MODEL FOR OPTIMAL BLOCK REPLACEMENT DECISION OF AIR CONDITIONERS USING FIRST ORDER MARKOV CHAINS WITH & WITHOUT CONSIDERING INFLATION

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Abstract:

In this paper, a mathematical model has been developed for group replacement of a block of ***Air Conditioners*** using discrete-time First Order Markov Chains. To make the model more realistic, three intermediate states viz., Minor Repair State, Semi-Major Repair State and Major Repair States have been introduced between Functioning State & Complete Failure States of the system. The Transition Probabilities for future periods for First Order Markov Chain (FOMC) are estimated by Spectral Decomposition Method. Using these probabilities, the number of systems in each state and accordingly the corresponding average maintenance cost is computed.

The forecasted inflation for Air Conditioners and the real value of money using Fisherman's relation are employed to study and develop the real time mathematical model for block replacement decision making.

Key words: Replacement, Markov Chains, Transition Probability, Spectral Decomposition Method, Inflation, Forecasting.

Introduction

The replacement decisions in most of the multinational companies, Software Development Centers, Star Hotels and other major food processing industries are mostly with the air conditioners. The primary decision is generally whether to replace the existing block of air conditioning system containing a large number of air conditioners or use for some more period of time.

The activity of maintenance involve repairs & service ranging from minor to major which cannot be defined and computed exactly in specific, various costs, and influence of various economic variables such as Inflation, value of money etc.

Several researchers investigated the optimal age replacement models with repairs to reduce the cost. Nuthall et *al* (1983) studied the impact of inflation on replacement costs along with the impact of some other parameters viz. financing method and increased or decreased hours of use. Chein et *al* (2007) presented an age-replacement model with minimal repair based on cumulative repair cost limit. In this they considered the complete repair cost data in order to decide whether to repair the unit or to replace. Bagai *et al* (1994) discussed optimal replacement time under the age replacement policy for a system with minimal repair that involves the replacement of only a very small part of the system. Rupe et al (2000) explored the maintenance models for finite time missions by considering net present value of costs.

There are some studies on the replacement decisions for warranted products. Zuo *et al* (2000) discussed replacement policy for multi state Markov deterioration of machines that are under warranty. Pan *et al* (2010) extended the work of Zuo *et al* (2000) by considering more general state space with time parameters at each state.

Archibald *et al* (1996) studied and compared the optimal age-Replacement, Standard Block Replacement and Modified Block Replacement (MBR) Policies with an inference of MBR policy is appreciably better than the remaining two.

However there is no much literature on block or group replacement model with Markov chain transition probabilities that is being used in many applications.

Markov chains concept was used by Stelios et al (1980), Jianming et al (2003), Shamsad et al (2005), Ying-Zi Li et al (2009), Avik Gosh et al (2010), Carpinone et al (2010), for forecasting of different parameters viz. Power generation, monsoon rainfall, manpower supplying, wind speed. With the increasing popularity of use of Markov chains, some studies by Bruce Craig et al (1998) and Liana Cazacioc et al (2004) are made on estimation and evaluation of transition probabilities using Markov chains.

Markov Chain forecasts as observed by Ying-Zi Li et al (2009) have some practical value that yielded relatively satisfied results. Shamsad et al (2005) and Carpinone et al (2010) observed that Second order Markov chains resulted in better forecast performance than first order Markov forecasts.

As the estimation of transition probabilities for bigger state space S =1,2,3,…,m is much time consuming one, Bruce A Criag et al (1998) and Sutawanir *et al* (2008) studied the spectral representation of transition probabilities and Zhenqing Li *et al* (2005) tried computer aided program to estimate the high-order Transition Probability Matrix of the Markov Chain.

This paper discusses a mathematical model for group replacement of block of air conditioners with three intermediary states viz., minor repair state, semi-major repair state and major repair states between working and failure states using first Markov Chains. Though it is difficult to identify the specific repairable intermediate states, to make the model simplistic the various repairs pertaining to air conditioners are grouped as shown in Table-1.

