

Quantum Mechanics_force

In physics, a **force** is any external effort that causes an object to undergo a certain change, either concerning its movement, direction, or geometrical construction. In other words, a force can cause an object with Mass to change its Velocity (which includes to begin moving from a state of rest), i.e., to accelerate, or a flexible object to deform, or both. Force can also be described by intuitive concepts such as a push or a pull. A force has both magnitude and direction, making it a vector quantity. It is measured in the SI unit of newtons and represented by the symbol **F**.

The original form of Newton's second law states that the net force acting upon an object is equal to the rate at which its Momentum changes with time. If the mass of the object is constant, this law implies that the Acceleration of an object is directly proportional to the net force acting on the object, is in the direction of the net force, and is inversely proportional to the Mass of the object. As a formula, this is expressed as:

$$\vec{F} = m\vec{a}$$

where the arrows imply a vector quantity possessing both magnitude and direction.

Related concepts to force include: thrust, which increases the velocity of an object; drag, which decreases the velocity of an object; and Torque which produces changes in rotational speed of an object. In an extended body, each part usually applies forces on the adjacent parts; the distribution of such forces through the body is the so-called mechanical stress. Pressure is a simple type of stress. Stress usually causes deformation of solid materials, or flow in fluids.

Development of the concept

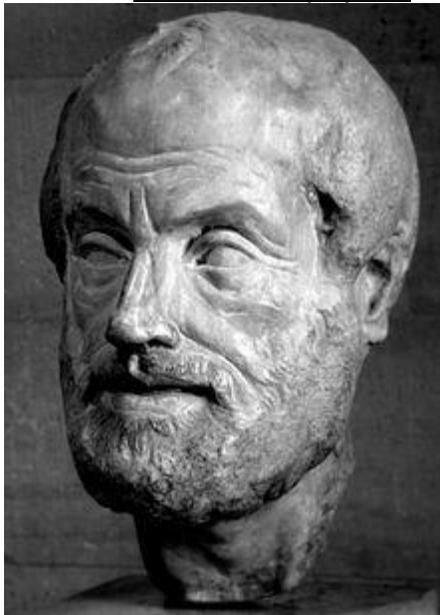
Philosophers in antiquity used the concept of force in the study of stationary and moving objects and simple machines, but thinkers such as Aristotle and Archimedes retained fundamental errors in understanding force. In part this was due to an incomplete understanding of the sometimes non-obvious force of Friction, and a consequently inadequate view of the nature of natural motion.[1] A fundamental error was the belief that a force is required to maintain motion, even at a constant velocity. Most of the previous misunderstandings about motion and force

were eventually corrected by Sir Isaac Newton; with his mathematical insight, he formulated laws of motion that were not improved-on for nearly three hundred years.[2] By the early 20th century, Einstein developed a theory of relativity that correctly predicted the action of forces on objects with increasing momenta near the speed of light, and also provided insight into the forces produced by gravitation and Inertia.

With modern insights into quantum mechanics and technology that can accelerate particles close to the speed of light, particle physics has devised a Standard Model to describe forces between particles smaller than atoms. The Standard Model predicts that exchanged particles called gauge bosons are the fundamental means by which forces are emitted and absorbed. Only four main interactions are known: in order of decreasing strength, they are: strong, electromagnetic, weak, and gravitational. [3]:2-10 [4]:79 High-energy particle physics observations made during the 1970s and 1980s confirmed that the weak and electromagnetic forces are expressions of a more fundamental electroweak interaction. [5]

Pre-Newtonian concepts

See also: Aristotelian physics and Theory of impetus



Aristotle famously described a force as anything that causes an object to undergo "unnatural motion"

Since antiquity the concept of force has been recognized as integral to the functioning of each of the simple machines. The mechanical advantage given by a simple machine

allowed for less force to be used in exchange for that force acting over a greater distance for the same amount of work. Analysis of the characteristics of forces ultimately culminated in the work of Archimedes who was especially famous for formulating a treatment of buoyant forces inherent in fluids.^[1]

Aristotle provided a philosophical discussion of the concept of a force as an integral part of Aristotelian cosmology. In Aristotle's view, the terrestrial sphere contained four elements that come to rest at different "natural places" therein. Aristotle believed that motionless objects on Earth, those composed mostly of the elements earth and water, to be in their natural place on the ground and that they will stay that way if left alone. He distinguished between the innate tendency of objects to find their "natural place" (e.g., for heavy bodies to fall), which led to "natural motion", and unnatural or forced motion, which required continued application of a force.^[6] This theory, based on the everyday experience of how objects move, such as the constant application of a force needed to keep a cart moving, had conceptual trouble accounting for the behavior of projectiles, such as the flight of arrows. The place where the archer moves the projectile was at the start of the flight, and while the projectile sailed through the air, no discernible efficient cause acts on it. Aristotle was aware of this problem and proposed that the air displaced through the projectile's path carries the projectile to its target. This explanation demands a continuum like air for change of place in general.^[7]

Aristotelian physics began facing criticism in Medieval science, first by John Philoponus in the 6th century.

The shortcomings of Aristotelian physics would not be fully corrected until the 17th century work of Galileo Galilei, who was influenced by the late Medieval idea that objects in forced motion carried an innate force of impetus. Galileo constructed an experiment in which stones and cannonballs were both rolled down an incline to disprove the Aristotelian theory of motion early in the 17th century. He showed that the bodies were accelerated by gravity to an extent which was independent of their mass and argued that objects retain their Velocity unless acted on by a force, for example Friction.^[8]

Newtonian mechanics

Main article: Newton's laws of motion

Sir Isaac Newton sought to describe the motion of all objects using the concepts of inertia and force, and in doing so he found that they obey certain conservation laws. In 1687, Newton went on to publish his thesis *Philosophiæ Naturalis Principia Mathematica*.^{[2][9]} In this work Newton set out three laws of motion that to this day are the way forces are described in physics.^[9]

First law

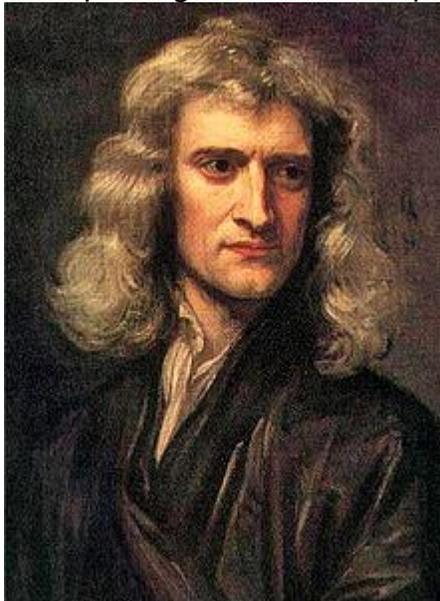
Main article: Newton's first law

Newton's First Law of Motion states that objects continue to move in a state of constant velocity unless acted upon by an external net force or *resultant force*.^[9] This law is an extension of Galileo's insight that constant velocity was associated with a lack of net force (see a more detailed description of this below). Newton proposed that every object with mass has an innate Inertia that functions as the fundamental equilibrium "natural state" in place of the Aristotelian idea of the "natural state of rest". That is, the first law contradicts the intuitive Aristotelian belief that a net force is required to keep an object moving with constant velocity. By making *rest* physically indistinguishable from *non-zero constant velocity*, Newton's First Law directly connects inertia with the concept of relative velocities. Specifically, in systems where objects are moving with different velocities, it is impossible to determine which object is "in motion" and which object is "at rest". In other words, to phrase matters more technically, the laws of physics are the same in every inertial frame of reference, that is, in all frames related by a Galilean transformation.

