Simulation of Fluid Flow and Heat Transfer in the Inclined Enclosure

A. Karimipour, M. Afrand, M. Akbari, M.R. Safaei

Abstract**—**Mixed convection in two-dimensional shallow rectangular enclosure is considered. The top hot wall moves with constant velocity while the cold bottom wall has no motion. Simulations are performed for Richardson number ranging from Ri = 0.001 to 100 and for Reynolds number keeping fixed at Re = 408.21. Under these conditions cavity encompasses three regimes: dominating forced, mixed and free convection flow. The Prandtl number is set to 6 and the effects of cavity inclination on the flow and heat transfer are studied for different Richardson number. With increasing the inclination angle, interesting behavior of the flow and thermal fields are observed. The streamlines and isotherm plots and the variation of the Nusselt numbers on the hot wall are presented. The average Nusselt number is found to increase with cavity inclination for $Ri \geq 1$. Also it is shown that the average Nusselt number changes mildly with the cavity inclination in the dominant forced convection regime but it increases considerably in the regime with dominant natural convection.

Keywords**—**Mixed convection- inclined driven cavity- Richardson number

I. Introduction

IN recent decades mixed convection problems have been in notice of researchers. Mixed convection in cavities is used in the designing of heat exchangers and making solar cells.

Mixed convection is consisted of two main parts. The first is natural convection which some of researchers have investigated it in cavities. For example, Davis [1] in 1983 solved flow and heat transfer inside a horizontal caviy by numerical method, assuming its vertical walls adiabatic, its horizontal walls at different temperatures and air as a fluid. With these assumptions he studied the effect of the Rayleigh number (Ra = Gr.Pr) on flow and heat transfer inside the cavity. Some other researchers have studied natural convection in geometries (rectangular cavities) assuming the walls to be at constant temperature or to be heated (including constant heat flux). Among them we can mention authors of references [2] and [3]. In reference [2] the authors studied the effect of the thermal boundary conditions at the sidewalls upon natural convection in rectangular enclosures which heated from below and cooled from above. And In reference [3] the authors studied the control of laminar natural convection in differentially heated square enclosures using solid inserts at the corners. They also studied the effect of Prandtl number in natural convection and concluded that it is proportional with inverse of Prandtl number.

[4] studied numerically natural convection around a square horizontal heated cylinder which was placed in an enclosure. They showed that the rate of natural convection can be increased if the enclosure has a partial hot blockage inside. Also, they studied the effect of Ra number in a cavity with different cavity wall boundary conditions and showed that constant wall temperature heating is efficient in comparison to the constant wall heat flux with regard to overall heat transfer for different locations of the heated blockage. Some researches have studied cavities with periodic boundary conditions. For example [5] studied the natural convection in a 2D enclosure with sinusoidal upper wall temperature. He assumed adiabatic conditions on the other walls. Also [6] studied the natural convection in enclosure with heating and cooling by sinusoidal temperature profiles on one side.

Chaotic mixing inside a two dimensional cavity can be achieved with time dependent natural convection if the motion of a fluid is generated by imposing alternating hot and cold wall temperatures. With this set up no moving walls are required to mix the fluid inside the container. Cruz [7] illustrated this idea by numerically solving the governing equations of natural convection in a two dimensional square cavity with sections of its upper and lower horizontal walls cooled and heated in a periodic manner. These conditions generate a vortex of time dependent intensity that moves its center in a closed loop around the geometrical center of the container.

After studying natural convection in different geometries and boundary conditions a great number of researchers began studying mixed convection in the cavity in order to compare the power of natural convection with the power of forced convection in the cavities. There are many ways to make mixed convection in cavity. One of those is such that a hot (or cold) fluid enters to the cavity and after passing over a surface with constant temperature, leaves the cavity from other side (like fluid passing through a duct). For example numerical simulations of laminar, forced convection heat transfer for reciprocating, two dimensional channel flows were performed as a function of the penetration length, and Prandtl number by [8]. He assumed uniform heat flux and constant temperature imposed on certain regions of the top surface while keeping the bottom surface insulated. In these conditions the heat flux could be assumed in periodic mood with different oscillation

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frequencies. It was seen that forced convection increases with the penetration length and Pr and that steady unidirectional forced convection is more effective than the reciprocating flow forced convection.

[9] studied mixed convection in two-sided lid-driven in a differentially heated square cavity. They assumed the left and right moving walls at different constant temperatures and the upper and bottom walls being thermally insulated. They studied the effect of variation of Richardson number on fluid behavior and found that both Richardson number and direction of moving walls have the same effects on the heat transfer when the walls move in opposite direction regardless of which side moves upwards. They showed that heat transfer is reduced when both walls move upwards. [10] numerically studied the simulation of combined thermal and mass transport in a square lid-driven cavity. He found maximum heat and mass transfer rate occurs at Ri <1.

