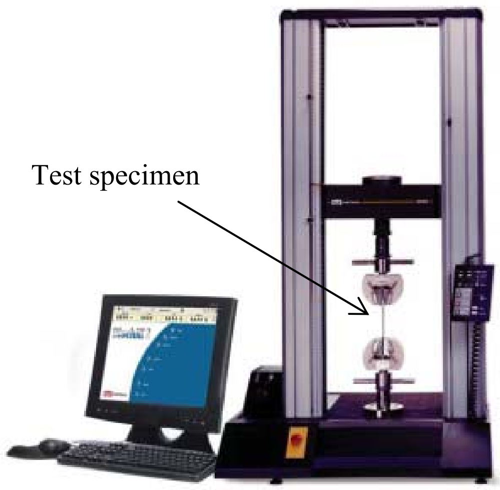
The Response of Real Materials

The constitutive equation was introduced in the previous section. The means by which the constitutive equation is determined is by carrying out experimental tests on the material in question. This topic is discussed in what follows.

5.2.1 The Tension Test

Consider the following key experiment, the **tensile test**, in which a small, usually cylindrical, specimen is gripped and stretched, usually at some given rate of stretching. A typical specimen would have diameter about 1cm and length 5cm, and larger ends so that it can be easily gripped, Fig. 5.2.1a. Specialised machines are used, for example the Instron testing machine shown in Fig. 5.2.1b.

1



2

Figure 5.2.1: the tension test; (a) test specimen, (b) testing machine

As the specimen is stretched, the force required to hold the specimen at a given displacement/stretch is recorded[1].

The Engineering Materials

For many of the (hard) engineering materials, the force/displacement curve will look something like that shown in Fig. 5.2.2. It will be found that the force is initially proportional to displacement as with the linear portion OA in Fig. 5.2.2. The following observations will also be made:

(1) if the load has not reached point *A*, and the material is then unloaded, the force/displacement curve will trace back along the line OA down to zero force and zero displacement; further loading and unloading will again be up and down OA.

(2) the loading curve remains linear up to a certain force level, the **elastic limit** of the material (point *A*). Beyond this point, **permanent deformations** are induced[2]; on

[1] the very precise details of how the test should be carried out are contained in the special standards for materials testing developed by the American Society for Testing and Materials (ASTM)

unloading to zero force (from point *B* to *C*), the specimen will have a permanent elongation. An example of this response (although not a tension test) can be seen with a paper clip – gently bend the clip and it will "spring back" (this is the *OA* behaviour); bend the clip too much (*AB*) and it will stay bent after you let go (*C*).

(3) above the elastic limit (from *A* to *B*), the material **hardens**, that is, the force required to maintain further stretching, unsurprisingly, keeps increasing. (However, some materials can **soften**, for example granular materials such as soils).

(4) the rate (speed) at which the specimen is stretched makes little difference to the results observed (at least if the speed and/or temperature is not too high).

(5) the strains up to the elastic limit are small, less than 1% (see below for more on strains).

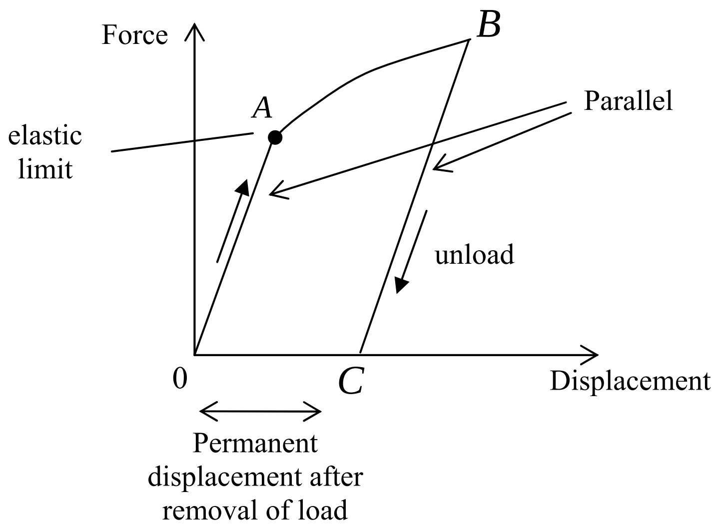


Figure 5.2.2: force/displacement curve for the tension test; typical response for engineering materials

Stress-Strain Curve

There are two definitions of stress used to describe the tension test. First, there is the force divided by the *original* cross sectional area of the specimen A_0; this is the **nominal stress** or **engineering stress**,

$$\sigma_n = \frac{F}{A_0} \tag{5.2.1}$$

Alternatively, one can evaluate the force divided by the (smaller) *current* cross-sectional area *A*, leading to the **true stress**

---

[2] if the tension tests are carried out extremely carefully, one might be able to distinguish between a point where the stress-strain curve ceases to be linear (the **proportional limit**) and the elastic limit (which will occur at a slightly higher stress)

$$\sigma = \frac{F}{A} \tag{5.2.2}$$

in which *F* and *A* are both changing with time. For small elongations, within the linear range OA, the cross-sectional area of the material undergoes negligible change and both definitions of stress are more or less equivalent.

