

# Quantum Mechanics\_mass

In physics, **mass** (from Greek  $\mu\acute{\alpha}\zeta\alpha$  (*maza*), meaning "barley cake, lump [of dough]") is a property of a physical body which determines the body's resistance to being accelerated by a Force and the strength of its mutual gravitational attraction with other bodies. The SI unit of mass is the kilogram (kg). As mass is difficult to measure directly, usually balances or scales are used to measure the weight of an object, and the weight is used to calculate the object's mass. For everyday objects and energies well-described by Newtonian physics, mass describes the amount of Matter in an object. However, at very high speeds or for subatomic particles, special relativity shows that Energy is an additional source of mass. Thus, any stationary body having mass has an equivalent amount of energy, and all forms of energy resist acceleration by a force and have gravitational attraction.

There are several distinct phenomena which can be used to measure mass. Although some theorists have speculated some of these phenomena could be independent of each other,<sup>[1]</sup> current experiments have found no difference among any of the ways used to measure mass:

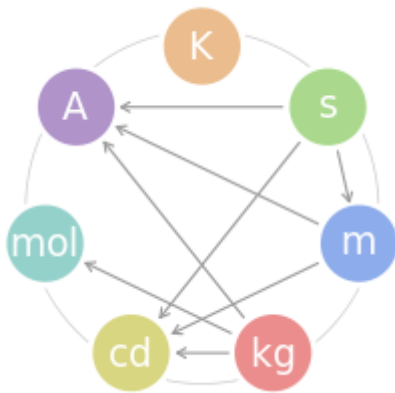
- *Inertial mass* measures an object's resistance to changes in velocity  $m=F/a$ . (the object's Acceleration)
- *Active gravitational mass* measures the gravitational force exerted by an object.
- *Passive gravitational mass* measures the gravitational force experienced by an object in a known gravitational field.
- *Mass-Energy* measures the total amount of Energy contained within a body, using  $E=mc^2$

The mass of an object determines its Acceleration in the presence of an applied force. This phenomenon is called Inertia. According to Newton's second law of motion, if a body of fixed mass  $m$  is subjected to a single force  $F$ , its acceleration  $a$  is given by  $F/m$ . A body's mass also determines the degree to which it generates or is affected by a gravitational field. If a first body of mass  $m_A$  is placed at a distance  $r$  (center of mass to center of mass) from a second body of mass  $m_B$ , each body experiences an attractive force  $F_g = Gm_A m_B / r^2$ , where  $G = 6.67 \times 10^{-11} \text{ N kg}^{-2} \text{ m}^2$  is the "universal gravitational constant". This is sometimes referred to as gravitational mass.<sup>[note 1]</sup> Repeated

experiments since the 17th century have demonstrated that inertial and gravitational mass are identical; since 1915, this observation has been entailed *a priori* in the equivalence principle of general relativity.

## Units of mass

Further information: Orders of magnitude (mass)



The kilogram is one of the seven SI base units; one of three which is defined *ad hoc*, without reference to another base unit.

The standard International System of Units (SI) unit of mass is the kilogram (kg). The kilogram is 1000 grams (g), which were first defined in 1795 as one cubic decimeter of water at the melting point of ice. Then in 1889, the kilogram was redefined as the mass of the international prototype kilogram, and as such is independent of the meter, or the properties of water. As of January 2013, there are several proposals for redefining the kilogram yet again, including a proposal for defining it in terms of the Planck constant.<sup>[2]</sup>

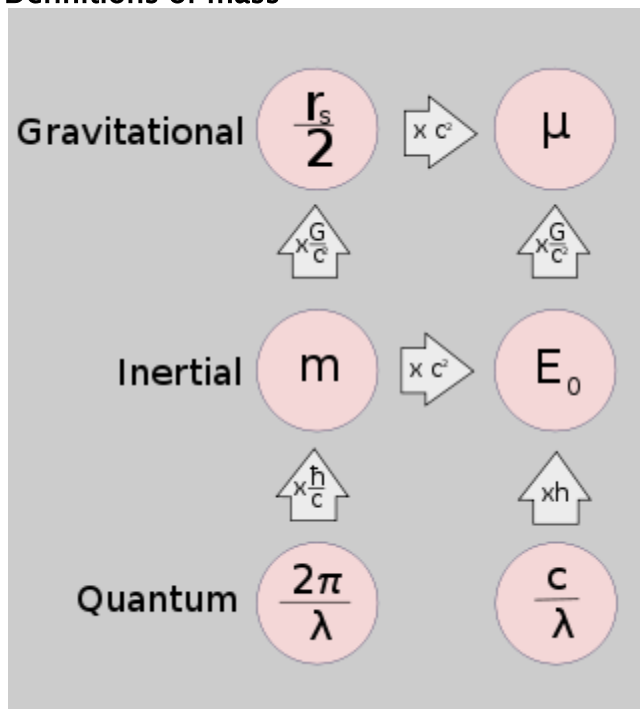
Other units are accepted for use in SI:

- The tonne (t) (or "metric ton") is equal to 1000 kg.
- The electronvolt (eV) is a unit of Energy, but because of the mass-energy equivalence it can easily be converted to a unit of mass, and is often used like one. In this context, the mass has units of eV/c<sup>2</sup>. The electronvolt is common in particle physics.
- The atomic mass unit (u) is 1/12 of the mass of a carbon-12 atom, approximately  $1.66 \times 10^{-27}$  kg.<sup>[note 2]</sup> The atomic mass unit is convenient for expressing the masses of atoms and molecules.

Outside SI system, other units include:

- The slug (sl) is an Imperial unit of mass, (about 14.6 kg) similar to the kilogram.
- The pound (lb) is a unit of both mass and force, used mainly in the United States. (about 0.45 kg or 4.5 N) In scientific contexts where pound (force) and pound (mass) need to be distinguished, SI units are usually used instead.
- The Planck mass ( $m_p$ ) is the maximum mass of point particles. (about  $2.18 \times 10^{-8}$  kg) it is used in particle physics.
- The solar mass is defined as the mass of the sun. It is primarily used in astronomy to compare large masses such as stars or galaxies. ( $\approx 1.99 \times 10^{30}$  kg)
- The mass of a very small particle may be identified with its inverse Compton wavelength ( $1 \text{ cm}^{-1} \approx 3.52 \times 10^{-41}$  kg).
- The mass of a very large star or black hole may be identified with its Schwarzschild radius ( $1 \text{ cm} \approx 6.73 \times 10^{24}$  kg).