In the above cases moving wall as a shear forces together with buoyancy forces create mixed convection in cavities. Studying the inclined cavities has been in notice in the last few years. In 2007 Zekeria and his colleague presented a paper about this case [11]. They studied the effect of inclination on the natural convection in a cavity.

In this paper the effect of inclination of the cavity on the flow and thermal field is investigated numerically. To achieve this purpose, laminar mixed convection heat transfer in an inclined, two dimensional, shallow rectangular driven cavity of aspect ratio 5 and fluid with Prandtl number 6, representing water, is studied.

II. Problem Description

In this paper the numerical analysis of mixed convective flow in a shallow lid-driven cavity cooled from the bottom and heated from the top moving lid is presented. The shallow configuration is chosen because of its possible practical applications. The effects of the cavity inclination on the convective process for a range of Richardson number are analyzed. Computations are performed for Richardson number $(Ri = Gr / Re^2)$ ranging from 0.001 to 100 and for Reynolds number keeping fixed at 408.21. It means by changing Richardson number we can change Grashof number which represents natural convection power.

The physical model considered here is shown in Fig. 1, along with the important geometric parameters. It consists of a rectangular cavity of dimensions L and H, whose sidewalls are adiabatic. The isothermal top and bottom walls are maintained at temperatures T_h and T_c respectively, with $T_h > T_c$. The aspect ratio of the cavity is defined as AR = L/H. With this geometry and boundary conditions, the present study reports the computations for cavities at a fixed aspect ratio of 5, inclination angles ranging from $0°$ to $60°$. The effect of inclination angle on the heat transfer process is analyzed and the results are presented using streamline and isotherm plots in the cavity. Also, the variation of the local and average Nusselt number at the hot surface is studied.

III. Mathematical Modeling

The governing equations using Boussinesq approximation are as allows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p_m}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + g\beta(T - T_0)\times\sin\gamma \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p_m}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta(T - T_0)\times\cos\gamma \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

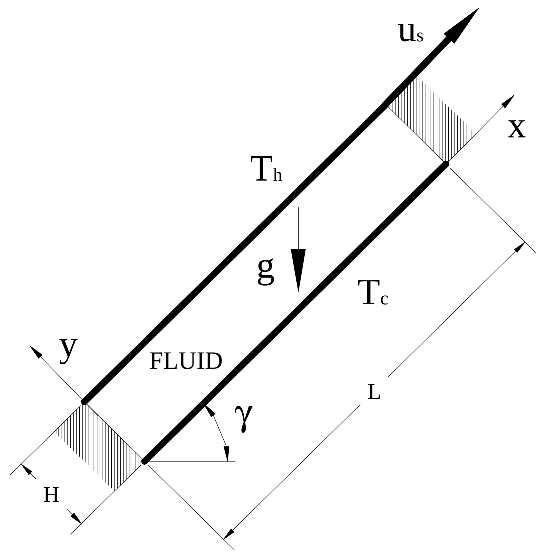

Fig. 1 Introducing geometry and parameters of the cavity and coordinate system

Where $\alpha = k / \rho c$ is thermal diffusivity, ρ is the density, γ is the inclination angle of the cavity with the horizontal direction, υ is the kinematic viscosity, t is the time, u and v are the velocity components in the x and y directions, g is the gravity, T is the temperature and P is the pressure.

By using the non-dimensional variables as below we can derive the non-dimensional form of equations (1) to (4):

Dimensionless coordinates:

$$Y = y / H, X = x / H \tag{5}$$

Dimensionless velocity components:

$$U = u / u_s, \ V = v / u_s \tag{6}$$

Dimensionless temperature:

$$\theta = (T - T_c)/(T_h - T_c) \tag{7}$$

Dimensionless pressure:

$$P = \overline{P} / \rho u_s^2 \tag{8}$$

Modified pressure:

$$p_m = p - p_0 ,\ \overline{P} = P + \rho g h \tag{9}$$

Dimensionless time:

$$\tau = \frac{t u_s}{H} \tag{10}$$

In these equations u_s is a constant and denote lid velocity. Other dimensionless numbers are defined as: $Re = u_s H / \nu$ Reynolds number, $Gr = g\beta H^3 (T_h - T_c)/\nu^2$ Grashof number, $Pr = \nu/\alpha$ Prandtl number and $Ri = Gr / Re^2$ Richardson number. Now by substitution of equations (5) to (10) into the equations (1) to (4) we have the non-dimensional form of governing equations as below:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{11}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) + \frac{Gr}{Re^2}\theta \times \sin\gamma \tag{12}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{Gr}{Re^2}\theta \times \cos\gamma \tag{13}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{14}$$