Similarly, one can describe the deformation in two alternative ways. As discussed in Section 4.1.1, one can use the engineering strain

$$\varepsilon = \frac{l - l_0}{l_0} \tag{5.2.3}$$

or the true strain

$$\varepsilon_t = \ln\left(\frac{l}{l_0}\right) \tag{5.2.4}$$

Here, l_0 is the original specimen length and l is the current length. Again, at small deformations, the difference between these two strain measures is negligible.

The stress-strain diagram for a tension test can now be described using the true stress/strain or nominal stress/strain definitions, as in Fig. 5.2.3. The shape of the nominal stress/strain diagram, Fig. 5.2.3a, is of course the same as the graph of force versus displacement. *C* here denotes the point at which the maximum force the specimen can withstand has been reached. The nominal stress at *C* is called the **Ultimate Tensile Strength** (UTS) of the material.

After the UTS is reached, the specimen "necks", that is, the specimen begins to deform locally – with a very rapid reduction in cross-sectional area somewhere about the centre of the specimen until the specimen breaks, as indicated by the asterisk in Fig. 5.2.3. The appearance of a test specimen at each of these stages of the stress-strain curve is shown top of Fig. 5.2.3a.

For many materials, it will be observed that there is little or no volume change during the permanent deformation phase, so $A_0 l_0 = Al$ and $\sigma = \sigma_N (1 + \varepsilon)$. This nominal stress to true stress conversion formula will only be valid up to the point of necking.

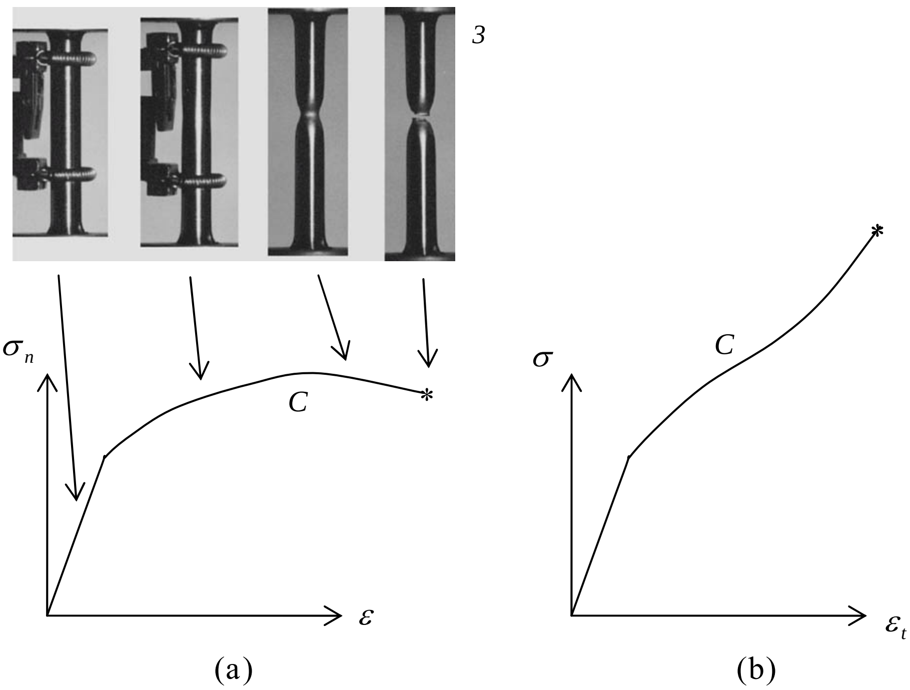


Figure 5.2.3: typical stress-strain curve for an engineering material; (a) engineering stress and strain, (b) true stress and strain

The stress-strain curves for mild steel and aluminium are shown in Fig. 5.2.4. For mild steel, the stress at first increases after reaching the elastic limit, but then decreases. The curve contains a distinct **yield point**; this is where a large increase in strain begins to occur with little increase in required stress[3], i.e. little hardening. There is no distinct yield point for aluminium (or, in fact, for most materials), Fig. 5.2.4b. In this case, it is useful to define a **yield strength** (or **offset yield point**). This is the maximum stress that can be applied without exceeding a specified value of permanent strain. This offset strain is usually taken to be 0.1 or 0.2% and the yield strength is found by following a line parallel to the linear portion until it intersects the stress-strain curve.