#### Definitions of mass



The relation between properties of mass and their associated physical constants. Every massive object is believed to exhibit all five properties. However, due to extremely large or extremely small constants, it is generally impossible to verify more than two or three properties for any object.

- The Schwarzschild radius ( $r_s$ ) represents the ability of mass to cause curvature in space and time.
- The standard gravitational parameter ( $\mu$ ) represents the ability of a massive body to exert Newtonian gravitational forces on other bodies.
- Inertial **mass** ( $m$ ) represents the Newtonian response of mass to forces.
- Rest energy ( $E_0$ ) represents the ability of mass to be converted into other forms of energy.
- The Compton wavelength ( $\lambda$ ) represents the quantum response of mass to local geometry.

In physical science, one may distinguish conceptually between at least seven different aspects of *mass*, or seven physical notions that involve the concept of *mass*:<sup>[3]</sup> Every experiment to date has shown these seven values to be proportional, and in some cases equal, and this proportionality gives rise to the abstract concept of mass.

- The amount of Matter in certain types of samples can be exactly determined through electrodeposition<sup>*clarification needed*</sup> or other precise processes. The mass of an exact sample is determined in part by the number and type of atoms or molecules it contains, and in part by the energy involved in binding it together (which contributes a negative "missing mass," or mass deficit).
- Inertial mass is a measure of an object's resistance to changing its state of motion when a Force is applied. It is determined by applying a force to an object and measuring the acceleration that results from that force. An object with small inertial mass will accelerate more than an object with large inertial mass when acted upon by the same force. One says the body of greater mass has greater Inertia.
- Active gravitational mass <sup>[note 3]</sup> is a measure of the strength of an object's gravitational flux (gravitational flux is equal to the surface integral of gravitational field over an enclosing surface). Gravitational field can be measured by allowing a small 'test object' to freely fall and measuring its free-fall acceleration. For example, an object in free-fall near the Moon will experience less gravitational field, and hence accelerate slower than the same object would if it were in free-fall near the Earth. The gravitational field near the Moon is weaker because the Moon has less active gravitational mass.

- Passive gravitational mass is a measure of the strength of an object's interaction with a gravitational field. Passive gravitational mass is determined by dividing an object's weight by its free-fall acceleration. Two objects within the same gravitational field will experience the same acceleration; however, the object with a smaller passive gravitational mass will experience a smaller force (less weight) than the object with a larger passive gravitational mass.
- Energy also has mass according to the principle of mass-energy equivalence. This equivalence is exemplified in a large number of physical processes including pair production, nuclear fusion, and the gravitational bending of light. Pair production and nuclear fusion are processes through which measurable amounts of mass and energy are converted into each other. In the gravitational bending of light, photons of pure energy are shown to exhibit a behavior similar to passive gravitational mass.
- Curvature of spacetime is a relativistic manifestation of the existence of mass. Curvature is extremely weak and difficult to measure. For this reason, curvature was not discovered until after it was predicted by Einstein's theory of general relativity. Extremely precise atomic clocks on the surface of the earth, for example, are found to measure less time (run slower) when compared to similar clocks in space. This difference in elapsed time is a form of curvature called gravitational time dilation. Other forms of curvature have been measured using the Gravity Probe B satellite.
- Quantum mass manifests itself as a difference between an object's quantum frequency and its wave number. The quantum mass of an electron, the Compton wavelength, can be determined through various forms of spectroscopy and is closely related to the Rydberg constant, the Bohr radius, and the classical electron radius. The quantum mass of larger objects can be directly measured using a watt balance. In relativistic quantum mechanics, mass is one of the irreducible representation labels of the Poincaré group.

### **Weight vs. mass**

Main article: Mass versus weight

In everyday usage, mass and "weight" are often used interchangeably. For instance, a person's weight may be stated as 75 kg. In a constant gravitational field, the weight of an object is proportional to its mass, and it is unproblematic to use the same unit for

both concepts. But because of slight differences in the strength of the Earth's gravitational field at different places, the distinction becomes important for measurements with a precision better than a few percent, and for places far from the surface of the Earth, such as in space or on other planets. Conceptually, "mass" (measured in kilograms) refers to an intrinsic property of an object, whereas "weight" (measured in newtons) measures an object's resistance to deviating from its natural course of free fall, which can be influenced by the nearby gravitational field. No matter how strong the gravitational field, objects in free fall are weightless, though they still have mass.[4]

The force known as "weight" is proportional to mass and Acceleration in all situations where the mass is accelerated away from free fall. For example, when a body is at rest in a gravitational field (rather than in free fall), it must be accelerated by a force from a scale or the surface of a planetary body such as the Earth or the Moon. This force keeps the object from going into free fall. Weight is the opposing force in such circumstances, and is thus determined by the acceleration of free fall. On the surface of the Earth, for example, an object with a mass of 50 kilograms weighs 491 newtons, which means that 491 newtons is being applied to keep the object from going into free fall. By contrast, on the surface of the Moon, the same object still has a mass of 50 kilograms but weighs only 81.5 newtons, because only 81.5 newtons is required to keep this object from going into a free fall on the moon. Restated in mathematical terms, on the surface of the Earth, the weight  $W$  of an object is related to its mass  $m$  by  $W = mg$ , where  $g = 9.80665 \text{ m/s}^2$  is the Earth's gravitational field, (expressed as the acceleration experienced by a free-falling object).

For other situations, such as when objects are subjected to mechanical accelerations from forces other than the resistance of a planetary surface, the weight force is proportional to the mass of an object multiplied by the total acceleration away from free fall, which is called the proper acceleration. Through such mechanisms, objects in elevators, vehicles, centrifuges, and the like, may experience weight forces many times those caused by resistance to the effects of gravity on objects, resulting from planetary surfaces. In such cases, the generalized equation for weight  $W$  of an object is related to its mass  $m$  by the equation  $W = -ma$ , where  $a$  is the proper acceleration of the object caused by all influences other than gravity. (Again, if gravity is the only influence, such as occurs when an object falls freely, its weight will be zero).