Table – 1: Identification of possible repairs and their category in Air Conditioners

Minor repairs	**Semi - Major repairs**	**Major repairs**
Running capacitor problem Fan capacitor problem Temperature sensor problem Condenser fan motor problem Condenser blade problem Blower problem etc.	Capillary problem Gas leak problem Relay board problem Display board problem	Compressor Problem Evaporator problem Condenser coil problem etc.

The transition probabilities are estimated using First Order Markov Chains. Transition probabilities for future periods are estimated by Spectral Decomposition. Also the influence of macroeconomic variables such as inflation and time value of money are considered to make the model yield better results.

Methodology of Markov chain

M*arkov process* is a stochastic or random process, which has property that the probability of transition from a given state to any future state depends on the present state and not on the manner in which it was reached.

First Order Markov Chain (FOMC)

The First Order Markov Chain (FOMC) assumes the probability of next state depends only on the immediately preceding state. Thus if $t_0 < t_1 < \ldots < t_n$ represents the points on time scale then the family of random variables $\{X(t_n)\}$ whose state space S = 1,2, …,m is said to be a Markov process provided it holds the Markovian property:

$$P\{X(t_n) = X_n \mid X(t_{n-1}) = X_{n-1}, \ldots, X(t_0) = X_0\} = P\{X(t_n) = X_n \mid X(t_{n-1}) = X_{n-1}\} \text{ for all } X(t_0), X(t_1), \ldots, X(t_n)$$

If the random process at time t_n is in the state x_n, the future state of the random process X_{n+1} at time depends only on the present state x_n and not on the past states $x_{n-1}, x_{n-2}, \ldots, x_0$.

The simplest of the Markov Process is discrete and constant over time. A system is said to be discrete in time if it is examined at regular intervals, e.g. daily, monthly or yearly.

Transition Probability: The probability of moving from one state to another future state or retaining in the same state during a particular time period is called transition probability.

Mathematically the transition probability can be expressed as:

$$P_{x(n-1),x(n)} = P\{X(t_n) = X_n \mid X(t_{n-1}) = X_{n-1}\}$$

and is called FOMC transition probability that represents the probability of moving from one state to another future state. The transition probabilities can be arranged in a matrix of size mxm and such a matrix can be called as one step Transition Probability Matrix (TPM), represented as below:

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1m} \\ P_{21} & P_{22} & \cdots & P_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mm} \end{bmatrix}$$ Where 'm' represents the number of states.

The matrix P is a square matrix of which each element is non-negative and sum of the elements in each row is unity i.e. $\sum_{j=1}^{m} P_{ij} = 1$; i= 1 to m and $0 \le P_{ij} \le 1$.

The initial estimates of P_{ij} can be computed as, $P_{ij} = \frac{N_{ij}}{N_i}$, (i, j = 1 to m) where N_{ij} is the raw data sample that refer the number of items or units or observations transitioned from the state i to state j. N_i is the raw data sample in state i.

Model Development:

Notations:

N = Total Number of items in the System
C_1 = Individual Replacement Cost Per Unit
C_2 = Minor Repair Cost
C_3 = Semi-Major Repair Cost
C_4 = Major Repair Cost
C_5 = Group Replacement Cost
X_0^I = Proportion of units in functioning state initially
X_0^{II} = Proportion of units in minor repair state initially
X_0^{III} = Proportion of units in semi-major repair state initially
X_0^{IV} = Proportion of units in major repair state initially
X_0^V = Proportion of units in complete failure state initially
X_i^I = proportion of units in functioning state at the end of i^{th} time period
X_i^{II} = proportion of units in minor repair state at the end of i^{th} time period
X_i^{III} = proportion of units in semi major repair state at the end of i^{th} time period,
X_i^{IV} = proportion of units in major repair state at the end of i^{th} time period,
X_i^V = proportion of units in complete failure state at the end of i^{th} time period
P_{ij} = Probability of items switching from i^{th} state to j^{th} state in a period
TPM = Transition Probability Matrix
φ_t = Rate of Inflation during time 't'

r_n = Nominal Rate of Interest
r_t = Real Rate of Interest = $\frac{(r_n-\varphi_t)}{(1+\varphi_t)}$, from Fisherman's Relation
Present Value Factor (PVF) = $\frac{1}{(1+r_t)}$
P_{ij} = Probability of items switching over from i^{th} state to j^{th} state in a period
TPM = Transition Probability Matrix
W(t) = Weighted average cost per period in group replacement policy,
AC(t) = average cost per period in group replacement policy