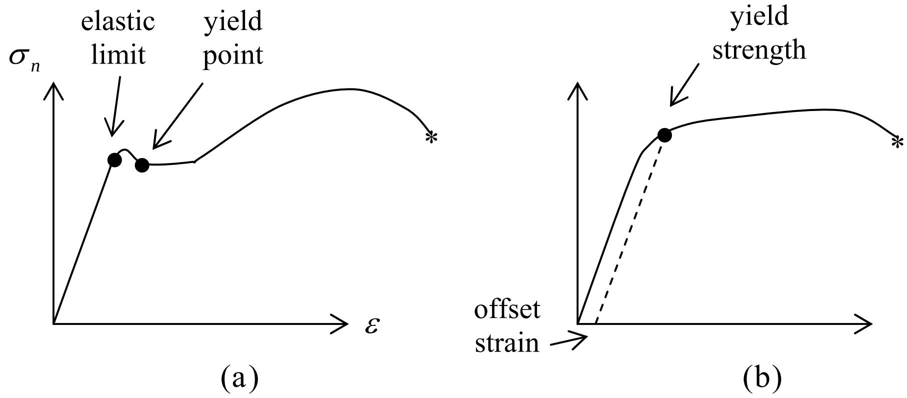


Figure 5.2.4: typical stress-strain curves for (a) mild steel, (b) aluminium

[3] this is also called the **lower yield point**; the **upper yield point** is then the higher stress value just above the elastic limit

The Young's Modulus

The slope of the stress-strain curve over the linear region, before the elastic limit is reached, is the **Young's Modulus** *E*:

$$E = \frac{\sigma}{\varepsilon} \tag{5.2.5}$$

The Young's Modulus has units of stress and is a measure of how "stiff" a material is.

Eqn. 5.2.5 is a constitutive relation (see Eqn. 5.1.2); it is the **one-dimensional linear elastic** constitutive relation.

Use of the Tension Test Data

What is the data from the tension test used for? First of all, it is of direct use in many structural applications. Many structures, such as bridges, buildings and the human skeleton, are composed in part of relatively long and slender components. In service, these components undergo tension and/or compression, very much like the test specimen in the tension test. The tension test data (the Young's Modulus, the Yield Strength and the UTS) then gives direct information on the amount of stress that these components can safely handle, before undergoing dangerous straining or all-out failure.

More importantly, the tension test data (and similar test data – see below) can be used to predict what will happen when a component of complex three-dimensional shape is loaded in a complex way, nothing like as in the simple tension test. This can be put another way: one must be able to predict the world around us without having to resort to complex, expensive, time-consuming materials testing – one should be able to use the test data from the tension test (and similar simple tests) to achieve this. How this is actually done is a major theme of mechanics modelling and these Books.

Test data for a number of metals are listed in Table 5.2.1 below. Note that although some materials can have similar stiffnesses, for example Nickel and Steel, their relative strengths can be very different.

	Young's Modulus *E* (GPa)	0.2% Yield Strength (MPa)	Ultimate Tensile Strength (MPa)
Ni	200	70	400
Mild steel	203	220	430
Steel (AISI 1144)	210	540	840
Cu	120	60	400
Al	70	40	200
Al Alloy (2014-T651)	73	415	485

Table 5.2.1: Tensile test data for some metals (at room temperature)

Data as listed above should be treated with caution – it should be used only as a rough guide to the actual material under study; the data can vary wildly depending on the purity and precise nature of the material. For example, the tensile strength of glass as found in a typical glass window is about 50MPa. For fine glass fibres as used in fibre-reinforced plastics and composite materials, the tensile strength can be 4000MPa. In fact, glass is a good reminder as to why the tensile values differ from material to material – it is due to the difference in microstructure. The glass window has many very fine flaws and cracks in it, invisible to the naked eye, and so this glass is not very strong; very fine slivers of glass have no such flaws and are extremely strong – hence their use in engineering applications.

The Poisson's Ratio

Another useful material parameter is the **Poisson's ratio** ν.[4] As the material stretches in the tension test, it gets thinner; the Poisson's ratio is a measure of the ease with which it thins:

$$\nu = -\frac{\Delta w / w_0}{\Delta l / l_0} = -\frac{\varepsilon_w}{\varepsilon} \tag{5.2.6}$$

Here, $\Delta w = w - w_0$, w_0 are the change in thickness and original thickness of the specimen, Fig. 5.2.5; $\Delta l = l - l_0$, l_0 are the change in length and original length of the specimen; $\varepsilon_w = (w - w_0)/w_0$ is the strain in the thickness direction. A negative sign is included because Δw is negative, making the Poisson's ratio a positive number. (It is implicitly assumed here that the material is getting thinner by the same amount in all directions; see below in the context of anisotropy for when this is not the case.)