Macroscopically, mass is associated with Matter—although matter, unlike mass, is poorly defined in science. On the sub-atomic scale, not only fermions, the particles often associated with matter, but also some bosons, the particles that act as force carriers, have rest mass. Another problem for easy definition is that much of the rest mass of ordinary matter derives from the invariant mass contributed to matter by particles and kinetic energies which have no rest mass themselves (only 1% of the rest mass of matter is accounted for by the rest mass of its fermionic quarks and electrons). From a fundamental physics perspective, mass is the number describing under which the representation of the little group of the Poincaré group a particle transforms. In the Standard Model of particle physics, this symmetry is described as arising as a consequence of a coupling of particles with rest mass to a postulated additional field, known as the Higgs field.

The total mass of the observable universe is estimated at between  $10^{52}$  kg and  $10^{53}$  kg, corresponding to the rest mass of between  $10^{79}$  and  $10^{80}$  protons.<sup>[*citation needed*]</sup>

### **Inertial vs. gravitational mass**

Although inertial mass, passive gravitational mass and active gravitational mass are conceptually distinct, no experiment has ever unambiguously demonstrated any difference between them. In Classical mechanics, Newton's third law implies that active and passive gravitational mass must always be identical (or at least proportional), but the classical theory offers no compelling reason why the gravitational mass has to equal the inertial mass. That it does is merely an empirical fact.

Albert Einstein developed his general theory of relativity starting from the assumption that this correspondence between inertial and (passive) gravitational mass is not accidental: that no experiment will ever detect a difference between them (the weak version of the equivalence principle). However, in the resulting theory, gravitation is not a force and thus not subject to Newton's third law, so "the equality of inertial and *active* gravitational mass [...] remains as puzzling as ever".<sup>[5]</sup>

The equivalence of inertial and gravitational masses is sometimes referred to as the "Galilean equivalence principle" or the "weak equivalence principle". The most important consequence of this equivalence principle applies to freely falling objects. Suppose we have an object with inertial and gravitational masses  $m$  and  $M$ , respectively. If the only force acting on the object comes from a gravitational field  $\mathbf{g}$ , combining Newton's second law and the gravitational law yields the acceleration

$$\mathbf{a} = \frac{M}{m}\mathbf{g}.$$

This says that the ratio of gravitational to inertial mass of any object is equal to some constant  $K$  if and only if all objects fall at the same rate in a given gravitational field. This phenomenon is referred to as the "universality of free-fall". (In addition, the constant  $K$  can be taken to be 1 by defining our units appropriately.)

The first experiments demonstrating the universality of free-fall were conducted by Galileo. It is commonly stated that Galileo obtained his results by dropping objects from the Leaning Tower of Pisa, but this is most likely apocryphal; actually, he performed his experiments with balls rolling down nearly frictionless inclined planes to slow the motion and increase the timing accuracy. Increasingly precise experiments have been performed, such as those performed by Loránd Eötvös,<sup>[6]</sup> using the torsion balance pendulum, in 1889. As of 2008, no deviation from universality, and thus from Galilean equivalence, has ever been found, at least to the precision  $10^{-12}$ . More precise experimental efforts are still being carried out.

The universality of free-fall only applies to systems in which gravity is the only acting force. All other forces, especially Friction and air resistance, must be absent or at least negligible. For example, if a hammer and a feather are dropped from the same height through the air on Earth, the feather will take much longer to reach the ground; the feather is not really in *free-fall* because the force of air resistance upwards against the feather is comparable to the downward force of gravity. On the other hand, if the experiment is performed in a vacuum, in which there is no air resistance, the hammer and the feather should hit the ground at exactly the same time (assuming the acceleration of both objects towards each other, and of the ground towards both objects, for its own part, is negligible). This can easily be done in a high school laboratory by dropping the objects in transparent tubes that have the air removed with a vacuum pump. It is even more dramatic when done in an environment that naturally has a vacuum, as David Scott did on the surface of the Moon during Apollo 15.

A stronger version of the equivalence principle, known as the *Einstein equivalence principle* or the *strong equivalence principle*, lies at the heart of the general theory of relativity. Einstein's equivalence principle states that within sufficiently small regions of space-time, it is impossible to distinguish between a uniform acceleration and a uniform gravitational field. Thus, the theory postulates that the force acting on a massive object caused by a gravitational field is a result of the object's tendency to



move in a straight line (in other words its inertia) and should therefore be a function of its inertial mass and the strength of the gravitational field.

### Origin of mass

Main article: [Mass generation mechanism](#)

In [theoretical physics](#), a [Mass generation mechanism](#) is a theory which attempts to explain the origin of mass from the most fundamental laws of [physics](#). To date, a number of different models have been proposed which advocate different views at the origin of mass. The problem is complicated by the fact that the notion of mass is strongly related to the [gravitational interaction](#) but a theory of the latter has not been yet reconciled with the currently popular model of [particle physics](#), known as the [Standard Model](#).

### Pre-Newtonian concepts

#### Weight as an amount

Main article: [weight](#)



Depiction of early [balance scales](#) in the [Papyrus of Hunefer](#) (dated to the [19th dynasty](#), ca. 1285 BC). The scene shows [Anubis](#) weighing the heart of Hunefer.

The concept of [amount](#) is very old and [predates recorded history](#). Humans, at some early era, realized that the weight of a collection of similar objects was [directly proportional](#) to the number of objects in the collection:

$$W_n \propto n,$$

where  $W$  is the weight of the collection of similar objects and  $n$  is the number of objects in the collection. Proportionality, by definition, implies that two values have a constant [ratio](#):

$$\frac{W_n}{n} = \frac{W_m}{m}, \text{ or equivalently } \frac{W_n}{W_m} = \frac{n}{m}.$$

An early use of this relationship is a balance scale, which balances the force of one object's weight against the force of another object's weight. The two sides of a balance scale are close enough that the objects experience similar gravitational fields. Hence, if they have similar masses then their weights will also be similar. This allows the scale, by comparing weights, to also compare masses.