In this paper, a group replacement model for (N) items that fail completely on usage, considering three intermediate states i.e. minor repair, semi-major and major repair, is developed by using first order Markov chain. So as it consists m = 5 states, the FOMC Transition Probability Matrix(TPM) can be written as:

$$\text{TPM} = \text{P} = \begin{array}{c|ccccc} & I & II & III & IV & V \\ I & P_{11} & P_{12} & P_{13} & P_{14} & P_{15} \\ II & P_{21} & P_{22} & P_{23} & P_{24} & P_{25} \\ III & P_{31} & P_{32} & P_{33} & P_{34} & P_{35} \\ IV & P_{41} & P_{42} & P_{43} & P_{44} & P_{45} \\ V & P_{51} & P_{52} & P_{53} & P_{54} & P_{55} \end{array}$$

Where I, II, III, IV and V represents Working, Minor repair, Semi-Major repair, Major repair and Complete failure States respectively.

Sum of the transition probabilities in each row is unity i.e. $\sum_{j=1}^{5} P_{ij} = 1$; i= 1 to 5

The initial estimates of P_{ij} can be computed as, $P_{ij} = \frac{N_{ij}}{N_i}$, (i, j = 1 to 5) where

N_{ij} is the number of Air Conditioners transitioned from the state i to state j.
N_i is the raw data sample in state i.

FOMC Transition probability, $P^i = V D^i V^{-1}$, where i = 1 to n for the future 'n' time periods are computed using Spectral Decomposition Method [Sutawanir *et al* (2008)].

Spectral Decomposition Method:
As the estimation of high–order Markov chain transition probabilities for bigger state space S =1,2,3,…,m is much time consuming one, Bruce A Criag et al (1998) and Sutawanir *et al* (2008) studied the advantage of spectral representation of transition probabilities for multi state process and Zhenqing Li *et al* (2005) tried computer aided program to estimate the high-order Transition Probability Matrix of the Markov Chain. As several software for spectral decomposition are widely available [Sutawanir *et al* (2008)], this method provides flexibility for the computation of transition probabilities for multi state process.

Spectral Decomposition is based on eigen values. It is applicable to square matrix that will be decomposed into a product of three matrices, only one of which is diagonal. As a result, the decomposition of a matrix into matrices composed of its eigen values and eigen vectors is called Eigen or Spectral decomposition.

An nxn matrix 'P' always has 'n' eigen values, which can be ordered (in more than one way) to form an nxn diagonal matrix D formed from the eigen values and a corresponding matrix, V, of non zero columns (eigen vectors) that satisfies the eigen value equation: PV = VD.

This gives the amazing decomposition of 'P' into a similarity transformation involving V and D as

$$P = V D V^{-1}$$

Furthermore, squaring both sides of above equation gives

$$\begin{aligned} P^2 &= (V D V^{-1})(V D V^{-1}) \\ &= V D (V^{-1} V) D V^{-1} \\ &= V D^2 V^{-1} \end{aligned}$$

Mathematically, Spectral Decomposition can be represented as $P^i = V D^i V^{-1}$ where i= 1 to n.

Therefore higher order Transition Probability Matrix (TPM) of four state Markov chain can be computed using the equation, $P^i = V D^i V^{-1}$ where i= 1 to n

Calculation of number of items in each state:
The proportion (X_i) of units during i^{th} period in various states i.e., the state probabilities of items in different states can be computed as follows.