For instance, while traveling in a moving vehicle at a constant Velocity, the laws of physics do not change from being at rest. A person can throw a ball straight up in the air and catch it as it falls down without worrying about applying a force in the direction the vehicle is moving. This is true even though another person who is observing the moving vehicle pass by also observes the ball follow a curving parabolic path in the same direction as the motion of the vehicle. It is the inertia of the ball associated with its constant velocity in the direction of the vehicle's motion that ensures the ball continues to move forward even as it is thrown up and falls back down. From the perspective of the person in the car, the vehicle and everything inside of it is at rest: It is the outside world that is moving with a constant speed in the opposite direction. Since there is no experiment that can distinguish whether it is the vehicle that is at rest or the outside world that is at rest, the two situations are considered to be physically

indistinguishable. Inertia therefore applies equally well to constant velocity motion as it does to rest.

The concept of inertia can be further generalized to explain the tendency of objects to continue in many different forms of constant motion, even those that are not strictly constant velocity. The rotational inertia of planet Earth is what fixes the constancy of the length of a day and the length of a year. Albert Einstein extended the principle of inertia further when he explained that reference frames subject to constant acceleration, such as those free-falling toward a gravitating object, were physically equivalent to inertial reference frames. This is why, for example, astronauts experience weightlessness when in free-fall orbit around the Earth, and why Newton's Laws of Motion are more easily discernible in such environments. If an astronaut places an object with mass in mid-air next to himself, it will remain stationary with respect to the astronaut due to its inertia. This is the same thing that would occur if the astronaut and the object were in intergalactic space with no net force of gravity acting on their shared reference frame. This principle of equivalence was one of the foundational underpinnings for the development of the general theory of relativity.^[10]



Though Sir Isaac Newton's most famous equation is $\vec{F}=m\vec{a}$, he actually wrote down a different form for his second law of motion that did not use differential calculus.

Second law

Main article: [Newton's second law](#)

A modern statement of Newton's Second Law is a vector differential equation:[\[Note 1\]](#)

$$\vec{F} = \frac{d\vec{p}}{dt},$$

where \vec{p} is the Momentum of the system, and \vec{F} is the net (vector sum) force. In equilibrium, there is zero *net* force by definition, but (balanced) forces may be present nevertheless. In contrast, the second law states an *unbalanced* force acting on an object will result in the object's momentum changing over time.[\[9\]](#)

By the definition of momentum,

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt},$$

where m is the Mass and \vec{v} is the Velocity.[\[3\]](#):9-1,9-2

Newton's second law applies only to a system of constant mass,[\[Note 2\]](#) and hence m may be moved outside the derivative operator. The equation then becomes

$$\vec{F} = m \frac{d\vec{v}}{dt}.$$

By substituting the definition of Acceleration, the algebraic version of Newton's second law is derived:

$$\vec{F} = m\vec{a}.$$

Newton never explicitly stated the formula in the reduced form above.[\[11\]](#)

Newton's Second Law asserts the direct proportionality of acceleration to force and the inverse proportionality of acceleration to mass. Accelerations can be defined through kinematic measurements. However, while kinematics are well-described through reference frame analysis in advanced physics, there are still deep questions that remain as to what is the proper definition of mass. General relativity offers an equivalence between space-time and mass, but lacking a coherent theory of quantum gravity, it is unclear as to how or whether this connection is relevant on microscales. With some justification, Newton's second law can be taken as a quantitative definition of *mass* by writing the law as an equality; the relative units of force and mass then are fixed.

The use of Newton's Second Law as a *definition* of force has been disparaged in some of the more rigorous textbooks,[\[3\]](#):12-1[\[4\]](#):59[\[12\]](#) because it is essentially a mathematical truism. Notable physicists, philosophers and mathematicians who have sought a more explicit definition of the concept of force include Ernst Mach,Clifford Truesdell[\[citation needed\]](#) and Walter Noll.[\[13\]](#)[\[14\]](#)

Newton's Second Law can be used to measure the strength of forces. For instance, knowledge of the masses of planets along with the accelerations of their orbits allows scientists to calculate the gravitational forces on planets.

Third law

Main article: Newton's third law

Newton's Third Law is a result of applying symmetry to situations where forces can be attributed to the presence of different objects. The third law means that all forces are *interactions* between different bodies,^{[15][Note 3]} and thus that there is no such thing as a unidirectional force or a force that acts on only one body. Whenever a first body exerts a force F on a second body, the second body exerts a force $-F$ on the first body. F and $-F$ are equal in magnitude and opposite in direction. This law is sometimes referred to as the action-reaction law, with F called the "action" and $-F$ the "reaction". The action and the reaction are simultaneous:

$$\vec{F}_{1,2} = -\vec{F}_{2,1}.$$

If object 1 and object 2 are considered to be in the same system, then the net force on the system due to the interactions between objects 1 and 2 is zero since

$$\begin{aligned}\vec{F}_{1,2} + \vec{F}_{2,1} &= 0 \\ \sum \vec{F} &= 0.\end{aligned}$$

This means that in a closed system of particles, there are no internal forces that are unbalanced. That is, the action-reaction force shared between any two objects in a closed system will not cause the center of mass of the system to accelerate. The constituent objects only accelerate with respect to each other, the system itself remains unaccelerated. Alternatively, if an external force acts on the system, then the center of mass will experience an acceleration proportional to the magnitude of the external force divided by the mass of the system.^{[3]:19-1[4]}

Combining Newton's Second and Third Laws, it is possible to show that the linear momentum of a system is conserved. Using

$$\vec{F}_{1,2} = \frac{d\vec{p}_{1,2}}{dt} = -\vec{F}_{2,1} = -\frac{d\vec{p}_{2,1}}{dt}$$

and integrating with respect to time, the equation:

$$\Delta\vec{p}_{1,2} = -\Delta\vec{p}_{2,1}$$

is obtained. For a system which includes objects 1 and 2,

$$\sum \Delta\vec{p} = \Delta\vec{p}_{1,2} + \Delta\vec{p}_{2,1} = 0$$

which is the conservation of linear momentum.[16] Using the similar arguments, it is possible to generalize this to a system of an arbitrary number of particles. This shows that exchanging momentum between constituent objects will not affect the net momentum of a system. In general, as long as all forces are due to the interaction of objects with mass, it is possible to define a system such that net momentum is never lost nor gained.[3][4]

Special theory of relativity

In the special theory of relativity, mass and Energy are equivalent (as can be seen by calculating the work required to accelerate an object). When an object's velocity increases, so does its energy and hence its mass equivalent (inertia). It thus requires more force to accelerate it the same amount than it did at a lower velocity. Newton's Second Law

$$\vec{F} = d\vec{p}/dt$$

remains valid because it is a mathematical definition.[17]:855-876 But in order to be conserved, relativistic momentum must be redefined as:

$$\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - v^2/c^2}}$$

where

v is the velocity and

c is the speed of light

m_0 is the rest mass.

