Material Acceleration

- Derivation and Review
  The material acceleration is defined as the acceleration following a fluid particle. Since acceleration is the time derivative of velocity, the material acceleration can be derived from the definition of material derivative as follows:
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  $$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy}{dt} + \frac{\partial \vec{V}}{\partial z}\frac{dz}{dt}$$
  $$= \frac{\partial \vec{V}}{\partial t}(1) + \frac{\partial \vec{V}}{\partial x}(u) + \frac{\partial \vec{V}}{\partial y}(v) + \frac{\partial \vec{V}}{\partial z}(w)$$
  $$= \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$
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  Note that dt/dt = 1 by definition, and since a fluid particle is being followed, dx/dt = u, i.e. the x-component of the velocity of the fluid particle. Similarly, dy/dt = v, and dz/dt = w following a fluid particle.
- Explanation of the material acceleration:
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  $$\underbrace{\vec{a} = \frac{d\vec{V}}{dt}}_{\text{total accel.}} = \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{local accel.}} + \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\text{convective accel.}}$$
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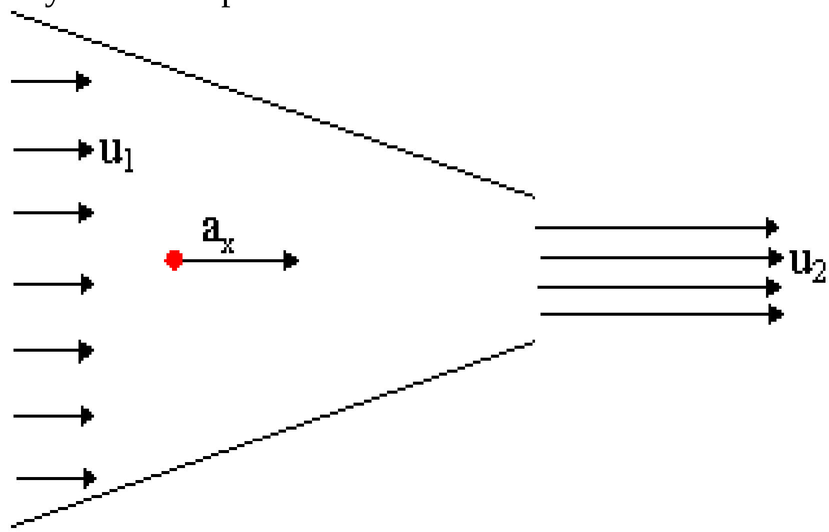
  The term on the left is the ***total acceleration*** following a fluid particle. It represents the actual acceleration vector experienced by whatever fluid particle happens to reside at the location and time of interest. The first term on the right hand side is called the ***local acceleration*** or the ***unsteady acceleration***. It is only non-zero in an unsteady flow. The last three terms make up the ***convective acceleration***, which is defined as the acceleration due to convection or movement of the fluid particle to a different part of the flow field. The convective acceleration can be non-zero even in a steady

flow! In other words, even when the velocity field is not a function of time (i.e. a steady flow), a fluid particle is still accelerated from one location to another.

- Examples
  - Physical example -

    Physical example of material acceleration:

    Consider steady flow in a converging duct (like at the end of a fireman's hose). The flow must accelerate through the nozzle in order to conserve mass. I.e. the u-component of velocity increases as a fluid particle passes through the nozzle. Thus, the acceleration vector is non-zero, even though this is a steady-state flow.

  - Mathematical example of material acceleration:
    Given: A steady, 2-D velocity field,

$$\vec{V} = 3x\vec{i} - 3y\vec{j}$$

Find: The acceleration field.
Solution: Plug this velocity into the acceleration equation.

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u \cdot \frac{\partial \vec{V}}{\partial x} + v \cdot \frac{\partial \vec{V}}{\partial y} + w \frac{\partial \vec{V}}{\partial z}$$

$$= 0 + 3x \cdot (3\vec{i}) - 3y \cdot (-3\vec{j}) + 0$$

$$= 9x\vec{i} + 9y\vec{j} \neq 0$$

Comments: This 2-D steady flow field is described by velocity components u = 3x and v = -3y. When the derivatives are calculated, as above, the final result is an acceleration vector that is clearly not zero. Thus, the acceleration vector can be non-zero, even for a steady-state flow.

Source:

http://www.mne.psu.edu/cimbala/Learning/Fluid/Material_Acc/material_acceleration.htm