Boundary conditions:

Top wall: U =1, V = 0, $\theta = 1$

Bottom wall: U = V = 0, $\theta = 0$

Right and left wall: U = V = 0, $\partial\theta / \partial X = 0$

The local Nusselt number and the rate of heat transfer along the hot surface can be obtained as below:

$$Nu_x = \frac{hH}{k} = \frac{\left[q_s'' / (T_h - T_c)\right] H}{k} \tag{15}$$

$$q_s'' = -k\frac{\partial T}{\partial y}\Big|_{y=H} \tag{16}$$

By substituting equation (16) into equation (15) and by considering $\partial\theta = \frac{1}{T_h - T_c}(\partial T)$, $\partial Y = \frac{1}{H}(\partial y)$ the local Nusselt number along the hot surface can be written as equation (17),

$$Nu_x = -H\frac{1}{T_h - T_c}\frac{\partial T}{\partial y}\Big|_{y=H} = -H\frac{\partial\theta}{\partial y}\Big|_{y=H} = -\frac{\partial\theta}{\partial Y}\Big|_{Y=1} \tag{17}$$

The overall Nusselt number along the hot surface can be derived by integration from upper equation along the surface.

$$Nu_m = \frac{1}{1 \times L}\int_0^L Nu_x dx = \frac{1}{AR}\int_0^{AR}\left(-\frac{\partial\theta}{\partial Y}\right)_{Y=1} dX \tag{18}$$

The stream function is calculated from its definition as:

$$u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x} \tag{19}$$

The set of governing equations are integrated over the control volumes, which produces a set of algebraic equations. Using finite volume method, the SIMPLE algorithm is used to solve the coupled system of governing equations using finite volume method. To obtain finite volume equations, Power low scheme was used. It is assumed that the flow is two-dimensional, steady state, laminar and the fluid is incompressible. The thermo physical properties of the fluid at a reference temperature are assumed to be constant, except in the buoyancy term of the momentum equations, i.e., the Boussinesq approximation. The streamlines and isotherm plots and the variation of the Nusselt number at hot wall are presented.

Numerical experiments are performed in order to check the grid independency of the solution. The average Nusselt number and the maximum of stream function by three different grid sizes are shown in table.1. This table shows that 140×28 grid points can be adopted for grid free solution throughout the calculations in the present study. This grid dimension shows a negligible deviation in Nusselt number and stream function (0.7 %).

TABLE I
THE AVERAGE NUSSELT NUMBER AND THE MAXIMUM OF STREAM FUNCTION WITH THE NUMBER OF GRIDS IN RI = 1 AND γ =0

number of grids	Nu_m	ψ_{max}
130×26	4.39	0.141
140×28	4.43	0.144
150×30	4.45	0.145

In Fig. 2 the average Nusselt number obtained from the present work and the average Nusselt number resulted from Oztop [9] (two dimensional mixed convection steady flow in a cavity with two adiabatic walls and two moving walls at different temperature) is compared and good agreement is observed.

IV. RESULTS

The working fluid is chosen as water with Prandtl number, Pr = 6. The Reynolds number, Re, is kept fixed at 408.21 and the cavity aspect ratio, A, is taken as 5. The Richardson number, Ri, is varied as 0.001, 1, and 100. The inclination angle, γ, is varied as $0°$, $30°$, $60°$ by tilting the horizontal cavity counter-clockwise.

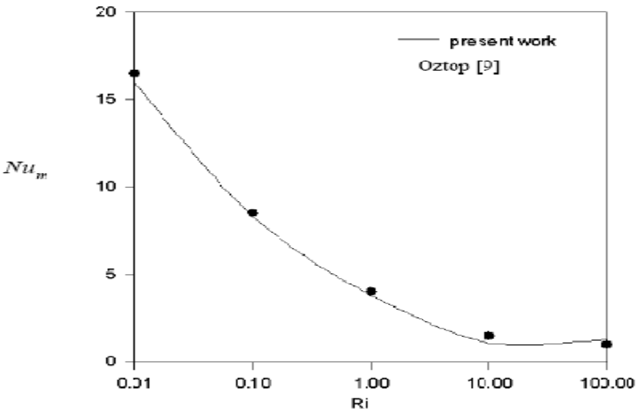

Fig. 2 The comparison of average Nusselt number values with Oztop [9]

Stream lines for different values of γ for Ri=1 are shown in Fig. 3. Mixed convection exists in this case. This figure shows that the forced convection plays a dominant role and moving lid generates most of the recirculation flow. The main cell of the cavity is close to right side and the recirculation is clockwise. Some perturbations are seen in streamlines near to the left side. With increasing γ it is observed that streamlines cover more places in the enclosure and the perturbations approach to the left side. It shows that with increasing γ power of natural convection increases and streamlines approach to the fixed bottom side. Isotherms for different values of γ for Ri=1 are shown in Fig. 4.