The relativistic expression relating force and acceleration for a particle with constant non-zero rest mass m moving in the x direction is:

$$F_x = \gamma^3 m a_x$$

$$F_y = \gamma m a_y$$

$$F_z = \gamma m a_z$$

where the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \quad [18]$$

In the early history of relativity, the expressions $\gamma^3 m$ and γm were called longitudinal and transverse mass. Relativistic force does not produce a constant acceleration, but an ever decreasing acceleration as the object approaches the speed of light. Note

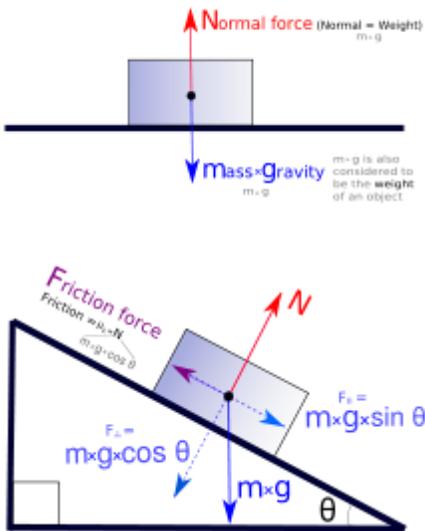
that γ is undefined for an object with a non-zero rest mass at the speed of light, and the theory yields no prediction at that speed.

One can, however, restore the form of

$$F^\mu = mA^\mu$$

for use in relativity through the use of four-vectors. This relation is correct in relativity when F^μ is the four-force, m is the invariant mass, and A^μ is the four-acceleration. [19]

Descriptions



Diagrams of a block on a flat surface and an inclined plane. Forces are resolved and added together to determine their magnitudes and the net force.

Since forces are perceived as pushes or pulls, this can provide an intuitive understanding for describing forces. [2] As with other physical concepts (e.g. temperature), the intuitive understanding of forces is quantified using precise operational definitions that are consistent with direct observations and compared to a standard measurement scale. Through experimentation, it is determined that laboratory measurements of forces are fully consistent with the conceptual definition of force offered by Newtonian mechanics.

Forces act in a particular direction and have sizes dependent upon how strong the push or pull is. Because of these characteristics, forces are classified as "vector quantities". This means that forces follow a different set of mathematical rules than physical quantities that do not have direction (denoted scalar quantities). For example,

when determining what happens when two forces act on the same object, it is necessary to know both the magnitude and the direction of both forces to calculate the result. If both of these pieces of information are not known for each force, the situation is ambiguous. For example, if you know that two people are pulling on the same rope with known magnitudes of force but you do not know which direction either person is pulling, it is impossible to determine what the acceleration of the rope will be. The two people could be pulling against each other as in tug of war or the two people could be pulling in the same direction. In this simple one-dimensional example, without knowing the direction of the forces it is impossible to decide whether the net force is the result of adding the two force magnitudes or subtracting one from the other. Associating forces with vectors avoids such problems. Historically, forces were first quantitatively investigated in conditions of static equilibrium where several forces canceled each other out. Such experiments demonstrate the crucial properties that forces are additive vector quantities: they have magnitude and direction.[2] When two forces act on a point particle, the resulting force, the *resultant* (also called the *net force*), can be determined by following the parallelogram rule of vector addition: the addition of two vectors represented by sides of a parallelogram, gives an equivalent resultant vector which is equal in magnitude and direction to the transversal of the parallelogram.[3][4] The magnitude of the resultant varies from the difference of the magnitudes of the two forces to their sum, depending on the angle between their lines of action. However, if the forces are acting on an extended body, their respective lines of application must also be specified in order to account for their effects on the motion of the body.

Free-body diagrams can be used as a convenient way to keep track of forces acting on a system. Ideally, these diagrams are drawn with the angles and relative magnitudes of the force vectors preserved so that graphical vector addition can be done to determine the net force.[20]

As well as being added, forces can also be resolved into independent components at right angles to each other. A horizontal force pointing northeast can therefore be split into two forces, one pointing north, and one pointing east. Summing these component forces using vector addition yields the original force. Resolving force vectors into components of a set of basis vectors is often a more mathematically clean way to describe forces than using magnitudes and directions.[21] This is because,

for orthogonal components, the components of the vector sum are uniquely determined by the scalar addition of the components of the individual vectors. Orthogonal components are independent of each other because forces acting at ninety degrees to each other have no effect on the magnitude or direction of the other. Choosing a set of orthogonal basis vectors is often done by considering what set of basis vectors will make the mathematics most convenient. Choosing a basis vector that is in the same direction as one of the forces is desirable, since that force would then have only one non-zero component. Orthogonal force vectors can be three-dimensional with the third component being at right-angles to the other two.[3][4]

Equilibrium

Equilibrium occurs when the resultant force acting on a point particle is zero (that is, the vector sum of all forces is zero). When dealing with an extended body, it is also necessary that the net torque in it is 0.

There are two kinds of equilibrium: static equilibrium and dynamic equilibrium.

Static

Main articles: Statics and static equilibrium

Static equilibrium was understood well before the invention of classical mechanics. Objects which are at rest have zero net force acting on them.[22]

The simplest case of static equilibrium occurs when two forces are equal in magnitude but opposite in direction. For example, an object on a level surface is pulled (attracted) downward toward the center of the Earth by the force of gravity. At the same time, surface forces resist the downward force with equal upward force (called the normal force). The situation is one of zero net force and no acceleration.[2]

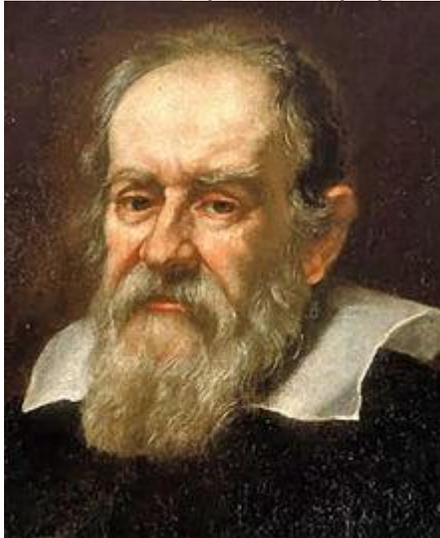
Pushing against an object on a frictional surface can result in a situation where the object does not move because the applied force is opposed by static friction, generated between the object and the table surface. For a situation with no movement, the static friction force *exactly* balances the applied force resulting in no acceleration. The static friction increases or decreases in response to the applied force up to an upper limit determined by the characteristics of the contact between the surface and the object.[2]

A static equilibrium between two forces is the most usual way of measuring forces, using simple devices such as weighing scales and spring balances. For example, an object suspended on a vertical spring scale experiences the force of gravity acting on the object balanced by a force applied by the "spring reaction force" which equals the

object's weight. Using such tools, some quantitative force laws were discovered: that the force of gravity is proportional to volume for objects of constant density (widely exploited for millennia to define standard weights); Archimedes' principle for buoyancy; Archimedes' analysis of the lever; Boyle's law for gas pressure; and Hooke's law for springs. These were all formulated and experimentally verified before Isaac Newton expounded his Three Laws of Motion.^{[2][3][4]}

Dynamic

Main article: Dynamics (physics)



Galileo Galilei was the first to point out the inherent contradictions contained in Aristotle's description of forces.