$[X_i^I \; X_i^{II} \; X_i^{III} \; X_i^{IV} \; X_i^{IV}] = [X_0^I \; X_0^{II} \; X_0^{III} \; X_0^{IV} \; X_0^V] * P^i$, where i = 1 to n

X_i = (Probability of items in different states during initial period) * $(TPM)^i$

In general, $X_i = X_0 P^i$

The TPMs, $P^1, P^2, \ldots, P^i$ of future periods for FOMC can be calculated by spectral decomposition method.
At the end of the first period, the state probabilities can be calculated from

$$X_1 = X_0 P \qquad (\because X_i = X_0 P^i)$$

$$\Rightarrow \left[X_i^I \quad X_i^{II} \quad X_i^{III} \quad X_i^{IV} \quad X_i^V\right] = \left[X_0^I \quad X_0^{II} \quad X_0^{III} \quad X_0^{IV} \quad X_0^V\right] \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} \\ P_{21} & P_{22} & P_{23} & P_{24} & P_{25} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} \\ P_{41} & P_{42} & P_{43} & P_{44} & P_{45} \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} \end{bmatrix}$$

Therefore, Probability of items in functional state, at the end of the first period

$$X_1^I = X_0^I P_{11} + X_0^{II} P_{21} + X_0^{III} P_{31} + X_0^{IV} P_{41} + X_0^V P_{51}$$

Probability of items in minor repair state, at the end of the first period

$$X_1^{II} = X_0^I P_{12} + X_0^{II} P_{22} + X_0^{III} P_{32} + X_0^{IV} P_{42} + X_0^V P_{52}$$

Probability of items in semi-major repair state, at the end of the first period

$$X_1^{III} = X_0^I P_{13} + X_0^{II} P_{23} + X_0^{III} P_{33} + X_0^{IV} P_{43} + X_0^V P_{53}$$

Probability of items in major repair state, at the end of the first period

$$X_1^{IV} = X_0^I P_{14} + X_0^{II} P_{24} + X_0^{III} P_{34} + X_0^{IV} P_{44} + X_0^V P_{54}$$

Probability of items in irreparable state, at the end of the first period

$$X_1^V = X_0^I P_{15} + X_0^{II} P_{25} + X_0^{III} P_{35} + X_0^{IV} P_{45} + X_0^V P_{55}$$

Similarly, the probabilities of items falling in different states in future time periods (i= 1 to n) are to be calculated by using the equation, $X_n = X_0 P^n$.

The TPMs (P^i) for future 'n' time periods i = 1,2,…, n are calculated by using Spectral Decomposition method. Using these state probabilities the number of individual replacements (α_i), minor repairs (β_i), semi-major repairs (γ_i) and major repairs (δ_i) in future time periods can be calculated as shown in the following tables – 2 & 3:

Table – 2: No. of Individual replacements and minor repairs during different time periods

Time period	**No. of individual replacements**	**No. of minor repairs**
1st	$\alpha_1 = NX_1^V$	$\beta_1 = NX_1^{II}$
2nd	$\alpha_2 = NX_2^V + \alpha_1 X_1^V$	$\beta_2 = NX_2^{II} + \beta_1 X_1^{II}$
3rd	$\alpha_3 = NX_3^V + \alpha_1 X_2^V + \alpha_2 X_1^V$	$\beta_3 = NX_3^{II} + \beta_1 X_2^{II} + \beta_2 X_1^{II}$
4th	$\alpha_4 = NX_4^V + \alpha_1 X_3^V + \alpha_2 X_2^V + \alpha_3 X_1^V$	$\beta_4 = NX_4^{II} + \beta_1 X_3^{II} + \beta_2 X_2^{II} + \beta_3 X_1^{II}$
5th	$\alpha_5 = NX_5^V + \alpha_1 X_4^V + \alpha_2 X_3^V + \alpha_3 X_2^V + \alpha_4 X_1^V$	$\beta_5 = NX_5^{II} + \beta_1 X_4^{II} + \beta_2 X_3^{II} + \beta_3 X_2^{II} + \beta_4 X_1^{II}$
. . . S o on	. . . S o on	. . . S o on

Table – 3: No. of Semi-Major repairs and major repairs during different time periods