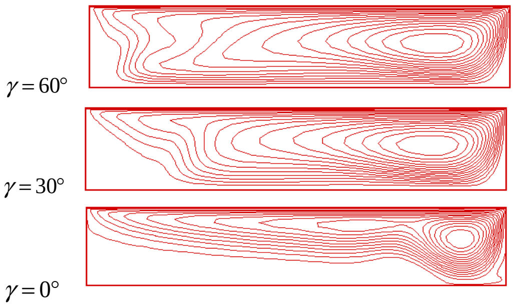

Fig. 3 Stream lines for Ri=1

The isotherms show that in the stagnant zone fluid is vertically stratified.It is observed that the vertical direction is not aligned with the side walls for nonzero inclination.

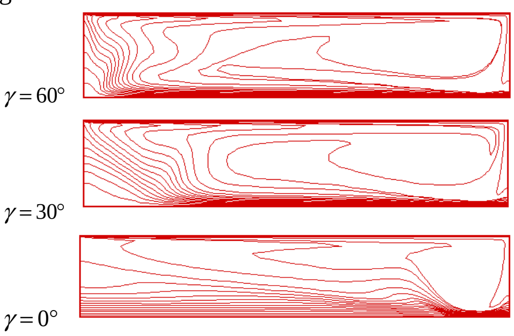

Fig. 4 Isotherms for Ri=1

For the natural convection dominated case with Ri=100 (Fig. 5), the cavity inclination has significant impact on the thermal and hydrodynamic flow fields.

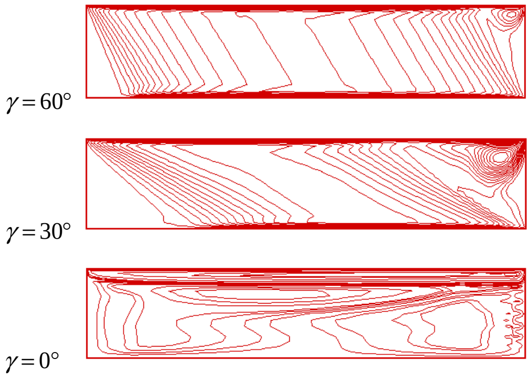

Fig. 5 Stream lines for Ri=100

The variation of the local Nusselt number along the hot moving lid at Ri=1 is plotted in Fig. 6.

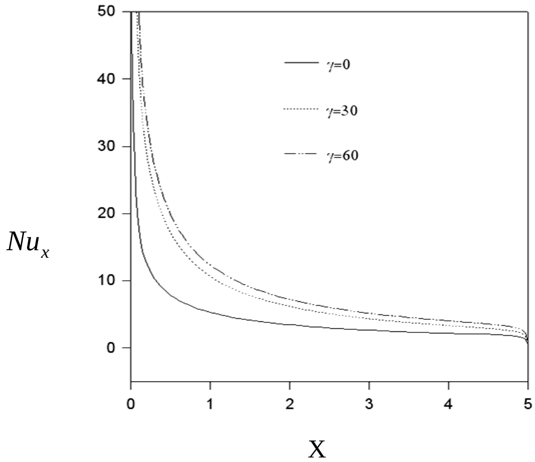

Fig. 6 Variation of the local Nusselt number along the hot surface, Ri =1

The effect of cavity inclination on the heat transfer process is clearly discernible in this plot. In general, the Nusselt number at the hot lid starts with a high value at the left end and decreases monotonically to a small value towards the right end. Getting more results show that for $Ri \geq 1$ the local Nusselt number increases with the cavity inclination. The high values of the local Nusselt number at the left end of the lid, can be attributed to the impinging effect of the recirculation flow streams in these locations. For the force convection dominated case, $Ri < 1$, the local Nusselt number plots don't have any important differences with the cavity inclination, so their figures haven't been plotted. Variation of the average Nusselt number at the hot surface with cavity inclination is shown in Fig.7.The effect of inclination on the overall heat transfer process is clearly depicted in this figure. The average Nusselt number decreases mildly with the cavity inclination for the dominant forced convection case (Ri = 0.001) but it increases mildly for the dominant natural convection case