Dynamic equilibrium was first described by Galileo who noticed that certain assumptions of Aristotelian physics were contradicted by observations and logic. Galileo realized that simple velocity addition demands that the concept of an "absolute rest frame" did not exist. Galileo concluded that motion in a constant Velocity was completely equivalent to rest. This was contrary to Aristotle's notion of a "natural state" of rest that objects with mass naturally approached. Simple experiments showed that Galileo's understanding of the equivalence of constant velocity and rest were correct. For example, if a mariner dropped a cannonball from the crow's nest of a ship moving at a constant velocity, Aristotelian physics would have the cannonball fall straight down while the ship moved beneath it. Thus, in an Aristotelian universe, the falling cannonball would land behind the foot of the mast of a moving ship. However, when this experiment is actually conducted, the cannonball

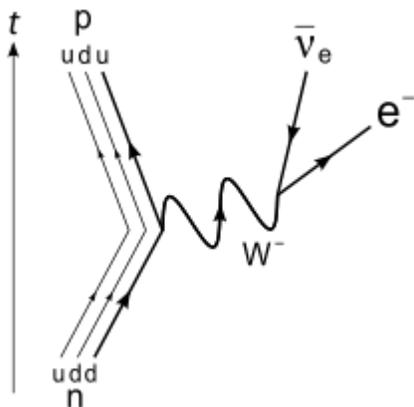
always falls at the foot of the mast, as if the cannonball knows to travel with the ship despite being separated from it. Since there is no forward horizontal force being applied on the cannonball as it falls, the only conclusion left is that the cannonball continues to move with the same velocity as the boat as it falls. Thus, no force is required to keep the cannonball moving at the constant forward velocity.[8]

Moreover, any object traveling at a constant velocity must be subject to zero net force (resultant force). This is the definition of dynamic equilibrium: when all the forces on an object balance but it still moves at a constant velocity.

A simple case of dynamic equilibrium occurs in constant velocity motion across a surface with kinetic friction. In such a situation, a force is applied in the direction of motion while the kinetic friction force exactly opposes the applied force. This results in zero net force, but since the object started with a non-zero velocity, it continues to move with a non-zero velocity. Aristotle misinterpreted this motion as being caused by the applied force. However, when kinetic friction is taken into consideration it is clear that there is no net force causing constant velocity motion.[3][4]

Feynman diagrams

Main article: Feynman diagrams



Feynman diagram for the decay of a neutron into a proton. The W boson is between two vertices indicating a repulsion.

In modern particle physics, forces and the acceleration of particles are explained as a mathematical by-product of exchange of momentum-carrying gauge bosons. With the development of quantum field theory and General relativity, it was realized that force is a redundant concept arising from conservation of momentum (4-momentum in

relativity and momentum of virtual particles in quantum electrodynamics). The conservation of momentum, can be directly derived from homogeneity (=shift symmetry) of Space and so is usually considered more fundamental than the concept of a force. Thus the currently known fundamental forces are considered more accurately to be "fundamental interactions".[5]:199-128 When particle A emits (creates) or absorbs (annihilates) virtual particle B, a momentum conservation results in recoil of particle A making impression of repulsion or attraction between particles A A' exchanging by B. This description applies to all forces arising from fundamental interactions. While sophisticated mathematical descriptions are needed to predict, in full detail, the accurate result of such interactions, there is a conceptually simple way to describe such interactions through the use of Feynman diagrams. In a Feynman diagram, each matter particle is represented as a straight line (see world line) traveling through time which normally increases up or to the right in the diagram. Matter and anti-matter particles are identical except for their direction of propagation through the Feynman diagram. World lines of particles intersect at interaction vertices, and the Feynman diagram represents any force arising from an interaction as occurring at the vertex with an associated instantaneous change in the direction of the particle world lines. Gauge bosons are emitted away from the vertex as wavy lines and, in the case of virtual particle exchange, are absorbed at an adjacent vertex.[23]

The utility of Feynman diagrams is that other types of physical phenomena that are part of the general picture of fundamental interactions but are conceptually separate from forces can also be described using the same rules. For example, a Feynman diagram can describe in succinct detail how a neutron decays into an electron, proton, and neutrino, an interaction mediated by the same gauge boson that is responsible for the weak nuclear force. [23]

Fundamental models

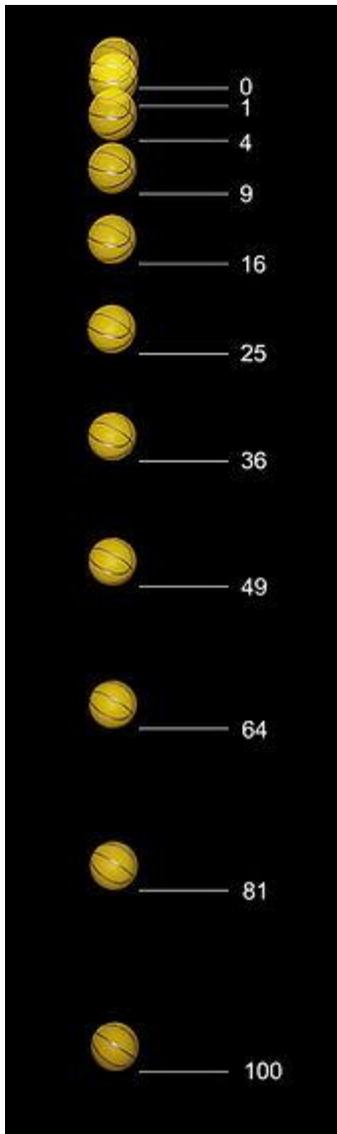
All of the forces in the universe are based on four fundamental interactions. The strong and weak forces are nuclear forces that act only at very short distances, and are responsible for the interactions between subatomic particles, including nucleons and compound nuclei. The electromagnetic force acts between electric charges, and the gravitational force acts between masses. All other forces in nature derive from these four fundamental interactions. For

example, Friction is a manifestation of the electromagnetic force acting between the atoms of two surfaces, and the Pauli exclusion principle,^[24] which does not permit atoms to pass through each other. Similarly, the forces in springs, modeled by Hooke's law, are the result of electromagnetic forces and the Exclusion Principle acting together to return an object to its Equilibrium position. Centrifugal forces are Acceleration forces which arise simply from the acceleration of rotating frames of reference.^{[3]:12–11}^{[4]:359}