Consequently, historical weight standards were often defined in terms of amounts. The Romans, for example, used the carob seed (carat or siliqua) as a measurement standard. If an object's weight was equivalent to 1728 carob seeds, then the object was said to weigh one Roman pound. If, on the other hand, the object's weight was equivalent to 144 carob seeds then the object was said to weigh one Roman ounce (uncia). The Roman pound and ounce were both defined in terms of different sized collections of the same common mass standard, the carob seed. The ratio of a Roman ounce (144 carob seeds) to a Roman pound (1728 carob seeds) was:

$$\frac{\text{ounce}}{\text{pound}} = \frac{W_{144}}{W_{1728}} = \frac{144}{1728} = \frac{1}{12}$$

### **Planetary motion**

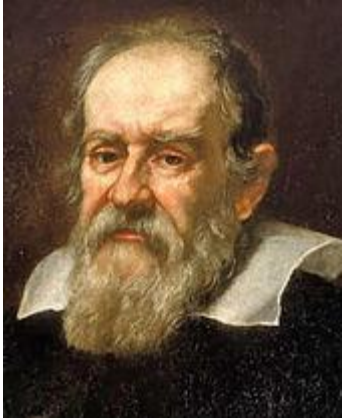
See also: Kepler's laws of planetary motion

In 1600 AD, Johannes Kepler sought employment with Tycho Brahe, who had some of the most precise astronomical data available. Using Brahe's precise observations of the planet Mars, Kepler spent the next five years developing his own method for characterizing planetary motion. In 1609, Johannes Kepler published his three laws of planetary motion, explaining how the planets orbit the Sun. In Kepler's final planetary model, he described planetary orbits as following elliptical paths with the Sun at a focal point of the ellipse. Kepler discovered that the square of the orbital period of each planet is directly proportional to the cube of the semi-major axis of its orbit, or equivalently, that the ratio of these two values is constant for all planets in the Solar System.[\[note 4\]](#)

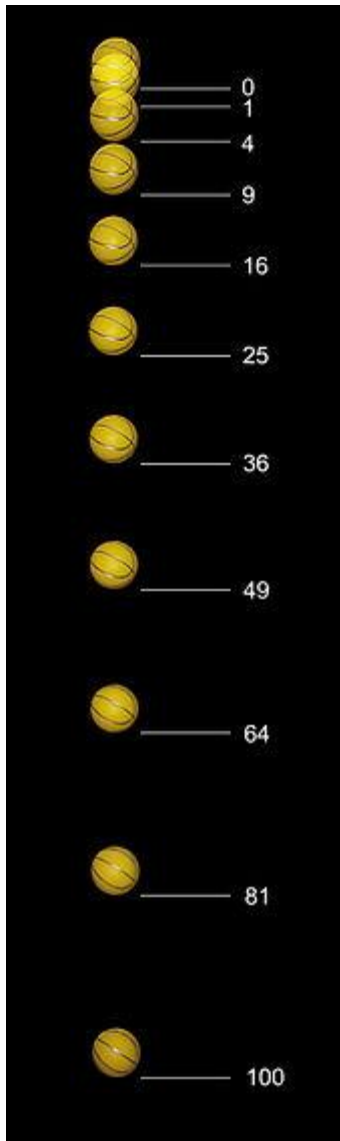
On 25 August 1609, Galileo Galilei demonstrated his first telescope to a group of Venetian merchants, and in early January of 1610, Galileo observed four dim objects near Jupiter, which he mistook for stars. However, after a few days of observation, Galileo realized that these "stars" were in fact orbiting Jupiter. These four objects (later named the Galilean moons in honor of their discoverer) were the first celestial bodies observed to orbit something other than the Earth or Sun. Galileo continued to observe

these moons over the next eighteen months, and by the middle of 1611 he had obtained remarkably accurate estimates for their periods.

### Galilean free fall



Galileo Galilei 1636



Distance traveled by a freely falling ball is proportional to the square of the elapsed time

Sometime prior to 1638, Galileo turned his attention to the phenomenon of objects in free fall, attempting to characterize these motions. Galileo was not the first to investigate Earth's gravitational field, nor was he the first to accurately describe its fundamental characteristics. However, Galileo's reliance on scientific experimentation to establish physical principles would have a profound effect on future generations of scientists. It is unclear if these were just hypothetical experiments used to illustrate a concept, or if they were real experiments performed by Galileo,<sup>[7]</sup> but the results obtained from these experiments were both realistic and compelling. A biography by

Galileo's pupil Vincenzo Viviani stated that Galileo had dropped balls of the same material, but different masses, from the Leaning Tower of Pisa to demonstrate that their time of descent was independent of their mass.[note 5] In support of this conclusion, Galileo had advanced the following theoretical argument: He asked if two bodies of different masses and different rates of fall are tied by a string, does the combined system fall faster because it is now more massive, or does the lighter body in its slower fall hold back the heavier body? The only convincing resolution to this question is that all bodies must fall at the same rate.[8]

A later experiment was described in Galileo's *Two New Sciences* published in 1638. One of Galileo's fictional characters, Salviati, describes an experiment using a bronze ball and a wooden ramp. The wooden ramp was "12 cubits long, half a cubit wide and three finger-breadths thick" with a straight, smooth, polished groove. The groove was lined with "parchment, also smooth and polished as possible". And into this groove was placed "a hard, smooth and very round bronze ball". The ramp was inclined at various angles to slow the acceleration enough so that the elapsed time could be measured. The ball was allowed to roll a known distance down the ramp, and the time taken for the ball to move the known distance was measured. The time was measured using a water clock described as follows:

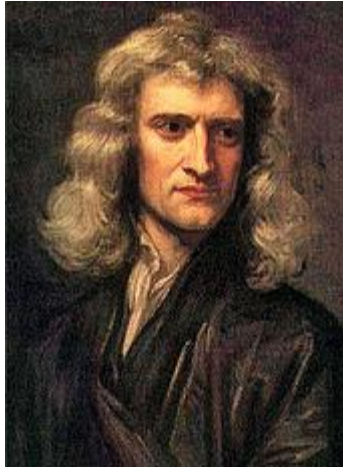
"a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small glass during the time of each descent, whether for the whole length of the channel or for a part of its length; the water thus collected was weighed, after each descent, on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the times, and this with such accuracy that although the operation was repeated many, many times, there was no appreciable discrepancy in the results."[9]

Galileo found that for an object in free fall, the distance that the object has fallen is always proportional to the square of the elapsed time:

$$\text{Distance} \propto \text{Time}^2$$

Galileo had shown that objects in free fall under the influence of the Earth's gravitational field have a constant acceleration, and Galileo's contemporary, Johannes Kepler, had shown that the planets follow elliptical paths under the influence of the Sun's gravitational mass. However, Galileo's free fall motions and Kepler's planetary motions remained distinct during Galileo's lifetime.

