Members Subjected to Combined Loads

Combined Bending & Twisting : In some applications the shaft are simultaneously subjected to bending moment M and Torque T.The Bending moment comes on the shaft due to gravity or Inertia loads. So the stresses are set up due to bending moment and Torque.

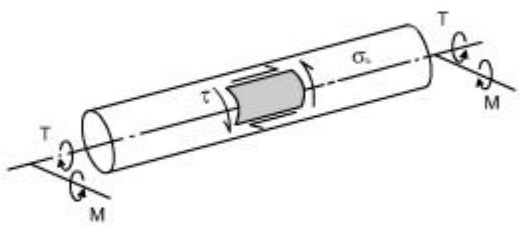
For design purposes it is necessary to find the principal stresses, maximum shear stress, which ever is used as a criterion of failure.

From the simple bending theory equation $\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$

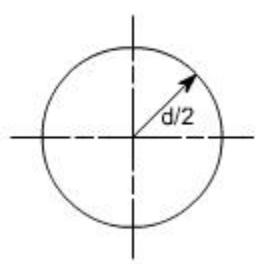
If σ_b is the maximum bending stresses due to bending.

$$\sigma_b = \frac{M.y}{I}$$

$$\sigma_b\Big|_{max^m} = \frac{M}{I}.y\,max^m$$



For the case of circular shafts y_{max^m} – equal to d/2 since y is the distance from the neutral axis.



I is the moment of inertia for circular shafts

$I = \pi d^4/64$

Hence then, the maximum bending stresses developed due to the application of bending moment M is

$$\sigma_b\Big|_{max^m} = \frac{M}{\pi d^4/64}.\frac{d}{2}$$

$$\sigma_b\Big|_{max^m} = \frac{32M}{\pi d^3} \qquad (1)$$

From the torsion theory, the maximum shear stress on the surface of the shaft is given by the torsion equation

$$\frac{T}{J} = \frac{\tau'}{r} = \frac{G.\theta}{L}$$

$$\Rightarrow \frac{\tau'}{r} = \frac{T}{J}$$

Where τ' is the shear stress at any radius r but when the maximum value is desired the value of r should be maximum and the value of r is maximum at r = d/2

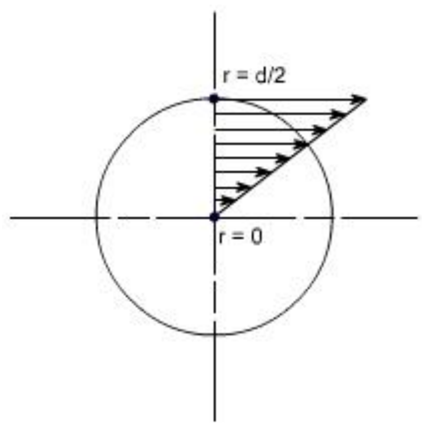
$$\text{Thus } \tau_{max^m} = \frac{T}{J}.\frac{d}{2}$$

$$J = \frac{\pi d^4}{32}$$

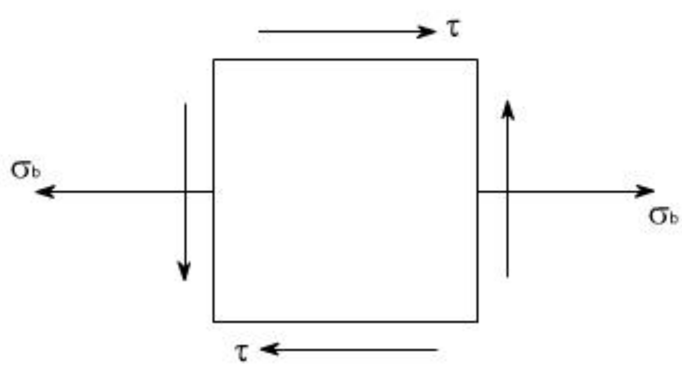
substituting the value of J, we get

$$\tau_{max^m} = \frac{16T}{\pi d^3} \qquad (2)$$

The nature of the shear stress distribution is shown below :