Time period	No. of semi-major repairs	No. of major repairs
1st	$\gamma_1 = NX_1^{III}$	$\delta_1 = NX_1^{IV}$
2nd	$\gamma_2 = NX_2^{III} + \gamma_1 X_1^{III}$	$\delta_2 = NX_2^{IV} + \delta_1 X_1^{IV}$
3rd	$\gamma_3 = NX_3^{III} + \gamma_1 X_2^{III} + \gamma_2 X_1^{III}$	$\delta_3 = NX_3^{IV} + \delta_1 X_2^{IV} + \delta_2 X_1^{IV}$
4th	$\gamma_4 = NX_4^{III} + \gamma_1 X_3^{III} + \gamma_2 X_2^{III} + \gamma_3 X_1^{III}$	$\delta_4 = NX_4^{IV} + \delta_1 X_3^{IV} + \delta_2 X_2^{IV} + \delta_3 X_1^{IV}$
5th	$\gamma_5 = NX_5^{III} + \gamma_1 X_4^{III} + \gamma_2 X_3^{III} + \gamma_3 X_2^{III} + \gamma_4 X_1^{III}$	$\delta_5 = NX_5^{IV} + \delta_1 X_4^{IV} + \delta_2 X_3^{IV} + \delta_3 X_2^{IV} + \delta_4 X_1^{IV}$
. . . S o on	. . . S o on	. . . S o on

Inflation prediction:

Inflation is predicted using regression model with trigonometric function and the influence of Inflation on the air conditioners in India from the year 2005 onwards is studied over a period of time, forecasted and compared with actual values for the known periods by employing various forecasting techniques to identify the underlying model that best fits the time series data. Subsequently the inflation is predicted for *Air Conditioners* for the future time periods by the developed Regression model with trigonometric function, which yielded relatively minimal errors.

A sinusoidal trigonometric function is used in the regression model to accommodate cyclical fluctuations of inflation. For this the following mathematical equation is considered.

$\phi = a + bt + c \sin(t\pi + \pi/4)$ - - - (1)

To find the constants a, b & c the following set of equations are used.

$\sum\phi = na + b\sum t + c\sum\sin(t\pi + \pi/4)$ --- (2)

$\sum(\phi t) = a\sum t + b\sum t^2 + c\sum[t \sin(t\pi + \pi/4)]$ --- (3)

$\sum(\phi t^2) = a\sum t^2 + b\sum t^3 + c\sum[t^2 \sin(t\pi + \pi/4)]$ --- (4)

where ϕ is the inflation, t is time period, n is the number of time periods and a, b & c are the coefficients.

Regression model with trigonometric function for predicting inflation for a time period t is

$F = -4.50352 + 0.30617T + 7.55993 \sin(T\pi + \pi/4)$

Influence of Inflation and time value of money

Conventional models are available to make the replacement decisions considering the value of money. Here the Net Present Worth criterion based on the nominal interest rate (r_n) does not reflect the real value of money. Real interest rates (r_r) are computed using Fisherman's relation. When the present worth factors are computed and multiplied with future money, it gives purchasing power of money.

Present worth factor= pwf= $\nu = \dfrac{1}{1+r_t}$

Real rate of interest = $r_t = \dfrac{r_n - \varphi_t}{1 + \varphi_t}$, from Fisherman's relation

To get more realistic results, the forecasted values of Inflation to get the real interest rates, using Fisherman's relation, are used. The Inflation values based on WPI for *air conditioners* are predicted by using a regression forecasting model using a sinusoidal trigonometric function.

Total cost (TC) for n time periods, without the influence of Inflation:

TC = Group replacement cost + Individual replacement cost + Minor repair cost + Semi - Major repair cost + Major repair cost

$$TC = NC_5 + C_1\sum_{i=1}^{n}\alpha_i + C_2\sum_{i=1}^{n}\beta_i + C_3\sum_{i=1}^{n}\gamma_i + C_4\sum_{i=1}^{n}\delta_i$$

Average cost per period = $\dfrac{\text{Total cost for 'n' periods}}{\text{number of periods}}$

Weighted average cost, W(t) = TC/n

Total cost (TC) for n time periods, with the influence of Inflation:

$$TC(n) = C_1\left[\alpha_1 + \alpha_2 \nu + \alpha_3 \nu^2 + \ldots + \alpha_n \nu^{n-1}\right] + C_2\left[\beta_1 + \beta_2 \nu + \beta_3 \nu^2 + \ldots + \beta_n \nu^{n-1}\right]$$
$$+ C_3\left[\gamma_1 + \gamma_2 \nu + \gamma_3 \nu^2 + \ldots + \gamma_n \nu^{n-1}\right] + C_4\left[\delta_1 + \delta_2 \nu + \delta_3 \nu^2 + \ldots + \delta_n \nu^{n-1}\right] + NC_5 \nu^{n-1}$$

$$TC(n)=C_1\sum(\alpha_n \nu^{n-1}) + C_2\sum(\beta_n \nu^{n-1}) + C_3\sum(\gamma_n \nu^{n-1}) + C_4\sum(\delta_n \nu^{n-1}) + NC_5 \nu^{n-1}$$