The development of fundamental theories for forces proceeded along the lines of unification of disparate ideas. For example, Isaac Newton unified the force responsible for objects falling at the surface of the Earth with the force responsible for the orbits of celestial mechanics in his universal theory of gravitation. Michael Faraday and James Clerk Maxwell demonstrated that electric and magnetic forces were unified through one consistent theory of electromagnetism. In the 20th century, the development of quantum mechanics led to a modern understanding that the first three fundamental forces (all except gravity) are manifestations of matter (fermions) interacting by exchanging virtual particles called gauge bosons.^[25] This standard model of particle physics posits a similarity between the forces and led scientists to predict the unification of the weak and electromagnetic forces in electroweak theory subsequently confirmed by observation. The complete formulation of the standard model predicts an as yet unobserved Higgs mechanism, but observations such as neutrino oscillations indicate that the standard model is incomplete. A Grand Unified Theory allowing for the combination of the electroweak interaction with the strong force is held out as a possibility with candidate theories such as supersymmetry proposed to accommodate some of the outstanding unsolved problems in physics. Physicists are still attempting to develop self-consistent unification models that would combine all four fundamental interactions into a theory of everything. Einstein tried and failed at this endeavor, but currently the most popular approach to answering this question is string theory.^{[5]:212–219}

Gravity

Main article: gravity



The distance units on the right are multiples of about 12 millimetres. The basketball starts at rest. At the time of the first flash (distance zero) it is released, after which the number of units fallen is equal to the square of the number of flashes.

What we now call gravity was not identified as a universal force until the work of Isaac Newton. Before Newton, the tendency for objects to fall towards the Earth was not understood to be related to the motions of celestial objects. Galileo was instrumental in describing the characteristics of falling objects by determining that the Acceleration of every object in free-fall was constant and independent of the mass of the object. Today, this acceleration due to gravity towards the surface of the Earth is usually designated as \vec{g} and has a magnitude of about 9.81 metersper second squared

(this measurement is taken from sea level and may vary depending on location), and points toward the center of the Earth.[26] This observation means that the force of gravity on an object at the Earth's surface is directly proportional to the object's mass. Thus an object that has a mass of m will experience a force:

$$\vec{F} = m\vec{g}$$

In free-fall, this force is unopposed and therefore the net force on the object is its weight. For objects not in free-fall, the force of gravity is opposed by the reactions of their supports. For example, a person standing on the ground experiences zero net force, since his weight is balanced by a normal force exerted by the ground.[3][4]

Newton's contribution to gravitational theory was to unify the motions of heavenly bodies, which Aristotle had assumed were in a natural state of constant motion, with falling motion observed on the Earth. He proposed a law of gravity that could account for the celestial motions that had been described earlier using Kepler's laws of planetary motion. [27]

Newton came to realize that the effects of gravity might be observed in different ways at larger distances. In particular, Newton determined that the acceleration of the Moon around the Earth could be ascribed to the same force of gravity if the acceleration due to gravity decreased as an inverse square law. Further, Newton realized that the acceleration due to gravity is proportional to the mass of the attracting body.[27] Combining these ideas gives a formula that relates the mass (m_{\oplus}) and the radius (R_{\oplus}) of the Earth to the gravitational acceleration:

$$\vec{g} = -\frac{Gm_{\oplus}}{R_{\oplus}^2}\hat{r}$$

where the vector direction is given by \hat{r} , the unit vector directed outward from the center of the Earth.[9]

In this equation, a dimensional constant G is used to describe the relative strength of gravity. This constant has come to be known as Newton's Universal Gravitation Constant, [28] though its value was unknown in Newton's lifetime. Not until 1798 was Henry Cavendish able to make the first measurement of G using a torsion balance; this was widely reported in the press as a measurement of the mass of the Earth since knowing G could allow one to solve for the Earth's mass given the above equation. Newton, however, realized that since all celestial bodies followed the same laws of motion, his law of gravity had to be universal. Succinctly stated, Newton's Law of

Gravitation states that the force on a spherical object of mass m_1 due to the gravitational pull of mass m_2 is

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

where r is the distance between the two objects' centers of mass and \hat{r} is the unit vector pointed in the direction away from the center of the first object toward the center of the second object.[9]

This formula was powerful enough to stand as the basis for all subsequent descriptions of motion within the solar system until the 20th century. During that time, sophisticated methods of perturbation analysis[29] were invented to calculate the deviations of orbits due to the influence of multiple bodies on a planet, moon, comet, or asteroid. The formalism was exact enough to allow mathematicians to predict the existence of the planet Neptune before it was observed.[30]

It was only the orbit of the planet Mercury that Newton's Law of Gravitation seemed not to fully explain. Some astrophysicists predicted the existence of another planet (Vulcan) that would explain the discrepancies; however, despite some early indications, no such planet could be found. When Albert Einstein formulated his theory of General relativity (GR) he turned his attention to the problem of Mercury's orbit and found that his theory added a correction which could account for the discrepancy. This was the first time that Newton's Theory of Gravity had been shown to be less correct than an alternative.[31]

Since then, and so far, general relativity has been acknowledged as the theory which best explains gravity. In GR, gravitation is not viewed as a force, but rather, objects moving freely in gravitational fields travel under their own inertia in straight lines through curved space-time - defined as the shortest space-time path between two space-time events. From the perspective of the object, all motion occurs as if there were no gravitation whatsoever. It is only when observing the motion in a global sense that the curvature of space-time can be observed and the force is inferred from the object's curved path. Thus, the straight line path in space-time is seen as a curved line in space, and it is called the ballistic trajectory of the object. For example, a basketball thrown from the ground moves in a parabola, as it is in a uniform gravitational field. Its space-time trajectory (when the extra ct dimension is added) is almost a straight line, slightly curved (with the radius of curvature of the order of

few light-years). The time derivative of the changing momentum of the object is what we label as "gravitational force".[4]

Electromagnetic forces

Main article: electromagnetic force

The electrostatic force was first described in 1784 by Coulomb as a force which existed intrinsically between two charges.^{[17]:519} The properties of the electrostatic force were that it varied as an inverse square law directed in the radial direction, was both attractive and repulsive (there was intrinsic polarity), was independent of the mass of the charged objects, and followed the superposition principle. Coulomb's law unifies all these observations into one succinct statement.^[32]

Subsequent mathematicians and physicists found the construct of the electric field to be useful for determining the electrostatic force on an electric charge at any point in space. The electric field was based on using a hypothetical "test charge" anywhere in space and then using Coulomb's Law to determine the electrostatic force.^{[33]:4-6 to 4-}

⁸ Thus the electric field anywhere in space is defined as

$$\vec{E} = \frac{\vec{F}}{q}$$

where q is the magnitude of the hypothetical test charge.

Meanwhile, the Lorentz force of magnetism was discovered to exist between two electric currents. It has the same mathematical character as Coulomb's Law with the proviso that like currents attract and unlike currents repel. Similar to the electric field, the magnetic field can be used to determine the magnetic force on an electric current at any point in space. In this case, the magnitude of the magnetic field was determined to be

$$B = \frac{F}{I\ell}$$

where I is the magnitude of the hypothetical test current and ℓ is the length of hypothetical wire through which the test current flows. The magnetic field exerts a force on all magnets including, for example, those used in compasses. The fact that the Earth's magnetic field is aligned closely with the orientation of the Earth's axis causes compass magnets to become oriented because of the magnetic force pulling on the needle.

