{r\theta} = -\frac{\sigma}{2}\sin 2\theta$$

such that 1st component in σ_r and σ_θ is constant and the second component varies with θ. Similar argument holds for $\tau_{r\theta}$ if we write $\tau_{r\theta} = -\frac{\sigma}{2}\sin 2\theta$. The stress distribution within the ring with inner radius $r_i = a$ and outer radius $r_o = b$ due to 1st component can be analyzed using the solutions of thick cylinders and the effect due to the 2nd component can be analyzed following the Stress-function approach. Using a stress function of the form $\phi = R(r)\cos 2\theta$ the stress distribution due to the 2nd component can be found and it was noted that the dominant stress is the Hoop Stress, given by

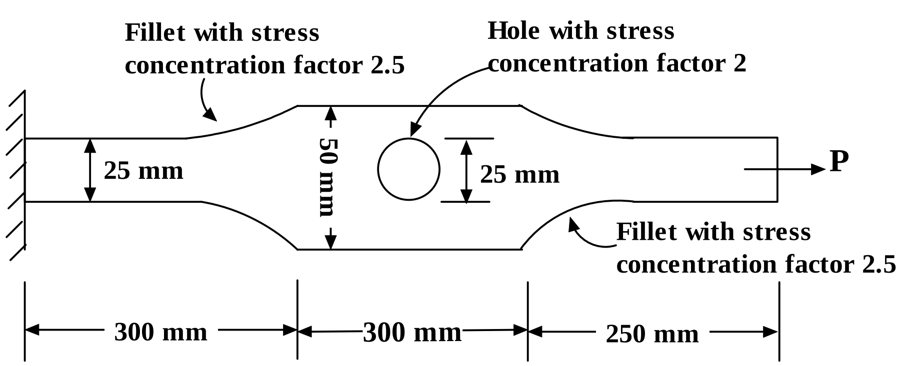
$$\sigma_\theta = \frac{\sigma}{2}\left(1+\frac{a^2}{r^2}\right) - \frac{\sigma}{2}\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta$$

This is maximum at $\theta = \pm\pi/2$ and the maximum value of $\sigma_\theta = \frac{\sigma}{2}\left(2+\frac{a^2}{r^2}+\frac{3a^4}{r^4}\right)$

Therefore at points P and Q where $r = a$ σ_θ is maximum and is given by $\sigma_\theta = 3\sigma$ i.e. stress concentration factor is 3.

3.2.4 Problems with Answers

Q.1: The flat bar shown in **figure- 3.2.4.1** is 10 mm thick and is pulled by a force P producing a total change in length of 0.2 mm. Determine the maximum stress developed in the bar. Take E= 200 GPa.



3.2.4.1F

A.1:

Total change in length of the bar is made up of three components and this is given by

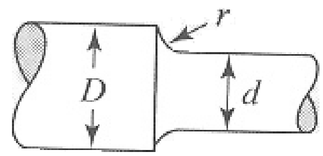
$$0.2\text{x}10^{-3} = \left[\frac{0.3}{0.025\text{x}0.01}+\frac{0.3}{0.05\text{x}0.01}+\frac{0.25}{0.025\text{x}0.01}\right]\frac{P}{200\text{x}10^9}$$

This gives P=14.285 KN.

Stress at the shoulder $\sigma_s = k\frac{16666}{(0.05-0.025)x0.01}$ where k=2.

This gives σ_h = 114.28 MPa.

Q.2: Find the maximum stress developed in a stepped shaft subjected to a twisting moment of 100 Nm as shown in **figure- 3.2.4.2**. What would be the maximum stress developed if a bending moment of 150 Nm is applied.



3.2.4.2F

r = 6 mm

d = 30 mm

D = 40 mm.

A.2:

Referring to the stress- concentration plots in **figure- 3.2.4.3** for stepped shafts subjected to torsion for r/d = 0.2 and D/d = 1.33, $K_t \approx 1.23$.

Torsional shear stress is given by $\tau = \frac{16T}{\pi d^3}$. Considering the smaller diameter and the stress concentration effect at the step, we have the maximum shear stress as

$$\tau_{max} = K_t \frac{16x100}{\pi(0.03)^3}$$

This gives τ_{max} = 23.201 MPa.