Weighted average cost, W(t) = $TC \;/ \sum \nu^{n-1}$

Policy: 'n' is optimal when the weighted average cost per period is minimum i.e. average cost per period should be minimum in n^{th} period, to block replace in n^{th} period.

Case Study:

In the present study, a block of 100 air conditioners in a software development centre have been studied and the cost data for various types of repairs based on the information given by the air conditioners service engineers is assumed as given below.

N = Total number of Air Conditioners in the system = 100
C_1 = Individual replacement cost per unit = Rs.23000
C_2 = Minor repair cost = Rs.4500
C_3 = Semi-major repair cost = Rs.8000
C_4 = Major repair cost = Rs.15500
C_5 = Group replacement cost = Rs.21000
r_n = Nominal rate of Interest = 20%

$$X_0 = \left[X_0^{I} \quad X_0^{II} \quad X_0^{III} \quad X_0^{IV} \quad X_0^{V}\right] = \left[0.850 \quad 0.090 \quad 0.030 \quad 0.020 \quad 0.010\right]$$

$$P^1 = TPM = \begin{bmatrix} 0.6588 & 0.1765 & 0.0471 & 0.1059 & 0.0118 \\ 0.6667 & 0.1111 & 0.1111 & 0.1111 & 0.0000 \\ 0.3333 & 0.3333 & 0.3333 & 0.0000 & 0.0000 \\ 0.5000 & 0.5000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

The calculations pertaining to optimal block replacement decisions of Air Conditioners without considering the influence of inflation are shown in the table-4 and considering the influence inflation are shown in table-5.

Results and Discussions

The average cost per year using First Order Markov Chain are shown in the Table-6. When the influence of inflation is not considered, First Order Markov Chain (FOMC) model resulted in the replacement age as 7 years. When the influence of predicted inflation and net worth of the money is considered, FOMC resulted in the early replacement of block of Air Conditioners at the age of 4 years. Therefore for the block of air conditioners considered for study in this work, the optimal replacement time is at the end of 4^{th} year.

Table - 6: Average Cost in lakhs per year

Time period(n)	FOMC Without inflation	FOMC with inflation
1	24.30	24.30
2	14.05	12.50
3	10.83	9.77
4	9.37	**7.01***
5	8.63	7.31
6	8.26	
7	**8.12***	
8	8.13	

* Minimum Cost giving Optimum period for replacement

Concluding remarks

A Markov chain based mathematical model for group replacement model has been developed for a block of computers system using First Order Markov Chains. This paper considers five discrete states- working condition, minor repair, semi-major repair, major repair and break down state of an air conditioning system to make the maintenance cost more realistic. FOMC has resulted the optimal replacement age of 7 years without considering inflation and money value, where as considering inflation and time value of money it was 4 years. Also the model was more realistic as the influence of macroeconomic variables viz. inflation and time value of money on replacement model are considered. The authors of this paper have checked this with the replacement decision when the influence of inflation is not considered.

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Interpretation of columns in the following tables is given below:

A=Time Period 't'
B=α_t = number of individual replacements
C=Individual replacement Cost
D=β_t = number of minor replacements
E=Minor replacement cost
F= γ_t = number of semi-major replacements
G=Semi - Major replacement Cost
H=δ_t = number of major replacements
I=Major replacement Cost
J= Total Maintenance Cost = C+E+G+I
K= Cumulative Maintenance Cost

L=Group Replacement Cost
M= Total Cost = K+L
N =Average Maintenance Cost/period
P = Inflation
Q = Real Interest rate
R = Present Worth Factor
S = Discounting Factor
T = Cumulative Discounting Factor

The calculations pertaining to optimal block replacement decisions of Air Conditioners without considering the influence of inflation are shown in the table-4 and considering the influence inflation are shown in table-5.