This can now be treated as the two – dimensional stress system in which the loading in a vertical plane in zero i.e. $\sigma_y = 0$ and $\sigma_x = \sigma_b$ and is shown below :



Thus, the principle stresses may be obtained as

$$\sigma_1, \sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

or

$$\sigma_1 = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau_{max}^2}$$

$$= \frac{32M}{\pi d^3 .2} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16M}{\pi d^3 2} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + \left(\frac{2.16T}{\pi d^3}\right)^2}$$

$$= \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right]$$

Equivalent Bending Moment :

Now let us define the term the equivalent bending moment which acting alone, will produce the same maximum principal stress or bending stress.Let M_e be the equivalent bending moment, then due to bending

$$\sigma_1 = \frac{32M_e}{\pi d^3}$$

Futher

$$\sigma_1 = \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right]$$

Thus, equating the two we get

$$\boxed{M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]}$$

Equivalent Torque :

At we here already proved that σ_1 and σ_2 for the combined bending and twisting case are expressed by the relations:

$$\sigma_1, \sigma_2 = \frac{16}{\pi d^3}\left\{M \pm \sqrt{M^2 + T^2}\right\}$$

$$\text{or } \sigma_1 = \frac{16}{\pi d^3}\left[\left\{M + \sqrt{M^2 + T^2}\right\}\right]$$

$$\sigma_2 = \frac{16}{\pi d^3}\left[\left\{M - \sqrt{M^2 + T^2}\right\}\right]$$

$$\text{As } \tau_{max^m} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\text{so } \tau_{max^m} = \frac{16}{\pi d^3}\left[\left\{M + \sqrt{M^2 + T^2}\right\}\right] - \frac{16}{\pi d^3}\left[\left\{M - \sqrt{M^2 + T^2}\right\}\right] \Big/ 2$$

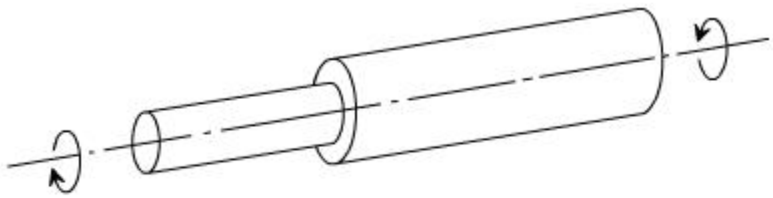
$$\tau_{max^m} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2} = \frac{16}{\pi d^3}.T_e$$

where $\sqrt{M^2 + T^2}$ is defined as the equivalent torque, which acting alone would produce the same maximum shear stress as produced by the pure torsion

Thus, $\boxed{T_e = \sqrt{M^2 + T^2}}$

Composite shafts: (in series)

If two or more shaft of different material, diameter or basic forms are connected together in such a way that each carries the same torque, then the shafts are said to be connected in series & the composite shaft so produced is therefore termed as series – connected.



Here in this case the equilibrium of the shaft requires that the torque 'T' be the same through out both the parts.

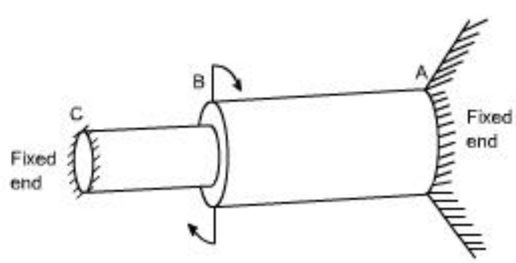
In such cases the composite shaft strength is treated by considering each component shaft separately, applying the torsion – theory to each in turn. The composite shaft will therefore be as weak as its weakest component. If relative dimensions of the various parts are required then a solution is usually effected by equating the torque in each shaft e.g. for two shafts in series

$$T = \frac{G_1 J_1 \theta_1}{L_1} = \frac{G_2 J_2 \theta_2}{L_2}$$

In some applications it is convenient to ensure that the angle of twist in each shaft are equal i.e. $\theta_1 = \theta_2$, so that for similar materials in each shaft $\frac{J_1}{L_1} = \frac{J_2}{L_2}$ or $\frac{L_1}{L_2} = \frac{J_1}{J_2}$

The total angle of twist at the free end must be the sum of angles $\theta_1 = \theta_2$ over each x - section

Composite shaft parallel connection: If two or more shafts are rigidly fixed together such that the applied torque is shared between them then the composite shaft so formed is said to be connected in parallel.