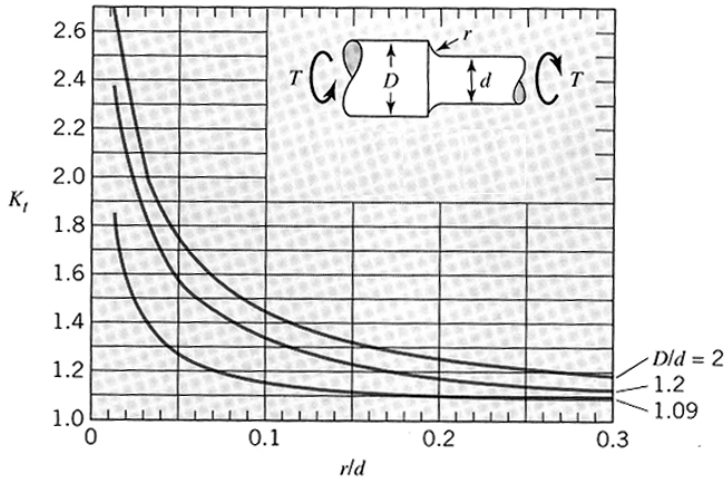
Similarly referring to stress-concentration plots in **figure- 3.2.4.4** for stepped shaft subjected to bending , for r/d = 0.2 and D/d = 1.33, $K_t \approx 1.48$

Bending stress is given by $\sigma = \frac{32M}{\pi d^3}$

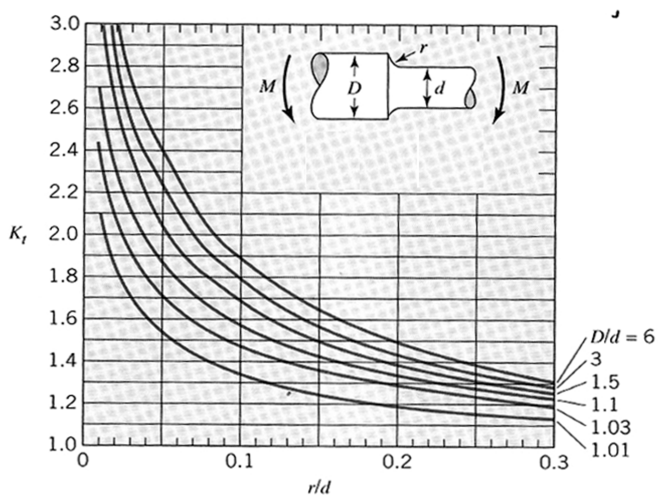
Considering the smaller diameter and the effect of stress concentration at the step, we have the maximum bending stress as

$$\sigma_{max} = K_t \frac{32x150}{\pi(0.03)^3}$$

This gives σ_{max} = 83.75 MPa.

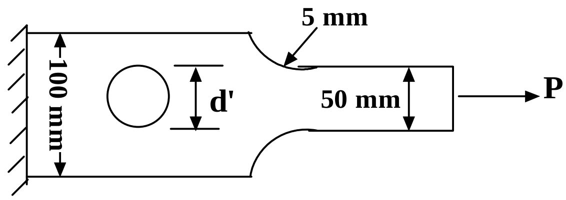


***3.2.4.3F**- Variation of theoretical stress concentration factor with r/d for a stepped shaft subjected to torsion (Ref.[5]).*



***3.2.4.4F**- Variation of theoretical stress concentration factor with r/d for a stepped shaft subjected to a bending moment (Ref.[5]) .*

Q.3: In the plate shown in **figure- 3.2.4.5** it is required that the stress concentration at Hole does not exceed that at the fillet. Determine the hole diameter.