Most engineering materials have a Poisson's Ratio of about 0.3. Values for a range of materials are listed in Table 5.2.2 further below.

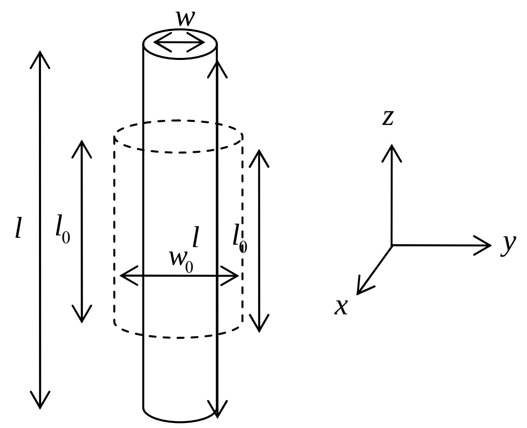


Figure 5.2.5: Change in dimensions of a test specimen

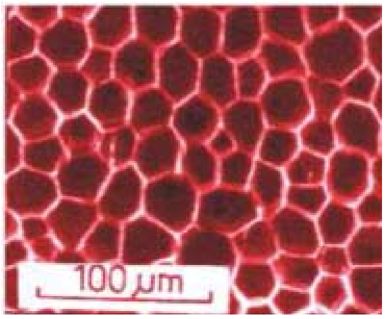
Recall from Section 4.3 that the volumetric strain is given by the sum of the normal strains. There is no harm in re-calculating this for the tensile test specimen of Fig. 5.2.5. One has $\Delta V / V = w^2 l / w_0^2 l_0 - 1$, so that, assuming the strains are small so that the terms $\varepsilon\varepsilon_w$, ε_w^2 and $\varepsilon\varepsilon_w^2$ can be neglected, $\Delta V / V = \varepsilon + 2\varepsilon_w$ (this is the sum of the normal

4 this is the Greek letter *nu*, not the letter "v"

strains, $\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$, Fig. 5.2.4). Using the definition of the Poisson's ratio, Eqn. 5.2.6, one has

$$\frac{\Delta V}{V} = \varepsilon(1 - 2\nu) \tag{5.2.7}$$

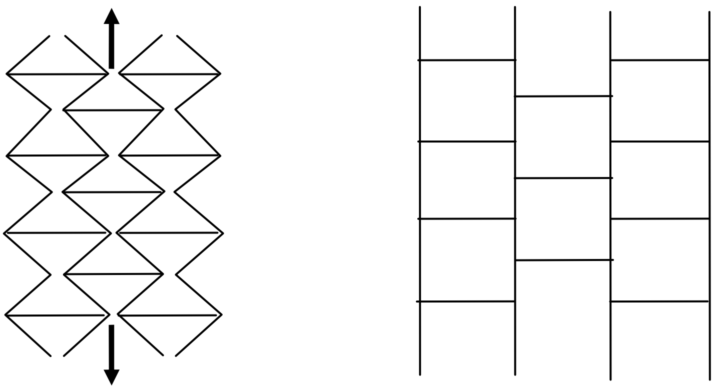
A material which undergoes little volume change thus has a Poisson's Ratio close to 0.5; rubber and other soft tissues, for example biological materials, have Poisson's Ratios very close to 0.5. A material which undergoes zero volume change ($\nu = 0.5$) is called **incompressible** (see more on incompressibility in Section 5.2.4 below). At the other extreme, materials such as cork can have Poisson's Ratios close to zero. The reason for this can be seen from the microstructure of cork shown in Fig. 5.2.6; when tested in compression, the hexagonal honeycomb structure simply folds down, with no necessary lateral expansion.



[4]

Figure 5.2.6: Microstructure of Cork

Auxetic materials are materials which have a negative Poisson's Ratio; when they are stretched, they get thicker. Examples can be found amongst polymers, foams, rocks and biological materials. These materials obviously have a very particular microstructure. A typical example is the network microstructure shown in Fig. 5.2.7.