Through combining the definition of electric current as the time rate of change of electric charge, a rule of vector multiplication called Lorentz's Law describes the force

on a charge moving in a magnetic field.[33] The connection between electricity and magnetism allows for the description of a unified *electromagnetic force* that acts on a charge. This force can be written as a sum of the electrostatic force (due to the electric field) and the magnetic force (due to the magnetic field). Fully stated, this is the law:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

where \vec{F} is the electromagnetic force, q is the magnitude of the charge of the particle, \vec{E} is the electric field, \vec{v} is the Velocity of the particle which is crossed with the magnetic field (\vec{B}).

The origin of electric and magnetic fields would not be fully explained until 1864 when James Clerk Maxwell unified a number of earlier theories into a set of 20 scalar equations, which were later reformulated into 4 vector equations by Oliver Heaviside and Josiah Willard Gibbs. [34] These "Maxwell Equations" fully described the sources of the fields as being stationary and moving charges, and the interactions of the fields themselves. This led Maxwell to discover that electric and magnetic fields could be "self-generating" through a wave that traveled at a speed which he calculated to be the speed of light. This insight united the nascent fields of electromagnetic theory with optics and led directly to a complete description of the electromagnetic spectrum. [35]

However, attempting to reconcile electromagnetic theory with two observations, the photoelectric effect, and the nonexistence of the ultraviolet catastrophe, proved troublesome. Through the work of leading theoretical physicists, a new theory of electromagnetism was developed using quantum mechanics. This final modification to electromagnetic theory ultimately led to quantum electrodynamics (or QED), which fully describes all electromagnetic phenomena as being mediated by wave-particles known as photons. In QED, photons are the fundamental exchange particle which described all interactions relating to electromagnetism including the electromagnetic force. [Note 4]

It is a common misconception to ascribe the stiffness and rigidity of solid matter to the repulsion of like charges under the influence of the electromagnetic force. However, these characteristics actually result from the Pauli exclusion principle. [citation needed] Since electrons are fermions, they cannot occupy the same quantum mechanical state as other electrons. When the electrons in a material are densely packed together, there are not enough lower energy quantum mechanical states for

them all, so some of them must be in higher energy states. This means that it takes energy to pack them together. While this effect is manifested macroscopically as a structural force, it is technically only the result of the existence of a finite set of electron states.

Nuclear forces

Main article: [Nuclear force](#)

See also: [Strong interaction](#) and [Weak interaction](#)

There are two "nuclear forces" which today are usually described as interactions that take place in quantum theories of particle physics. The [strong nuclear force](#)^{[17]:940} is the force responsible for the structural integrity of [atomic nuclei](#) while the [weak nuclear force](#)^{[17]:951} is responsible for the decay of certain [nucleons](#) into [leptons](#) and other types of [hadrons](#).^{[3][4]}

The strong force is today understood to represent the [interactions](#) between [quarks](#) and [gluons](#) as detailed by the theory of [quantum chromodynamics](#) (QCD).^[36] The strong force is the [fundamental force](#) mediated by [gluons](#), acting upon quarks, [antiquarks](#), and the [gluons](#) themselves. The (aptly named) strong interaction is the "strongest" of the four fundamental forces.

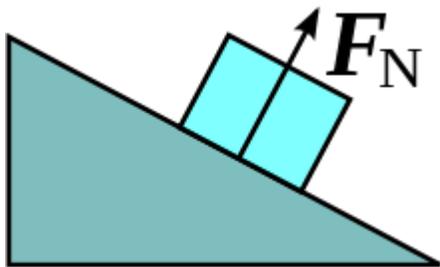
The strong force only acts *directly* upon elementary particles. However, a residual of the force is observed between [hadrons](#) (the best known example being the force that acts between [nucleons](#) in atomic nuclei) as the [Nuclear force](#). Here the strong force acts indirectly, transmitted as gluons which form part of the virtual pi and rho [mesons](#) which classically transmit the nuclear force (see this topic for more). The failure of many searches for [free quarks](#) has shown that the elementary particles affected are not directly observable. This phenomenon is called [color confinement](#).

The weak force is due to the exchange of the heavy [W and Z bosons](#). Its most familiar effect is [beta decay](#) (of neutrons in atomic nuclei) and the associated [radioactivity](#). The word "weak" derives from the fact that the field strength is some 10^{13} times less than that of the [strong force](#). Still, it is stronger than gravity over short distances. A consistent electroweak theory has also been developed which shows that electromagnetic forces and the weak force are indistinguishable at a temperatures in excess of approximately 10^{15} [kelvins](#). Such temperatures have been probed in modern [particle accelerators](#) and show the conditions of the [universe](#) in the early moments of the [Big Bang](#).

Non-fundamental forces

Some forces are consequences of the fundamental ones. In such situations, idealized models can be utilized to gain physical insight.

Normal force



F_N represents the normal force exerted on the object.

Main article: normal force

The normal force is due to repulsive forces of interaction between atoms at close contact. When their electron clouds overlap, Pauli repulsion (due to fermionic nature of electrons) follows resulting in the force which acts in a direction normal to the surface interface between two objects.^{[17]:93} The normal force, for example, is responsible for the structural integrity of tables and floors as well as being the force that responds whenever an external force pushes on a solid object. An example of the normal force in action is the impact force on an object crashing into an immobile surface.^{[3][4]}

Friction

Friction is a surface force that opposes relative motion. The frictional force is directly related to the normal force which acts to keep two solid objects separated at the point of contact. There are two broad classifications of frictional forces: static friction and kinetic friction.

The static friction force (F_{sf}) will exactly oppose forces applied to an object parallel to a surface contact up to the limit specified by the coefficient of static friction (μ_{sf}) multiplied by the normal force (F_N). In other words the magnitude of the static friction force satisfies the inequality:

$$0 \leq F_{sf} \leq \mu_{sf} F_N.$$

The kinetic friction force (F_{kf}) is independent of both the forces applied and the movement of the object. Thus, the magnitude of the force equals:

$$F_{kf} = \mu_{kf} F_N,$$

where μ_{kf} is the coefficient of kinetic friction. For most surface interfaces, the coefficient of kinetic friction is less than the coefficient of static friction.