3.2.4.5F

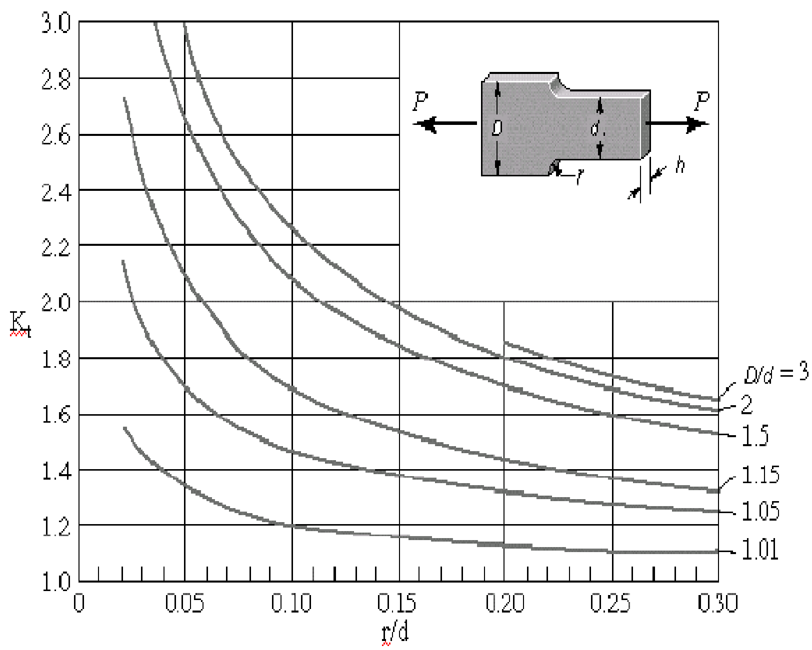
A.3:

Referring to stress-concentration plots for plates with fillets under axial loading (figure**- 3.2.4.6**) for r/d = 0.1 and D/d = 2,

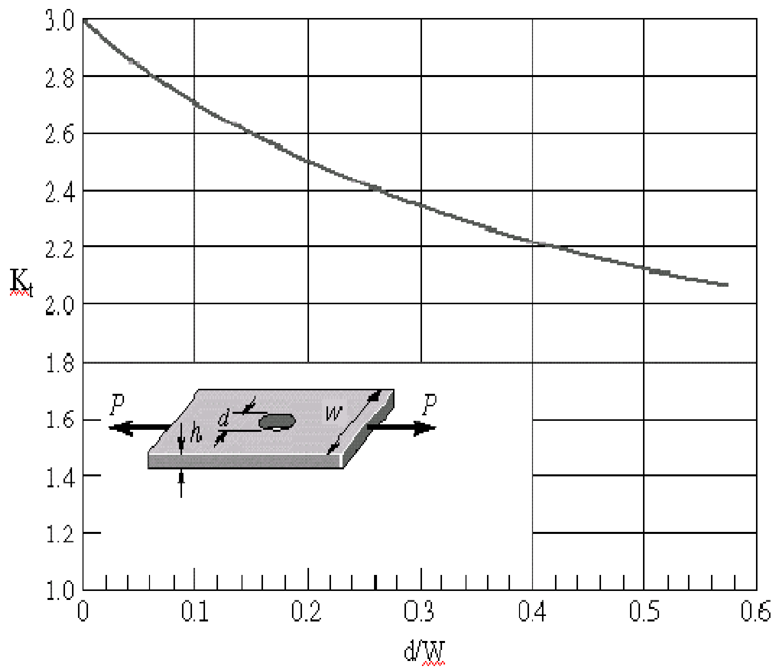
stress concentration factor, $K_t \approx 2.3$.

From stress concentration plots for plates with a hole of diameter 'd' under axial loading (**figure- 3.2.4.7**) we have for $K_t = 2.3$, d′/D = 0.35.

This gives the hole diameter d′ = 35 mm.



3.2.4.6F*- Variation of theoretical stress concentration factor with r/d for a plate with fillets subjected to a uni-axial loading (Ref.[5]).*



3.2.4.7F- *Variation of theoretical stress concentration factor with d/W for a plate with a transverse hole subjected to a uni-axial loading (Ref.[5]).*

3.2.5 Summary of this Lesson

Stress concentration for different geometric configurations and its relation to fatigue strength reduction factor and notch sensitivity have been discussed. Methods of reducing stress concentration have been demonstrated and a theoretical basis for stress concentration was considered.

Source:
http://nptel.ac.in/courses/Webcourse-contents/IIT%20Kharagpur/Machine%20design1/pdf/Module-3_lesson-2.pdf