(a) (b)

Figure 5.2.7: Auxetic material (a) before loading, (b) after loading

Ductile and Brittle Materials

The engineering materials can be grouped into two broad classes: the ductile materials and the brittle materials. The ductile materials undergo large permanent deformations, stretching and necking before failing[5]. The term ductile **rupture** is usually reserved for materials which fail in this way. The separate pieces of the specimen pull away from each other gradually, leaving rough surfaces. A simple measure of ductility is the engineering strain at failure. The brittle materials are generally more stiff and strong, but fail without undergoing much permanent deformation – the tension specimen undergoes a sudden clean break – a **fracture**. The UTS in the case of a brittle material is the same as the failure/fracture stress. Ceramics and glasses are extremely brittle – they fractures suddenly without undergoing any permanent deformation. The difference is illustrated schematically in Fig. 5.2.8 below.

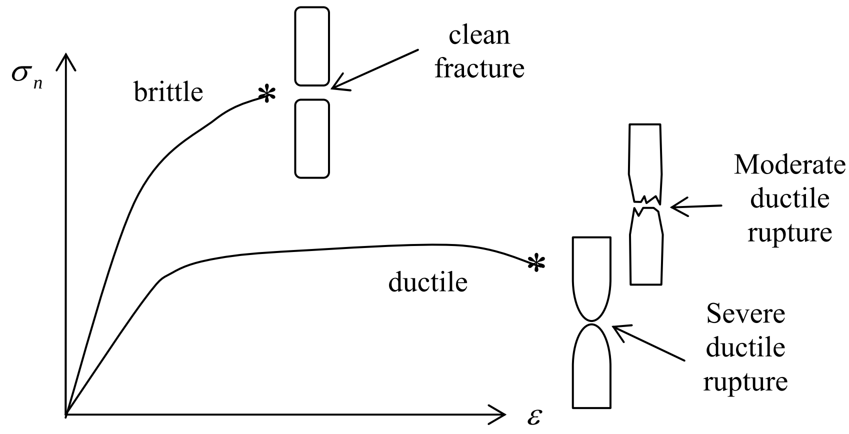


Figure 5.2.8: the difference between ductile and brittle materials

Ductility will depend on temperature – a very cold metal will tend to shatter suddenly, whereas it will stretch more easily when hot.

Soft Materials

Tension test data for non-engineering materials can be very different to that given above. For example, the typical response of a "soft" material, such as rubber, is shown in Fig. 5.2.9. For many soft materials, the elastic limit (or yield strength) can be very high on the stress-strain curve, close to failure. Most of the curve is elastic, meaning that when one unloads the material, the unloading curve traces over the loading curve back down to zero stress and zero strain: the material does not undergo any permanent deformation[6]. Note that the stress-strain curve is non-linear (curved), unlike the straight line elastic portion for a typical metal, Fig. 5.2.2-4, so these materials do not have a single Young's Modulus through which their response can be described.

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[5] the term ductile is used for a specimen in tension; the analogous term for compression is **malleability** – a malleable material is easily "squashed"

[6] here, as elsewhere, these statements should not be taken literally; a real rubber will undergo *some* permanent deformation, only it will often be so small that it can be discounted, and an unload curve will never "exactly" trace over a loading curve

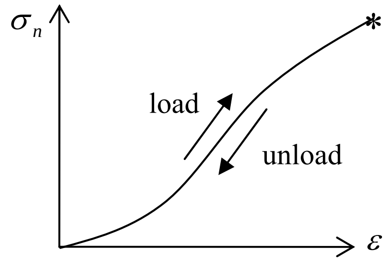
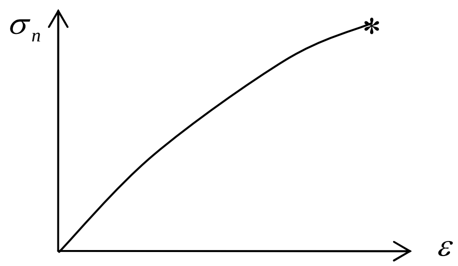


Figure 5.2.9: typical load/unload curves for rubber

5.2.2 Compression Tests

Many materials are used, or designed for use, in compression only, for example soils and concrete. These materials are tested in compression. A common testing method for concrete is to place a cylindrical specimen between two parallel plates and bring the plates together. The typical response of concrete is shown in Fig. 5.2.10a; at failure, the concrete crushes catastrophically, as in the specimen shown in Fig. 5.2.10b. Nominal stresses in the region 20-70MPa are typical and a good concrete would strain to much less than 1% at failure.



(a)


[5]

(b)

Figure 5.2.10: typical compressive response of concrete; (a) stress-strain curve, (b) specimen at failure

For many materials, e.g. metals, a compression test will lead to similar results as the tensile stress. The yield strength in compression will be approximately the same as (the negative of) the yield strength in tension. If one plots the true stress versus true strain curve for both tension and compression (absolute values for the compression), and the two curves more or less coincide, this would indicate that the behaviour of the material under compression is broadly similar to that under tension. However, if one were to use the nominal stress and strain, then the two curves would not coincide even if the real tensile/compressive behaviour was similar (although they would of course in the small-strain linear region); this is due to the definition of the engineering strain/stress.