## Newtonian mass



Isaac Newton 1689

Earth's Moon

semi-major axis Sidereal orbital period Mass of Earth

0.002 569 AU 0.074 802 sidereal year

Earth's Gravity Earth's Radius

9.806 65 m/s<sup>2</sup> 6 375 km

$$1.2\pi^2 \cdot 10^{-5} \frac{\text{AU}^3}{\text{y}^2} = 3.986 \cdot 10^{14} \frac{\text{m}^3}{\text{s}^2}$$

Robert Hooke had published his concept of gravitational forces in 1674, stating that, all Cœlestial Bodies whatsoever, have an attraction or gravitating power towards their own Centers [and] they do also attract all the other Cœlestial Bodies that are within the sphere of their activity. He further states that gravitational attraction increases by how much the nearer the body wrought upon is to their own center.[10] In a correspondence of 1679–1680 between Robert Hooke and Isaac Newton, Hooke conjectures that gravitational forces might decrease according to the double of the distance between the two bodies.[11] Hooke urged Newton, who was a pioneer in the development of calculus, to work through the mathematical details of Keplerian orbits to determine if Hooke's hypothesis was correct. Newton's own investigations verified that Hooke was correct, but due to personal differences between the two men, Newton chose not to reveal this to Hooke. Isaac Newton kept quiet about his discoveries until 1684, at which time he told a friend, Edmond Halley, that he had solved the problem of gravitational orbits, but had misplaced the solution in his office.[12] After being encouraged by Halley, Newton decided to develop his ideas about gravity and publish all of his findings. In November 1684, Isaac Newton sent a document to Edmund

Halley, now lost but presumed to have been titled *De motu corporum in gyrum* (Latin for "On the motion of bodies in an orbit").<sup>[13]</sup> Halley presented Newton's findings to the Royal Society of London, with a promise that a fuller presentation would follow. Newton later recorded his ideas in a three book set, entitled Philosophiæ Naturalis Principia Mathematica (Latin: "Mathematical Principles of Natural Philosophy"). The first was received by the Royal Society on 28 April 1685–6, the second on 2 March 1686–7, and the third on 6 April 1686–7. The Royal Society published Newton's entire collection at their own expense in May 1686–7.<sup>[14]:31</sup>

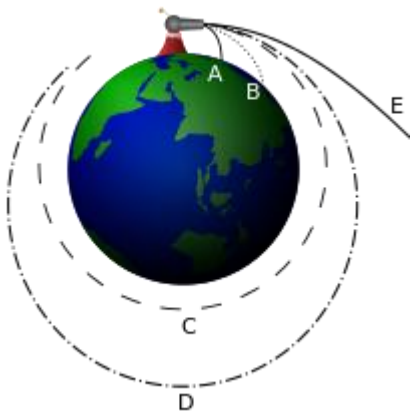
Isaac Newton had bridged the gap between Kepler's gravitational mass and Galileo's gravitational acceleration, and proved the following relationship:

$$\mathbf{g} = -\mu \frac{\hat{\mathbf{R}}}{|\mathbf{R}|^2}$$

where  $\mathbf{g}$  is the apparent acceleration of a body as it passes through a region of space where gravitational fields exist,  $\mu$  is the gravitational mass (standard gravitational parameter) of the body causing gravitational fields, and  $\mathbf{R}$  is the radial coordinate (the distance between the centers of the two bodies).

By finding the exact relationship between a body's gravitational mass and its gravitational field, Newton provided a second method for measuring gravitational mass. The mass of the Earth can be determined using Kepler's method (from the orbit of Earth's Moon), or it can be determined by measuring the gravitational acceleration on the Earth's surface, and multiplying that by the square of the Earth's radius. The mass of the Earth is approximately three millionths of the mass of the Sun. To date, no other accurate method for measuring gravitational mass has been discovered.<sup>[15]</sup>

### Newton's cannonball

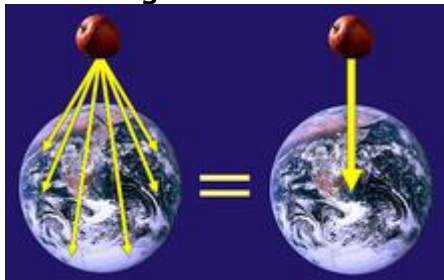


A cannon on top of a very high mountain shoots a cannonball horizontally. If the speed is low, the cannonball quickly falls back to Earth (A,B). At intermediate speeds, it will revolve around Earth along an elliptical orbit (C,D). At high speeds, it will leave Earth altogether (E).

Main article: [Newton's cannonball](#)

Newton's cannonball was a thought experiment used to bridge the gap between Galileo's gravitational acceleration and Kepler's elliptical orbits. It appeared in Newton's 1728 book *A Treatise of the System of the World*. According to Galileo's concept of gravitation, a dropped stone falls with constant acceleration down towards the Earth. However, Newton explains that when a stone is thrown horizontally (meaning sideways or perpendicular to Earth's gravity) it follows a curved path. "For a stone projected is by the pressure of its own weight forced out of the rectilinear path, which by the projection alone it should have pursued, and made to describe a curve line in the air; and through that crooked way is at last brought down to the ground. And the greater the velocity is with which it is projected, the farther it goes before it falls to the Earth."<sup>[14]:513</sup> Newton further reasons that if an object were "projected in an horizontal direction from the top of a high mountain" with sufficient velocity, "it would reach at last quite beyond the circumference of the Earth, and return to the mountain from which it was projected."<sup>*[citation needed]*</sup>