Table - 4 : Calculations of optimal block replacement decision using First Order Markov Chain without considering inflation

A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	2.00	46069.00	18.00	81010.35	6.00	48026.40	10.00	155021.70	330127.45	330127.45	2100000.00	2430127.45	2430127.45
2	2.80	64361.36	23.54	105921.58	7.38	59005.57	9.78	151555.59	380844.10	710971.55	2100000.00	2810971.55	1405485.77
3	3.59	82597.53	27.67	124510.88	8.34	66704.28	10.59	164118.46	437931.15	1148902.70	2100000.00	3248902.70	1082967.57
4	4.41	101506.30	33.06	148778.18	9.01	72058.22	11.40	176710.76	499053.45	1647956.15	2100000.00	3747956.15	936989.04
5	5.27	121222.55	39.37	177156.02	9.66	77274.59	12.33	191042.01	566695.18	2214651.33	2100000.00	4314651.33	862930.27
6	6.17	141940.45	46.90	211033.94	10.33	82642.09	13.33	206586.88	642203.36	2856854.69	2100000.00	4956854.69	826142.45
7*	7.12	163817.48	55.86	251361.13	11.04	88327.83	14.42	223446.53	726952.96	3583807.65	2100000.00	5683807.65	****811972.52**
8	8.13	187025.14	66.53	299397.21	11.80	94386.85	15.59	241691.83	822501.04	4406308.68	2100000.00	6506308.68	813288.59
9	9.21	211745.81	79.25	356611.81	12.61	100856.27	16.87	261431.01	930644.90	5336953.58	2100000.00	7436953.58	826328.18
10	10.36	238169.53	94.39	424759.62	13.47	107768.25	18.24	282783.25	1053480.65	6390434.23	2100000.00	8490434.23	849043.42

Table - 5 : Calculations optimal block replacement decision using First Order Markov Chain considering the influence of inflation

A	P	Q	R	S	T	B	C	D	E	F	G	H	I	J	K	L	M	N
1	2.07	0.18	0.85	1.00	1.00	2.00	46069.00	18.00	81010.35	6.00	48026.40	10.00	155021.70	330127.45	330127.45	2100000.00	2430127.45	2430127.45
2	-8.32	0.31	0.76	0.76	1.76	2.80	49172.98	23.54	80925.56	7.38	25358.10	9.78	115790.58	271247.22	601374.67	1604429.20	2205803.87	1250445.85
3	2.68	0.17	0.86	0.73	2.50	3.59	60474.12	27.67	91161.15	8.34	27471.27	10.59	120160.00	299266.53	900641.20	1537523.54	2438164.74	976763.09
4*	-7.71	0.30	0.77	0.45	2.95	4.41	46181.70	33.06	67688.69	9.01	18440.93	11.40	80397.01	212708.33	1113349.53	955424.08	2068773.61	****701010.23**
5	3.29	0.16	0.86	0.55	3.50	5.27	66545.33	39.37	97250.11	9.66	23861.26	12.33	104872.85	292529.55	1405879.08	1152798.69	2558677.77	731033.34
6	-7.09	0.29	0.77	0.28	3.78	6.17	39484.55	46.90	58704.77	10.33	12931.38	13.33	57467.70	168588.40	1574467.48	584171.48	2158638.96	571331.43
7	3.90	0.15	0.87	0.42	4.20	7.12	69034.11	55.86	105925.77	11.04	20937.44	14.42	94162.31	290059.64	1864527.12	884958.32	2749485.43	654690.96
8	-6.48	0.28	0.78	0.17	4.37	8.13	32652.83	66.53	52271.93	11.80	9269.47	15.59	42197.12	136391.35	2000918.47	366640.23	2367558.69	541247.85
9	4.52	0.15	0.87	0.33	4.71	9.21	70118.91	79.25	118090.80	12.61	18786.50	16.87	86572.00	293568.21	2294486.68	695407.95	2989894.63	635416.87
10	-5.87	0.27	0.78	0.11	4.82	10.36	26782.52	94.39	47764.85	13.47	6816.77	18.24	31799.40	113163.53	2407650.21	236148.12	2643798.33	548749.75