(Ri=1). This illustrates the relative insensitivity of the overall heat transfer process with inclination in these two extremes cases. In Ri=100 Nusselt number is more than the other two cases in vast range of the cavity inclination. We observe that at Ri>1 due to the enhancement of the buoyancy effects average Nusselt number increases. Another important thing about Fig. 4 is that in $\gamma = 0°$, no inclination, the Nusselt number has the least value in Ri=100 (free convection state) but by increasing the cavity inclination ($\gamma = 60°$) the Nusselt number has the least value in Ri=0.001 which illustrates the force convection state. Variation of the maximum stream function of the fluid with cavity inclination is shown in Fig. 8.

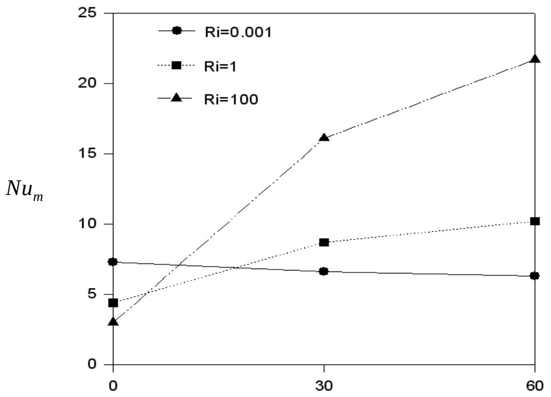

Fig. 7 Average Nusselt number at the hot surface

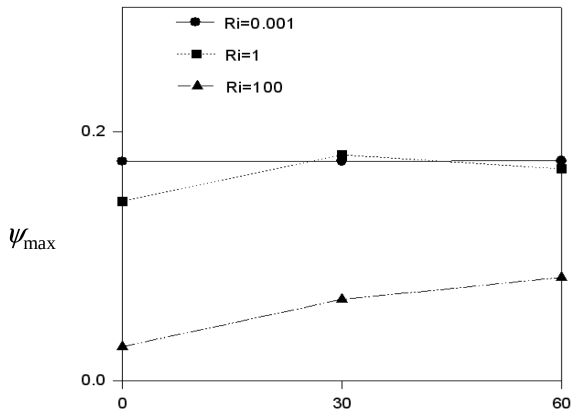

Fig. 8 Maximum stream function of the fluid with cavity inclination

The effect of inclination on the stream function is clearly depicted in this figure. The ψ_{max} increases mildly with the cavity inclination for the dominant natural convection case (Ri=100) but it doesn't change significantly for the dominant forced convection case (Ri=0.001). This illustrates the relative insensitivity of the stream function with inclination in these two extremes cases. So In general it can be mentioned that ψ_{max} have more differences in the dominant natural convection case than the dominant forced convection at different cavity inclinations and it's the result of stronger buoyancy forces in the higher cavity inclination.

V. Conclusion

A computer code was developed to investigate laminar mixed convection processes in shallow two dimensional rectangular cavities. The effects of inclination were investigated and the following results were achieved:

1) The local Nusselt number at the heated moving wall starts with a high value and decreases to a small value.
2) The average Nusselt number increases mildly with cavity inclination for Ri=1 while it increases much more for Ri = 100.
3) The average Nusselt number decreases very mildly with cavity inclination for the forced convection dominated case (Ri=0.001)
4) Maximum stream function has more differences in the dominant natural convection case than the dominant forced convection at the different cavity inclinations.
5) With increasing the cavity inclination the power of natural convection would increases (especially in the dominant natural convection)
6) In order to find a position of the cavity to have the most mixed convection heat transfer we should select an appropriate inclination according to the range of Richardson number. With variation of the cavity inclination, the average Nusselt number changes a little for the dominant forced convection but it increases considerably for the dominant natural convection.

List of Symbols

AR	aspect ratio of the cavity
g	gravitational acceleration
Gr	Grashof number
H	height of the cavity
K	thermal conductivity of fluid
L	length of the cavity
P	dimensionless local pressure
Pr	Prandtl number
Re	Reynolds number
T_c	temperature of the cold surface
T_h	temperature of the hot surface (lid)
U	dimensionless velocity component in X-direction
u_s	lid velocity
V	dimensionless velocity component in Y-direction

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
γ	cavity inclination angle
θ	dimensionless temperature
ρ	Density
υ	kinematic viscosity

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