### Universal gravitational mass



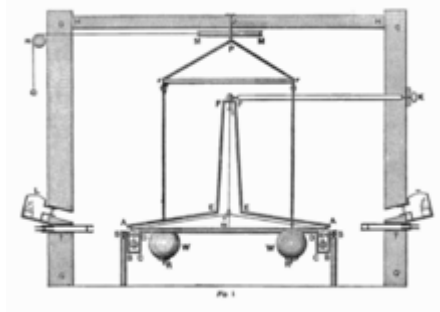
An apple experiences gravitational fields directed towards every part of the Earth; however, the sum total of these many fields produces a single gravitational field directed towards the Earth's center

In contrast to earlier theories (e.g. Celestial spheres) which stated that the heavens were made of entirely different material, Newton's theory of mass was groundbreaking partly because it introduced universal gravitational mass: every object has gravitational mass, and therefore, every object generates a gravitational field. Newton further assumed that the strength of each object's gravitational field would decrease according



to the square of the distance to that object. If a large collection of small objects were formed into a giant spherical body such as the Earth or Sun, Newton calculated the collection would create a gravitational field proportional to the total mass of the body,[14]:397 and inversely proportional to the square of the distance to the body's center.[14]:221[[note 6](#)]

For example, according to Newton's theory of universal gravitation, each carob seed produces a gravitational field. Therefore, if one were to gather an immense number of carob seeds and form them into an enormous sphere, then the gravitational field of the sphere would be proportional to the number of carob seeds in the sphere. Hence, it should be theoretically possible to determine the exact number of carob seeds that would be required to produce a gravitational field similar to that of the Earth or Sun. In fact, by [unit conversion](#) it is a simple matter of abstraction to realize that any traditional mass unit can theoretically be used to measure gravitational mass.



Vertical section drawing of Cavendish's torsion balance instrument including the building in which it was housed. The large balls were hung from a frame so they could be rotated into position next to the small balls by a pulley from outside. Figure 1 of Cavendish's paper.

Measuring gravitational mass in terms of traditional mass units is simple in principle, but extremely difficult in practice. According to Newton's theory all objects produce gravitational fields and it is theoretically possible to collect an immense number of small objects and form them into an enormous gravitating sphere. However, from a practical standpoint, the gravitational fields of small objects are extremely weak and difficult to measure. Newton's books on universal gravitation were published in the 1680s, but the first successful measurement of the Earth's mass in terms of traditional mass units, the [Cavendish experiment](#), did not occur until 1797, over a hundred years later. Cavendish found that the Earth's density was  $5.448 \pm 0.033$  times that of water.

As of 2009, the Earth's mass in kilograms is only known to around five digits of accuracy, whereas its gravitational mass is known to over nine significant figures. *[clarification needed]*

Given two objects A and B, of masses  $M_A$  and  $M_B$ , separated by a distance  $R_{AB}$ , Newton's law of gravitation states that each object exerts a gravitational force on the other, of magnitude

$$\mathbf{F}_{AB} = -GM_A M_B \frac{\widehat{R}_{AB}}{|R_{AB}|^2},$$

where  $G$  is the universal gravitational constant. The above statement may be reformulated in the following way: if  $g$  is the magnitude at a given location in a gravitational field, then the gravitational force on an object with gravitational mass  $M$  is  $\mathbf{F} = M\mathbf{g}$ .

This is the basis by which masses are determined by weighing. In simple spring scales, for example, the force  $F$  is proportional to the displacement of the spring beneath the weighing pan, as per Hooke's law, and the scales are calibrated to take  $g$  into account, allowing the mass  $M$  to be read off. Assuming the gravitational field is equivalent on both sides of the balance, a balance measures relative weight, giving the relative gravitation mass of each object.

### **Inertial mass**

*Inertial mass* is the mass of an object measured by its resistance to acceleration. The simple Classical mechanics definition of mass is slightly different than the definition in the theory of special relativity, but the essential meaning is the same.

In classical mechanics, according to Newton's second law, we say that a body has a mass  $m$  if, at any instant of time, it obeys the equation of motion

$$\mathbf{F} = m\mathbf{a},$$

where  $F$  is the resultant Force acting on the body and  $a$  is the Acceleration of the body's centre of mass. [note 7] For the moment, we will put aside the question of what "force acting on the body" actually means.

This equation illustrates how mass relates to the inertia of a body. Consider two objects with different masses. If we apply an identical force to each, the object with a bigger mass will experience a smaller acceleration, and the object with a smaller mass will experience a bigger acceleration. We might say that the larger mass exerts a greater "resistance" to changing its state of motion in response to the force.

However, this notion of applying "identical" forces to different objects brings us back to the fact that we have not really defined what a force is. We can sidestep this difficulty with the help of Newton's third law, which states that if one object exerts a force on a second object, it will experience an equal and opposite force. To be precise, suppose we have two objects of constant inertial masses  $m_1$  and  $m_2$ . We isolate the two objects from all other physical influences, so that the only forces present are the force exerted on  $m_1$  by  $m_2$ , which we denote  $F_{12}$ , and the force exerted on  $m_2$  by  $m_1$ , which we denote  $F_{21}$ . Newton's second law states that

$$\mathbf{F}_{12} = m_1 \mathbf{a}_1,$$

$$\mathbf{F}_{21} = m_2 \mathbf{a}_2,$$

where  $\mathbf{a}_1$  and  $\mathbf{a}_2$  are the accelerations of  $m_1$  and  $m_2$ , respectively. Suppose that these accelerations are non-zero, so that the forces between the two objects are non-zero. This occurs, for example, if the two objects are in the process of colliding with one another. Newton's third law then states that

$$\mathbf{F}_{12} = -\mathbf{F}_{21};$$

and thus

$$m_1 = \frac{m_2 |\mathbf{a}_2|}{|\mathbf{a}_1|}.$$

If  $|\mathbf{a}_1|$  is non-zero, the fraction is well-defined, which allows us to measure the inertial mass of  $m_1$ . In this case,  $m_2$  is our "reference" object, and we can define its mass  $m$  as (say) 1 kilogram. Then we can measure the mass of any other object in the universe by colliding it with the reference object and measuring the accelerations.