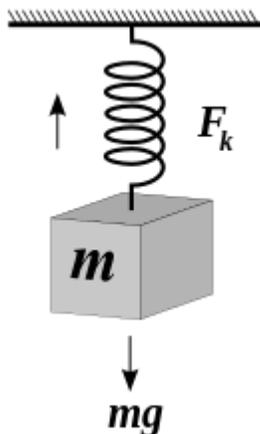
Tension

Main article: Tension (physics)

Tension forces can be modeled using ideal strings which are massless, frictionless, unbreakable, and unstretchable. They can be combined with ideal pulleys which allow ideal strings to switch physical direction. Ideal strings transmit tension forces instantaneously in action–reaction pairs so that if two objects are connected by an ideal string, any force directed along the string by the first object is accompanied by a force directed along the string in the opposite direction by the second object.[37] By connecting the same string multiple times to the same object through the use of a set–up that uses movable pulleys, the tension force on a load can be multiplied. For every string that acts on a load, another factor of the tension force in the string acts on the load. However, even though such machines allow for an increase in force, there is a corresponding increase in the length of string that must be displaced in order to move the load. These tandem effects result ultimately in the conservation of mechanical energy since the work done on the load is the same no matter how complicated the machine.[3][4][38]

Elastic force

Main articles: Elasticity (physics) and Hooke's law



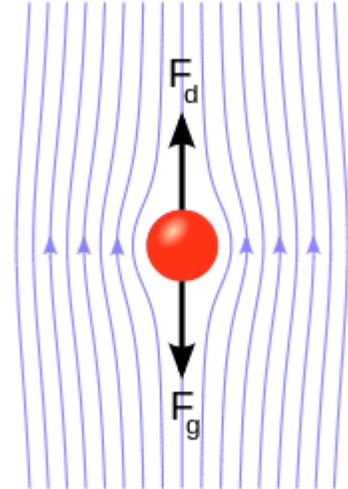
F_k is the force that responds to the load on the spring

An elastic force acts to return a spring to its natural length. An ideal spring is taken to be massless, frictionless, unbreakable, and infinitely stretchable. Such springs exert forces that push when contracted, or pull when extended, in proportion to the displacement of the spring from its equilibrium position.[39] This linear relationship was described by Robert Hooke in 1676, for whom Hooke's law is named. If Δx is the displacement, the force exerted by an ideal spring equals:

$$\vec{F} = -k\Delta\vec{x}$$

where k is the spring constant (or force constant), which is particular to the spring. The minus sign accounts for the tendency of the force to act in opposition to the applied load.[3][4]

Continuum mechanics



When the drag force (F_d) associated with air resistance becomes equal in magnitude to the force of gravity on a falling object (F_g), the object reaches a state of dynamic equilibrium at terminal velocity.

Main articles: Pressure, Drag (physics) and Stress (mechanics)

Newton's laws and Newtonian mechanics in general were first developed to describe how forces affect idealized point particles rather than three-dimensional objects. However, in real life, matter has extended structure and forces that act on one part of an object might affect other parts of an object. For situations where lattice holding together the atoms in an object is able to flow, contract, expand, or otherwise change shape, the theories of continuum mechanics describe the way forces affect the

material. For example, in extended fluids, differences in Pressure result in forces being directed along the pressure gradients as follows:

$$\frac{\vec{F}}{V} = -\vec{\nabla}P$$

where V is the volume of the object in the fluid and P is the scalar function that describes the pressure at all locations in space. Pressure gradients and differentials result in the buoyant force for fluids suspended in gravitational fields, winds in atmospheric science, and the lift associated with aerodynamics and flight.^{[3][4]}

A specific instance of such a force that is associated with dynamic pressure is fluid resistance: a body force that resists the motion of an object through a fluid due to viscosity. For so-called "Stokes' drag" the force is approximately proportional to the velocity, but opposite in direction:

$$\vec{F}_d = -b\vec{v}$$

where:

b is a constant that depends on the properties of the fluid and the dimensions of the object (usually the cross-sectional area), and

\vec{v} is the velocity of the object.^{[3][4]}

More formally, forces in continuum mechanics are fully described by a stress-tensor with terms that are roughly defined as

$$\sigma = \frac{F}{A}$$

where A is the relevant cross-sectional area for the volume for which the stress-tensor is being calculated. This formalism includes pressure terms associated with forces that act normal to the cross-sectional area (the matrix diagonals of the tensor) as well as shear terms associated with forces that act parallel to the cross-sectional area (the off-diagonal elements). The stress tensor accounts for forces that cause all strains (deformations) including also tensile stresses and compressions.^{[2][4]:133-}

¹³⁴^{[33]:38-1-38-11}

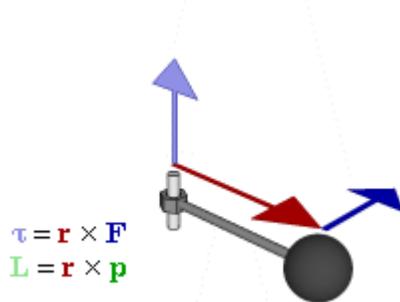
Fictitious forces

Main article: Fictitious forces

There are forces which are frame dependent, meaning that they appear due to the adoption of non-Newtonian (that is, non-inertial) reference frames. Such forces include the centrifugal force and the Coriolis force.^[40] These forces are considered fictitious because they do not exist in frames of reference that are not accelerating.^{[3][4]}

In General relativity, gravity becomes a fictitious force that arises in situations where spacetime deviates from a flat geometry. As an extension, Kaluza-Klein theory and string theory ascribe electromagnetism and the other fundamental forces respectively to the curvature of differently scaled dimensions, which would ultimately imply that all forces are fictitious.

Rotations and torque



Relationship between force (F), torque (τ), and momentum vectors (p and L) in a rotating system.

Main article: Torque

Forces that cause extended objects to rotate are associated with torques. Mathematically, the torque of a force \vec{F} is defined relative to an arbitrary reference point as the cross-product:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where

\vec{r} is the position vector of the force application point relative to the reference point.

Torque is the rotation equivalent of force in the same way that angle is the rotational equivalent for position, Angular velocity for Velocity, and Angular momentum for Momentum. As a consequence of Newton's First Law of Motion, there exists rotational inertia that ensures that all bodies maintain their angular momentum unless acted upon by an unbalanced torque. Likewise, Newton's Second Law of Motion can be used to derive an analogous equation for the instantaneous Angular acceleration of the rigid body:

$$\vec{\tau} = I\vec{\alpha}$$

where

I is the Moment of inertia of the body

$\vec{\alpha}$ is the angular acceleration of the body.

This provides a definition for the moment of inertia which is the rotational equivalent for mass. In more advanced treatments of mechanics, where the rotation over a time interval is described, the moment of inertia must be substituted by the tensor that, when properly analyzed, fully determines the characteristics of rotations including precession and nutation.

Equivalently, the differential form of Newton's Second Law provides an alternative definition of torque:

$$\vec{\tau} = \frac{d\vec{L}}{dt}, [41]$$

where \vec{L} is the angular momentum of the particle.

Newton's Third Law of Motion requires that all objects exerting torques themselves experience equal and opposite torques,[42] and therefore also directly implies the conservation of angular momentum for closed systems that experience rotations and revolutions through the action of internal torques.

Centripetal force

Main article: Centripetal force

For an object accelerating in circular motion, the unbalanced force acting on the object equals:[43]

$$\vec{F} = -\frac{mv^2\hat{r}}{r}$$

where m is the mass of the object, v is the velocity of the object and r is the distance to the center of the circular path and \hat{r} is the unit vector pointing in the radial direction outwards from the center. This means that the unbalanced centripetal force felt by any object is always directed toward the center of the curving path. Such forces act perpendicular to the velocity vector associated with the motion of an object, and therefore do not change the Speed of the object (magnitude of the velocity), but only the direction of the velocity vector. The unbalanced force that accelerates an object can be resolved into a component that is perpendicular to the path, and one that is tangential to the path. This yields both the tangential force which accelerates the object by either slowing it down or speeding it up and the radial (centripetal) force which changes its direction.[3][4]

Kinematic integrals

Main articles: Impulse, Mechanical work and Power (physics)

Forces can be used to define a number of physical concepts by integrating with respect to kinematic variables. For example, integrating with respect to time gives the definition of Impulse:^[44]

$$\vec{I} = \int_{t_1}^{t_2} \vec{F} dt$$

which, by Newton's Second Law, must be equivalent to the change in momentum (yielding the Impulse momentum theorem).