5.2.3 Shear Tests

In the **shear test**, the material is subjected to a shear strain $\gamma \equiv 2\varepsilon_{xy}$ by applying a shear stress[7] $\tau \equiv \sigma_{xy}$, Fig. 5.2.11a. The resulting shear stress-strain curve will be similar to the tensile stress-strain curve, Fig. 5.2.11b. The shear stress at failure, the **shear strength**, can be greater or smaller than the UTS. The shear yield strength, on the other hand, is usually in the region of 0.5-0.75 times the tensile yield strength. In the linear small-strain region, the shear stress will be proportional to the shear strain; the constant of proportionality is the **shear modulus** *G*:

$$G = \frac{\tau}{\gamma} \tag{5.2.8}$$

For many of the engineering materials, $G \approx 0.4E$.

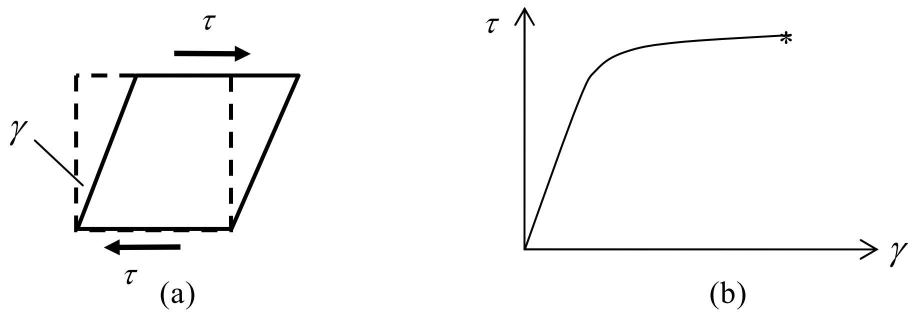


Figure 5.2.11: the shear test; (a) specimen subjected to shear stress and shear strain (dotted = undeformed), (b) shear stress-strain curve

5.2.4 Compressibility

In the **confined compression test**, a sample is placed in a container and a piston is used to compress it at some pressure *p*, Fig. 5.2.12a. This test can be used to determine how compressible a material is. When a material is compressed by equal pressures on all sides, the ratio of applied pressure *p* to (unit) volume change, i.e. volumetric strain $\Delta V / V$, is called the **Bulk Modulus** *K*, Fig. 5.2.12b (this is not quite the situation in Fig. 5.2.12a – the reaction pressures on the side walls will only be about half the applied surface pressure *p*; see Section 6.2):

$$K = -\frac{p}{\Delta V / V} \tag{5.2.9}$$

The negative sign is included since a positive pressure implies a negative volumetric strain, so that the Bulk Modulus is a positive value.

[7] there are many ways that this can be done, for example by pushing blocks of the material over each other, or using more sophisticated methods such as twisting thin tubes of the material (see Section 7.2)

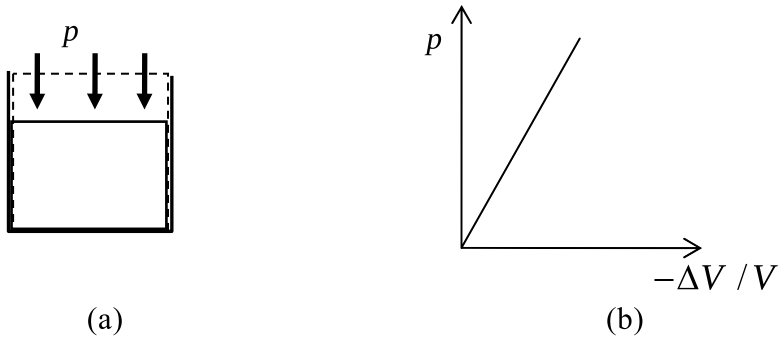


Figure 5.2.12: the confined compression test; (a) specimen subjected to confined compression, (b) pressure plotted against volume change

A material which can be easily compressed has a low Bulk Modulus. As mentioned earlier, a material which cannot be compressed at all is called incompressible ($K \to \infty$).