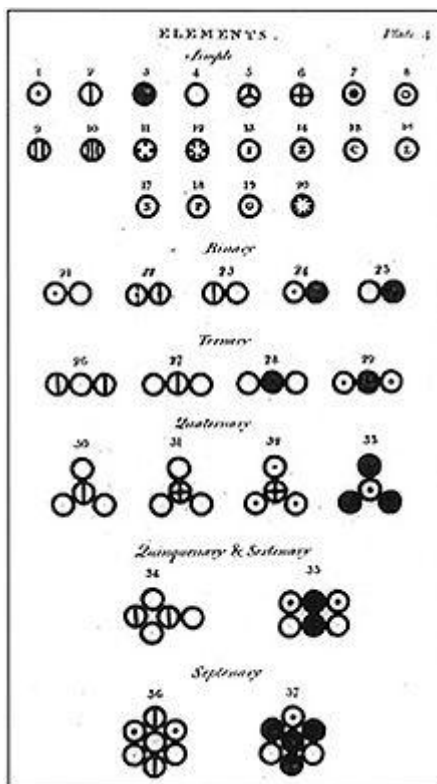
Additionally, mass relates a body's Momentum  $\mathbf{p}$  to its linear Velocity  $\mathbf{v}$ .

$$\mathbf{p} = m\mathbf{v},$$

and the body's kinetic energy  $K$  to its velocity:

$$K = \frac{1}{2} m |\mathbf{v}|^2.$$

**Atomic mass**



Various atoms and molecules as depicted in John Dalton's *A New System of Chemical Philosophy* (1808)

Main article: Atomic theory

The name atom comes from the Ancient Greek ἄτομος, which means "uncuttable", something that cannot be divided further. Despite its early origin, this philosophical assertion that matter is composed of discrete indivisible units remained very abstract, and was generally ignored. This changed in the late 18th century, when chemists uncovered increasing evidence for the existence of a law of multiple proportions. They noticed that when two or more elements combined to form a compound, the ingredients held to a fixed and definite ratio. For example, the mass ratio of nitrogen to oxygen in nitric oxide is seven eights, and ammonia has a hydrogen to nitrogen mass ratio of three fourteenths.

In order to explain the existence of these ratios, the chemist John Dalton proposed that all matter was made of tiny atoms. His first table of relative atomic weights in 1805 listed six elements: hydrogen, oxygen, nitrogen, carbon, sulfur, and phosphorus; he assigned hydrogen an atomic weight of 1. In 1815, the chemist William Prout concluded

that the hydrogen atom was in fact the fundamental mass unit from which all other atomic masses were derived.

Prout's hypothesis, however, was somewhat inaccurate. Although elemental masses were near-multiples of the hydrogen mass,(within about 1%) the discrepancies could not be ignored. The light isotope of hydrogen, for example, with a single proton, has an atomic mass of 1.007825 u. The most abundant isotope of iron has 26 protons and 30 neutrons, so one might expect its atomic mass to be 56 times that of the hydrogen atom, but in fact, its atomic mass is only 55.9383 u, which is clearly not an integer multiple of 1.007825 (In this case, iron-56 has a mass 0.89% less than 56 hydrogens).[\[note 8\]](#) Despite its shortcomings, the concepts of Prout's theory, such as atomic mass and amount, continue to play an influential role in chemistry, and the atomic mass unit continues to be the unit of choice for very small mass measurements.

### Mass in relativity

#### Special relativity

Main article: [Mass in special relativity](#)

In [special relativity](#), there are two kinds of mass: rest mass (or invariant mass),[\[note 9\]](#) and relativistic mass.(which increases with velocity) Rest mass is the Newtonian mass as measured by an observer moving along with the object.*Relativistic mass* is the total quantity of energy in a body or system divided by  $c^2$ . The two are related by the following equation:

$$m_{relative} = \gamma(m_{rest})$$

where  $\gamma$  is the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

The invariant mass of systems is the same for observers in all inertial frames, while the relativistic mass depends on the observer's Frame of reference. In order to formulate the equations of physics such that mass values do not change between observers, it is convenient to use rest mass. The rest mass of a body is also related to its energy  $E$  and the magnitude of its momentum  $p$  by the relativistic energy-momentum equation:

$$(m_{rest})c^2 = \sqrt{E_{total}^2 - (|p|c)^2}.$$

So long as the system is closed with respect to mass and energy, both kinds of mass are conserved in any given frame of reference. The conservation of mass holds even as

some types of particles are converted to others. Matter particles may be converted to types of energy which are not matter,(e.g. light, kinetic energy, and the potential energy in magnetic, electric, and other fields) but this does not affect the amount of mass. Although things like heat may not be matter, all types of energy still continue to exhibit mass.[note 10][16] Thus, mass and energy do not change into one another in relativity; rather, both are names for the same thing, and neither mass nor energy *appear* without the other.

Both rest and relativistic mass can be expressed as an energy by applying the well-known relationship  $E \equiv mc^2$ , yielding rest energy and "relativistic energy" (total system energy) respectively.

$$E_{rest} = (m_{rest})c^2$$
$$E_{total} = (m_{relative})c^2$$

The "relativistic" mass and energy concepts are related to their "rest" counterparts, but they do not have the same value as their rest counterparts in systems where there is a net momentum. Because the relativistic mass is proportional to the energy, it has gradually fallen into disuse among physicists.[17] There is disagreement over whether the concept remains pedagogically useful.[18][19][20]

In bound systems, the binding energy must often be subtracted from the mass of the unbound system, because binding energy commonly leaves the system at the time it is bound. Mass is not conserved in this process because the system is not closed during the binding process. For example, the binding energy of atomic nuclei is often lost in the form of gamma rays when the nuclei are formed, leaving nuclides which have less mass than the free particles (nucleons) of which they are composed.

### **General relativity**

Main article: Mass in general relativity

In general relativity, the equivalence principle is any of several related concepts dealing with the equivalence of gravitational and inertial mass. At the core of this assertion is Albert Einstein's idea that the gravitational force as experienced locally while standing on a massive body (such as the Earth) is the same as the pseudo-force experienced by an observer in a non-Inertial (accelerated) frame of reference.