Similarly, integrating with respect to position gives a definition for the work done by a force:^{[3]:13-3}

$$W = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F} \cdot d\vec{x}$$

which is equivalent to changes in kinetic energy (yielding the work energy theorem).^{[3]:13-3}

Power P is the rate of change dW/dt of the work W , as the trajectory is extended by a position change $d\vec{x}$ in a time interval dt :^{[3]:13-2}

$$dW = \frac{dW}{d\vec{x}} \cdot d\vec{x} = \vec{F} \cdot d\vec{x}, \quad \text{so} \quad P = \frac{dW}{dt} = \frac{dW}{d\vec{x}} \cdot \frac{d\vec{x}}{dt} = \vec{F} \cdot \vec{v},$$

with $\vec{v} = d\vec{x}/dt$ the Velocity.

Potential energy

Main article: Potential energy

Instead of a force, often the mathematically related concept of a Potential energy field can be used for convenience. For instance, the gravitational force acting upon an object can be seen as the action of the gravitational field that is present at the object's location. Restating mathematically the definition of energy (via the definition of work), a potential scalar field $U(\vec{r})$ is defined as that field whose gradient is equal and opposite to the force produced at every point:

$$\vec{F} = -\vec{\nabla}U.$$

Forces can be classified as conservative or nonconservative. Conservative forces are equivalent to the gradient of a potential while nonconservative forces are not.^{[3][4]}

Conservative forces

A conservative force that acts on a closed system has an associated mechanical work that allows energy to convert only between kinetic or potential forms. This means that

for a closed system, the net mechanical energy is conserved whenever a conservative force acts on the system. The force, therefore, is related directly to the difference in potential energy between two different locations in space,[45]and can be considered to be an artifact of the potential field in the same way that the direction and amount of a flow of water can be considered to be an artifact of the contour map of the elevation of an area.[3][4]

Conservative forces include gravity, the electromagnetic force, and the springforce. Each of these forces has models which are dependent on a position often given as a radial vector \vec{r} emanating from spherically symmetric potentials.[46]Examples of this follow:

For gravity:

$$\vec{F} = -\frac{Gm_1m_2\vec{r}}{r^3}$$

where G is the gravitational constant, and m_n is the mass of object n .

For electrostatic forces:

$$\vec{F} = \frac{q_1q_2\vec{r}}{4\pi\epsilon_0r^3}$$

where ϵ_0 is electric permittivity of free space, and q_n is the electric charge of object n .

For spring forces:

$$\vec{F} = -k\vec{r}$$

where k is the spring constant. [3][4]

Nonconservative forces

For certain physical scenarios, it is impossible to model forces as being due to gradient of potentials. This is often due to macrophysical considerations which yield forces as arising from a macroscopic statistical average of microstates. For example, friction is caused by the gradients of numerous electrostatic potentials between the atoms, but manifests as a force model which is independent of any macroscale position vector. Nonconservative forces other than friction include other contact forces, tension, compression, and drag. However, for any sufficiently detailed description, all these forces are the results of conservative ones since each of these macroscopic forces are the net results of the gradients of microscopic potentials.[3][4] The connection between macroscopic nonconservative forces and microscopic conservative forces is described by detailed treatment with statistical mechanics. In

macroscopic closed systems, nonconservative forces act to change the internal energies of the system, and are often associated with the transfer of heat. According to the Second law of thermodynamics, nonconservative forces necessarily result in energy transformations within closed systems from ordered to more random conditions as entropy increases.[3][4]

Units of measurement

The SI unit of force is the newton (symbol N), which is the force required to accelerate a one kilogram mass at a rate of one meter per second squared, or $\text{kg} \cdot \text{m} \cdot \text{s}^{-2}$. [47] The corresponding CGS unit is the dyne, the force required to accelerate a one gram mass by one centimeter per second squared, or $\text{g} \cdot \text{cm} \cdot \text{s}^{-2}$. A newton is thus equal to 100,000 dynes.

The gravitational foot-pound-second English unit of force is the pound-force (lbf), defined as the force exerted by gravity on a pound-mass in the standard gravitational field of $9.80665 \text{ m} \cdot \text{s}^{-2}$. [47] The pound-force provides an alternative unit of mass: one slug is the mass that will accelerate by one foot per second squared when acted on by one pound-force. [47]

An alternative unit of force in a different foot-pound-second system, the absolute fps system, is the poundal, defined as the force required to accelerate a one pound mass at a rate of one foot per second squared. [47] The units of slug and poundal are designed to avoid a constant of proportionality in Newton's second law.

The pound-force has a metric counterpart, less commonly used than the newton: the kilogram-force (kgf) (sometimes kilopond), is the force exerted by standard gravity on one kilogram of mass. [47] The kilogram-force leads to an alternate, but rarely used unit of mass: the metric slug (sometimes mug or hyl) is that mass which accelerates at $1 \text{ m} \cdot \text{s}^{-2}$ when subjected to a force of 1 kgf. The kilogram-force is not a part of the modern SI system, and is generally deprecated; however it still sees use for some purposes as expressing jet thrust, bicycle spoke tension, torque wrench settings and engine output torque. Other arcane units of force include the sthène which is equivalent to 1000 N and the kip which is equivalent to 1000 lbf.

- v
- t newton kilogram-force,
- e (SI unit) dyne kilopond pound-force poundal

	\equiv	1				
1 N	$\text{kg} \cdot \text{m}/\text{s}^2$	$= 10^5 \text{ dyn}$	$\approx 0.10197 \text{ kp}$	$\approx 0.22481 \text{ lb}_F$	$\approx 7.2330 \text{ pdl}$	
		\equiv	$1 \approx 1.0197$	$\times \approx 2.2481$	$\times \approx 7.2330$	\times
1 dyn	$= 10^{-5} \text{ N}$	$\text{g} \cdot \text{cm}/\text{s}^2$	10^{-6} kp	10^{-6} lb_F	10^{-5} pdl	
	$= 9.80665 =$	980665				
1 kp	N	dyn	$\equiv g_n \cdot (1 \text{ kg})$	$\approx 2.2046 \text{ lb}_F$	$\approx 70.932 \text{ pdl}$	
	$\approx 4.448222 \approx$	444822				
1 lb_F	N	dyn	$\approx 0.45359 \text{ kp}$	$\equiv g_n \cdot (1 \text{ lb})$	$\approx 32.174 \text{ pdl}$	
	$\approx 0.138255 \approx$	13825				
1 pdl	N	dyn	$\approx 0.014098 \text{ kp}$	$\approx 0.031081 \text{ lb}_F$	$\equiv 1 \text{ lb} \cdot \text{ft}/\text{s}^2$	

The value of g_n as used in the official definition of the kilogram–force is used here for all gravitational units.

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