No real material is incompressible, but some can be regarded as incompressible so as to make the mechanics modelling easier. For example, the Shear Modulus of rubber is very much smaller than its Bulk Modulus, Table 5.2.2. Essentially, this means that the shape of rubber can be easily changed as compared to its volume. Thus, in applications where a rubber component is being deformed or subjected to arbitrary stressing, it is perfectly reasonable to simply assume that rubber is incompressible. The same applies, only more so, to water; the Shear Modulus is effectively zero and there is no resistance to change in shape (which will be observed on pouring a glass of water on to the ground); it is thus regarded almost always as completely incompressible. On the other hand, even though the Bulk Modulus of the metals and other engineering materials is very much *larger* than that of water or rubber, they are still regarded as compressible in applications – the extremely small changes in volume are significant.

	Young's Modulus *E* (GPa)	Shear Modulus G (GPa)	Bulk Modulus *K* (GPa)	Poissons Ratio
Ni	200	76	180	0.31
Mild steel	203	78	138	0.30
Steel (AISI 1144)	210	80	140	0.31
Cu	120	46	142	0.34
Al	70	26	76	0.35
Rubber	14.9×10^{-4}	5×10^{-4}	1	0.49
Water	$\approx 10^{-14}$	$\approx 10^{-14}$	2.2	

Table 5.2.2: Moduli and Poisson's Ratios for a number of materials

5.2.5 Cyclic Tests

Many materials are subjected to complex loading regimes when in service, not simply a one-off stretching, shearing or compression. A classic example are the wings of an aircraft which are continually loaded in tension, then compression, then tension and so on, as in Fig. 5.2.13. Anything moving back and forward is likely to be subjected to this tension/compression-type cyclic loading. Another example would be the stresses experienced by cardiac tissue in a pumping heart.

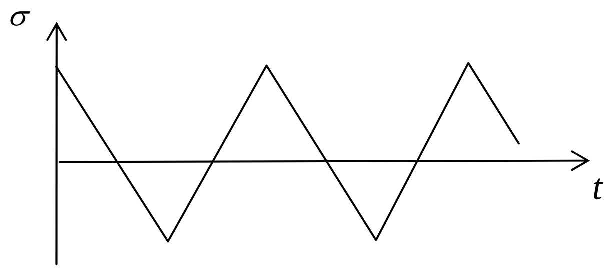


Figure 5.2.13: cyclic loading; alternating between tension (positive stress) and compression (negative stress) over time *t*

Cyclic tests can be carried out to determine the response of materials to such loading cycles. An example is shown in Fig. 5.2.14a, the stress-strain response of a Stainless Steel. The Steel is first cycled between two strain values (one positive, one negative, differing only in sign) a number of times. The stress is seen to increase on each successive cycle. The strain is then increased for a number of further cycles, and so on.

One does not have to move from tension to compression; many materials cycle in only tension or compression. For example, the response to cyclic (compressive) loading of polyurethane foam is shown in Fig. 5.2.14b (note how the loading curve is similar to that in 5.2.9).

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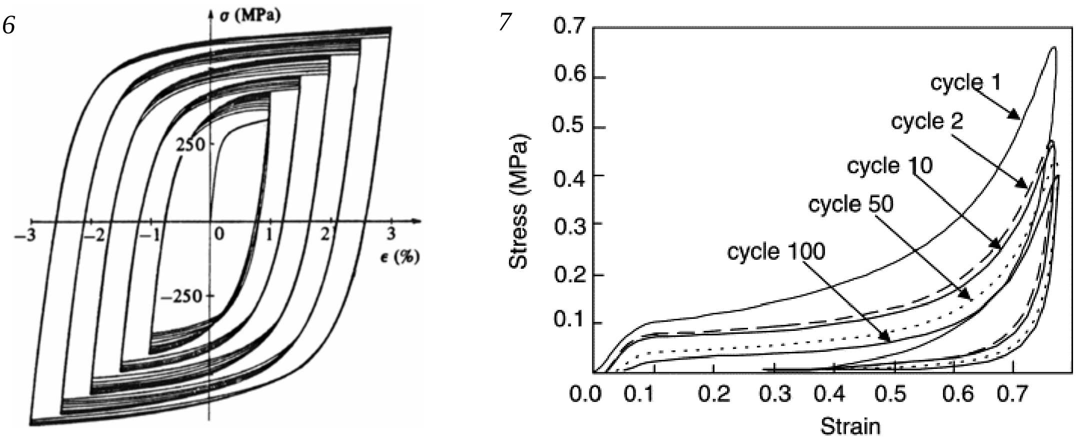

(a) (b)

Figure 5.2.14: cyclic loading; (a) cyclic straining of a Stainless Steel, (b) cyclic loading (in compression) of a polyurethane foam

5.2.6 Other Tests

There are other important tests, for example the Vickers and Brinell **hardness tests**, and the **three-point bending test**. The hardness tests will be discussed in Book II. The bending test is discussed in section 7.4.9, in the context of beam theory. Another two very important tests, the **creep test** and the **stress relaxation test**, will be discussed in Chapter 10.