However, it turns out that it is impossible to find an objective general definition for the concept of invariant mass in general relativity. At the core of the problem is the non-linearity of the Einstein field equations, which makes it impossible to write the gravitational field energy as part of the Stress-energy tensor in a way that is invariant

for all observers. For a given observer, this can be achieved by the Stress-energy-momentum pseudotensor. [21]

### Mass in quantum physics

In Classical mechanics, the inert mass of a particle appears in the Euler-Lagrange equation as a parameter  $m$ ,

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_i} \right) = m \ddot{x}_i$$

After quantization, replacing the position vector  $x$  with a wave function, the parameter  $m$  appears in the kinetic energy operator,

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left( -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right) \Psi(\mathbf{r}, t)$$

In the ostensibly covariant (relativistically invariant) Dirac equation, and in natural units, this becomes

$$(-i\gamma^\mu \partial_\mu + m)\psi = 0$$

Where the "mass" parameter  $m$  is now simply a constant associated with the quantum described by the wave function  $\psi$ .

In the Standard Model of particle physics as developed in the 1960s, there is the proposal that this term arises from the coupling of the field  $\psi$  to an additional field  $\Phi$ , the so-called Higgs field. In the case of fermions, the Higgs mechanism results in the replacement of the term  $m\psi$  in the Lagrangian with  $G_\psi \psi \phi \psi$ . This shifts the explanandum of the value for the mass of each elementary particle to the value of the unknown couplings  $G_\psi$ . The tentatively confirmed discovery of a massive Higgs boson is regarded as a strong confirmation of this theory. But there is indirect evidence for the reality of the Electroweak symmetry breaking as described by the Higgs mechanism, and the non-existence of Higgs bosons would indicate a "Higgsless" description of this mechanism.

### References

1. ^ "New Quantum Theory Separates Gravitational and Inertial Mass". MIT Technology Review. 14 Jun 2010. Retrieved 3 Dec 2013.
2. ^ Jacob Aron (10 Jan 2013). "Most fundamental clock ever could redefine kilogram". NewScientist. Retrieved 17 Dec 2013.

3. <sup>^</sup> W. Rindler (2006). *Relativity: Special, General, And Cosmological*. Oxford University Press. pp. 16–18. ISBN 0–19–856731–6.
4. <sup>^</sup> Kane, Gordon (September 4, 2008). "The Mysteries of Mass". *Scientific American* (Nature America, Inc.). pp. 32–39. Retrieved 2013–07–05.
5. <sup>^</sup> Rindler, W. (2006). *Relativity: Special, General, And Cosmological*. Oxford University Press. p. 22. ISBN 0–19–856731–6.
6. <sup>^</sup> Eötvös, R. V.; Pekár, D.; Fekete, E. (1922). "Beiträge zum Gesetz der Proportionalität von Trägheit und Gravität". *Annalen der Physik* **68**: 11.
7. <sup>^</sup> Drake, S. (1979). "Galileo's Discovery of the Law of Free Fall". *Scientific American* **228** (5): 84–92. Bibcode:1973SciAm.228e..84D. doi:10.1038/scientificamerican0573–84.
8. <sup>^</sup> Galileo, G. (1632). *Dialogue Concerning the Two Chief World Systems*.
9. <sup>^</sup> Galileo, G. (1638). *Discorsi e Dimostrazioni Matematiche, Intorno à Due Nuove Scienze* **213**. Louis Elsevier.
  - Translated in Crew, H.; de Salvio, A., eds. (1954). *Mathematical Discourses and Demonstrations, Relating to Two New Sciences*. Dover Publications. ISBN 1–275–10057–0.
  - Also available in Hawking, S., ed. (2002). *On the Shoulders of Giants*. Running Press. pp. 534–535. ISBN 0–7624–1348–4.
10. <sup>^</sup> Hooke, R. (1674). *An attempt to prove the motion of the earth from observations*. Royal Society.
11. <sup>^</sup> Turnbull, H. W., ed. (1960). *Correspondence of Isaac Newton, Volume 2 (1676–1687)*. Cambridge University Press. p. 297.
12. <sup>^</sup> Hawking, S., ed. (2005). *Principia*. Running Press. pp. 15 ff. ISBN 978–0–7624–2022–3.
13. <sup>^</sup> Whiteside, D. T., ed. (2008). *The Mathematical Papers of Isaac Newton, Volume VI (1684–1691)*. Cambridge University Press. ISBN 978–0–521–04585–8. Retrieved 12 March 2011.
14. <sup>^</sup> <sup>a</sup> <sup>b</sup> <sup>c</sup> <sup>d</sup> Sir Isaac Newton; N. W. Chittenden (1848). *Newton's Principia: The mathematical principles of natural philosophy*. D. Adee. Retrieved 12 March 2011.
15. <sup>^</sup> Cuk, M. (January 2003). "Curious About Astronomy: How do you measure a planet's mass?". *Ask an Astronomer*. Retrieved 2011–03–12.



16. <sup>^</sup> Taylor, E. F.; Wheeler, J. A. (1992). *Spacetime Physics*. W. H. Freeman. pp. 248–149. ISBN 0-7167-2327-1.
17. <sup>^</sup> G. Oas (2005). "On the Abuse and Use of Relativistic Mass". [arXiv:physics/0504110](https://arxiv.org/abs/physics/0504110) [physics.ed-ph].
18. <sup>^</sup> Okun, L. B. (1989). "The Concept of Mass". *Physics Today* **42** (6): 31–36. Bibcode:1989PhT....42f..31O.doi:10.1063/1.881171.
19. <sup>^</sup> Rindler, W.; Vandyck, M. A.; Murugesan, P.; Ruschin, S.; Sauter, C.; Okun, L. B. (1990). "Putting to Rest Mass Misconceptions". *Physics Today* **43** (5): 13–14, 115, 117. Bibcode:1990PhT....43e..13R.doi:10.1063/1.2810555.
20. <sup>^</sup> Sandin, T. R. (1991). "In Defense of Relativistic Mass". *American Journal of Physics* **59** (11): 1032. Bibcode:1991AmJPh..59.1032S.doi:10.1119/1.16642.
21. <sup>^</sup> Misner, C. W.; Thorne, K. S.; Wheeler, J. A. (1973). *Gravitation*. W. H. Freeman. p. 466. ISBN 978-0-7167-0344-0.

Source: <http://wateralkalinemachine.com/quantum-mechanics/?wiki-maping=Mass>