For parallel connection.

Total Torque $T = T_1 + T_2$

In this case the angle of twist for each portion are equal and $\frac{T_1 L_1}{G_1 J_1} = \frac{T_2 L_2}{G_2 J_2}$

for equal lengths(as is normaly the case for parallel shafts) $\frac{T_1}{T_2} = \frac{G_1 J_1}{G_2 J_2}$

This type of configuration is statically indeterminate, because we do not know how the applied torque is apportioned to each segment, To deal such type of problem the procedure is exactly the same as we have discussed earlier,

Thus two equations are obtained in terms of the torques in each part of the composite shaft and the maximun shear stress in each part can then be found from the relations.

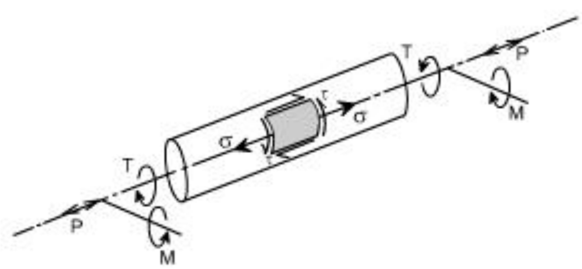
$$\tau_1 = \frac{T_1 R_1}{J_1}$$

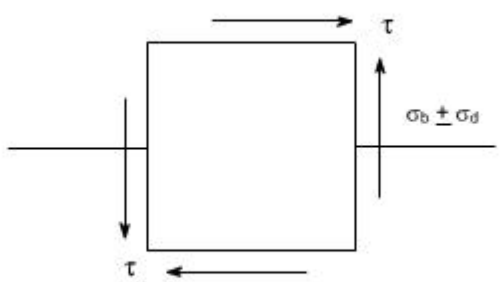
$$\tau_2 = \frac{T_2 R_2}{J_2}$$

Combined bending, Torsion and Axial thrust:

Sometimes, a shaft may be subjected to a combined bending, torsion and axial thrust. This type of situation arises in turbine propeller shaft

If P = Thrust load





Then σ_d = P / A (stress due to thrust)

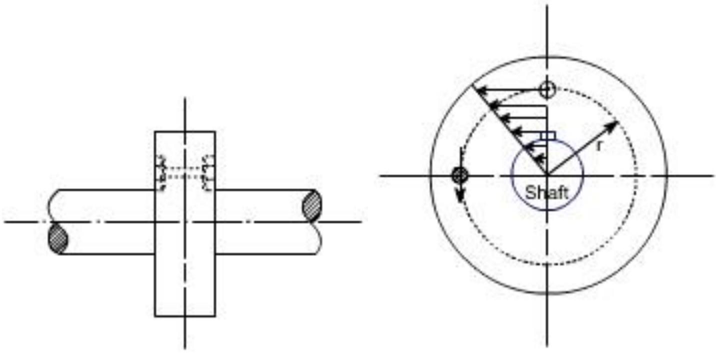
where σ_d is the direct stress depending on the whether the steam is tensile on the whether the stress is tensile or compressive

This type of problem may be analyzed as discussed in earlier case.

Shaft couplings: In shaft couplings, the bolts fail in shear. In this case the torque capacity of the coupling may be determined in the following manner

Assumptions:

The shearing stress in any bolt is assumed to be uniform and is governed by the distance from its center to the centre of coupling.



Thus, the torque capacity of the coupling is given as $T = \left(\frac{\pi}{4} d_b^2\right) . \tau_b ' . r . n$

where

d_b = diameter of bolt

τ'_b = maximum shear stress in bolt

n = no. of bolts

r = distance from center of bolt to center of coupling

Source: http://nptel.ac.in/courses/Webcourse-contents/IIT-ROORKEE/strength%20of%20materials/homepage.htm