Quantum Mechanics_ fractional quantum mechanics

In physics, **fractional quantum mechanics** is a generalization of standard Quantum mechanics, which naturally comes out when the Brownian-like quantum paths substitute with the Lévy-like ones in the Feynman path integral. It has been discovered by Nick Laskin who coined the term *fractional quantum mechanics*.[1]

Fundamentals

Standard quantum mechanics can be approached in three different ways: thematrix mechanics, the Schrödinger equation and the Feynman path integral.

The Feynman path integral[2] is the path integral over Brownian-like quantum-mechanical paths. Fractional quantum mechanics has been discovered by Nick Laskin (1999) as a result of expanding the Feynman path integral, from the Brownian-like to the Lévy-like quantum mechanical paths. A path integral over the Lévy-like quantum-mechanical paths results in a generalization of Quantum mechanics.[3] If the Feynman path integral leads to the well known Schrödinger equation, then the path integral over Lévy trajectories leads to the fractional Schrödinger equation.[4] The Lévy process is characterized by the Lévy index α, $0 < \alpha \leq 2$. At the special case when $\alpha = 2$ the Lévy process becomes the process ofBrownian motion. The fractional Schrödinger equation includes a space derivativeof fractional order α instead of the second order ($\alpha = 2$) space derivative in the standard Schrödinger equation. Thus, the fractional Schrödinger equation is afractional differential equation in accordance with modern terminology.[5] This is the main point of the term fractional Schrödinger equation or a more general term *fractional quantum mechanics*. As mentioned above, at $\alpha = 2$ the Lévy motion becomes Brownian motion. Thus, fractional quantum mechanics includes standard quantum mechanics as a particular case at $\alpha = 2$. The quantum-mechanical path integral over the Lévy paths at $\alpha = 2$ becomes the well-knownFeynman path integral and the fractional Schrödinger equation becomes the well-known Schrödinger equation.

Fractional Schrödinger equation

The fractional Schrödinger equation discovered by Nick Laskin has the following form (see, Refs.[1,3,4])

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = D_\alpha(-\hbar^2\Delta)^{\alpha/2}\psi(\mathbf{r},t) + V(\mathbf{r},t)\psi(\mathbf{r},t)\ ,$$

using the standard definitions:

- **r** is the 3-dimensional position vector,

- $\hbar$ is the reduced Planck constant,
- $\psi(\mathbf{r}, t)$ is the wavefunction, which is the quantum mechanical function that determines the probability amplitude for the particle to have a given position $\mathbf{r}$at any given time t,
- $V(\mathbf{r}, t)$ is a potential energy,
- $\Delta = \partial^2/\partial\mathbf{r}^2$ is the Laplace operator.

Further,

- D_α is a scale constant with physical dimension $[D_\alpha]$ = $[\text{energy}]^{1-\alpha}\cdot[\text{length}]^\alpha[\text{time}]^{-\alpha}$, at $\alpha = 2$, $D_2 = 1/2m$, where m is a particle mass,
- the operator $(-\hbar^2\Delta)^{\alpha/2}$ is the 3-dimensional fractional quantum Riesz derivative defined by (see, Ref.[4]);

$$(-\hbar^2\Delta)^{\alpha/2}\psi(\mathbf{r},t) = \frac{1}{(2\pi\hbar)^3}\int d^3p e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}|\mathbf{p}|^\alpha\varphi(\mathbf{p},t),$$

Here, the wave functions in the position and momentum spaces; $\psi(\mathbf{r},t)$ and $\varphi(\mathbf{p},t)$ are related each other by the 3-dimensional Fourier transforms:

$$\psi(\mathbf{r},t) = \frac{1}{(2\pi\hbar)^3}\int d^3p e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}\varphi(\mathbf{p},t), \qquad \varphi(\mathbf{p},t) = \int d^3r e^{-i\mathbf{p}\cdot\mathbf{r}/\hbar}\psi(\mathbf{r},t).$$

The index α in the fractional Schrödinger equation is the Lévy index, $1 < \alpha \leq 2$.

See also

- Quantum mechanics
- matrix mechanics
- Fractional calculus
- Fractional dynamics
- fractional Schrödinger equation
- Non-linear Schrödinger equation
- Path integral formulation
- Relation between Schrödinger's equation and the path integral formulation of quantum mechanics
- Lévy process

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