5.2.7 Isotropy and Anisotropy

Many materials have a strong direction-dependence. The classic example is wood, which has a clear structure –along the grain, along which fine lines can be seen, and against the grain, Fig. 5.2.15. The wood is stiffer and stronger along the grain than against the grain. A material which has this direction-dependence of mechanical (and physical) properties is called **anisotropic**.

8

Figure 5.2.15: Wood

Fig. 5.2.16 shows stress-strain curves for human ligament tissue; in one test, the ligament is stretched along its length (the **longitudinal** direction), in the second, across the width of the ligament (the **transverse** direction). It can be seen that the stiffness is much higher in the longitudinal direction. Another example is bone – it is much stiffer along the length of the bone than across the width of the bone. In fact, many biological materials are strongly anisotropic.

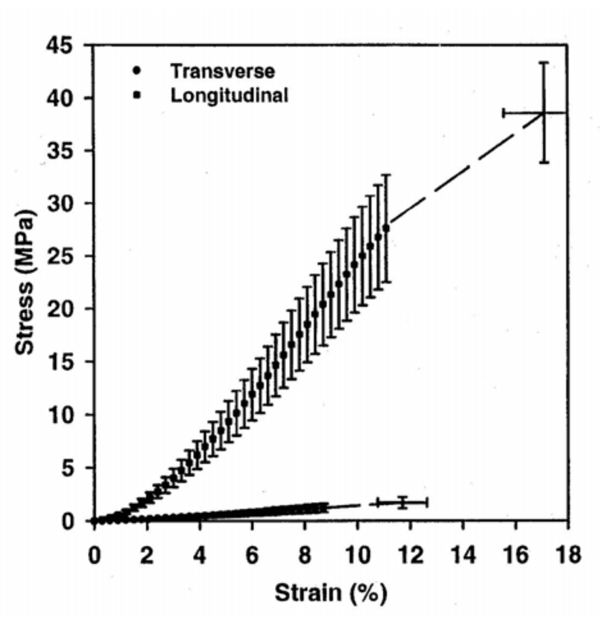

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Figure 5.2.16: Anisotropic response of human ligament

A material whose properties are the same in all directions is called **isotropic**. In particular, the relationship between stress and strain *at any single location* in a material is the same in all directions. This implies that if a specimen is cut from an isotropic material and subjected to a load, it would not matter in which orientation the specimen is cut, the

σ_o

resulting deformation would be the same – as illustrated in Fig. 5.2.17. Most metals and ceramics can be considered to be isotropic (see Section 5.4).

σ_o

σ_o

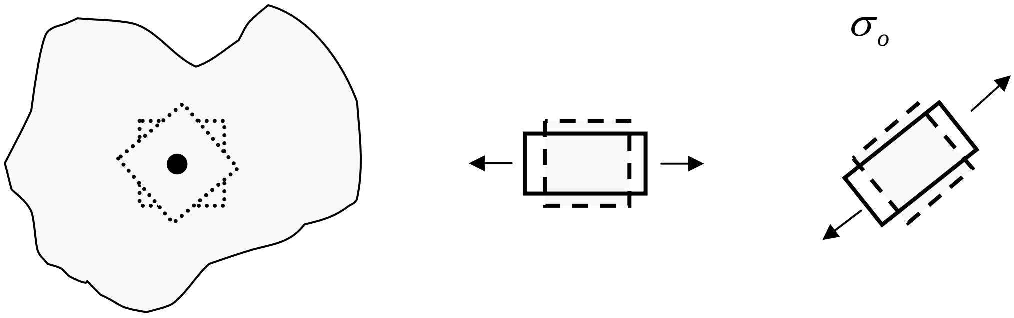

Figure 5.2.17: Illustration of Isotropy; the stress-strain response is the same no matter in what "direction" the test specimen is cut from the material

Anisotropy will be examined in more detail in §6.3. It will be shown there, for example, that an anisotropic material can have a Poisson's ratio greater than 0.5.[8]

5.2.8 Homogeneous Materials

The term homogeneous means that the mechanical properties are the same at each point throughout the material. In other words, the relationship between stress and strain is the same for all material particles. Most materials can be assumed to be homogeneous.

In engineering applications, it is sometimes beneficial to design materials/components which are specifically not homogeneous, i.e. **inhomogeneous**. Such materials whose properties vary gradually throughout are called **Functionally Graded Materials**, and have been gaining popularity since the 1980s-90s in advanced technologies.

Note that a material can be homogeneous and not isotropic, and *vice versa* – homogeneous refers to different locations whereas isotropy refers to the same location.