Principles of Passive Vibration Control: Isolators with stiffness and damping

INTRODUCTION

Noise and vibrations have undesirable effects on both human quality of life, and on our material goods. Vibration-generating machinery and processes contribute, to a large extent, to the total noise and vibration exposure. A very useful strategy to reduce noise and vibrations is to interrupt the propagation path between the source and the receiver. *Elastic mounting* is a simple method to hinder the spread of structural vibrations. In practice, an elastic mounting system is realized by incorporating so-called vibration isolators along the propagation path. Strongly vibrating machines in factories, dwellings, and office buildings can be placed on elastic elements. The objective of this chapter is both to provide the essential knowledge required to properly design vibration isolation systems, and to impart a physical understanding of the principles used in vibration isolation. A vibration isolation problem is often schematically described by division into substructures: a source structure which is coupled to a receiver structure. The vibration isolation is yet another substructure incorporated between the two structures.

The objective of vibration isolation is to reduce the vibrations in some specific portion of the receiver structure. It is apparent that vibration isolation can be realized in many different ways. It therefore falls upon the designer to arrive at an isolation system design well-suited to the specific situation. A vibration problem can also be nicely described by the same ***source – path – receiver*** model used to characterize the noise control problem. Source: a mechanical or fluid disturbance, generated internally by the machine, such as unbalance, torque pulsations, gear tooth meshing, fan blade passing, etc. Path: the structural or airborne path by which the disturbance is transmitted to the receiver Receiver: the responding system, generally having many natural frequencies which can potentially be excited by vibration frequencies generated by the source. The best solution to a vibration problem is to avoid it in the first place. Intelligent design is far more cost effective than building a bad design and having to repair it later. Minimizing the vibration transmission generally involves using isolator springs and/or inertia blocks. The basic principle is to make the natural frequency of the machine on its foundation as far below the excitation frequency as possible.

Consider a vibrating machine, bolted to a rigid floor (Figure 7.26 a). The force transmitted to the floor is equal to the force generated in the machine. The transmitted force can be decreased by adding a suspension and damping elements (often called vibration isolaters) Figure 7.26 b , or by adding what is called an inertia block, a large mass (usually a block of cast concrete), directly attached to the machine (Figure 7.26 c). Another option is to add an additional level of mass (sometimes called a seismic mass, again a block of cast concrete) and suspension (Figure 7.26 d).

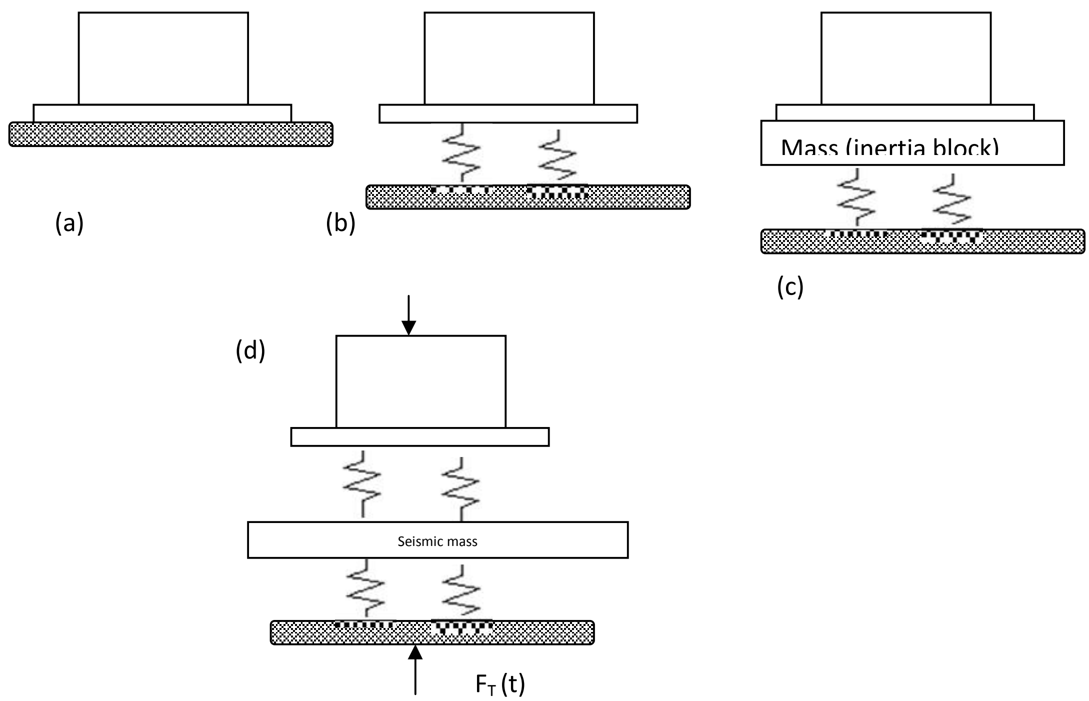

Figure 7.26 Vibration isolation systems: a) Machine bolted to a rigid foundation b) Supported on isolation springs, rigid foundation c) machine attached to an inertial block d) Supported on isolation springs, non-rigid foundation (such as a floor); or machine on isolation springs, seismic mass and second level of isolator springs

The equation of motion for the above mass spring system is:

$$m\ddot{x} + c\dot{x} + kx = F(t) \qquad (7.24)$$

The response of the system is:

$$x(t) = \frac{F_O/k}{1 - r^2} \sin \omega t \qquad (7.25)$$

Where $r = \dfrac{\omega}{\omega_n}$ $\qquad \omega_n = \sqrt{\dfrac{k}{m}}$

The critical frequency is

$$RPM_{critical} = 60 f_n = \frac{60}{2\pi}\sqrt{\frac{k}{m}} \qquad (7.26)$$

The ratio of transmitted force to the input force is called ***transmissibility, T***

$$T = \left|\frac{F_T}{F_O}\right| = \left|\frac{1}{r^2 - 1}\right| = \left|\frac{X}{Y}\right| \qquad (7.27)$$

This same equation can also be used to calculate the response of a machine ***X*** to displacement of the foundation, ***Y***. The effectiveness of the isolator, expressed in dB is:

$$E = 10 \log_{10} \frac{1}{T} \qquad (7.28)$$

The effectiveness of the isolator, expressed in percent is:

$$\%\, Isolation = (1 - T) \times 100 \qquad (7.29)$$

The transmissibility as a function of frequency ratio is shown in Figure 7.12. Vibration isolation (defined as T<1) occurs when the excitation frequency is > 1.4 *fn*. For minimum transmissibility (maximum isolation), the excitation frequency should be as high above the natural frequency as possible. The transmissibility above resonance has a slope of –20 dB/decade. The transmissibility including the effect of damping is:

$$T = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}} \tag{7.30}$$

$$\xi = \frac{c}{2\, m\, \omega_n} \tag{7.31}$$

Typical values for damping ratio ξ are .005 -.01 for steel, and .05-.10 for rubber. The inclusion of damping has the greatest effect in the vicinity of resonance, decreasing the vibration amplitude. A curious effect of damping is that it results in **increased** amplitude at frequencies > 1.4 *fn*. Typical vibration isolators employ a helical spring to provide stiffness, and an elastomeric layer (such as neoprene) to provide some damping. Other types use a solid elastomeric element for both the stiffness and the damping.

MEASURES OF TRANSMISSION ISOLATION

In order to be able to design in optimal vibration isolation, there is a need for, not only the determination of the vibration levels, but also for some measure of the vibration isolation obtained; that latter would permit comparison of alternative isolation strategies that may be applicable in a given situation. A number of different measures are in use for various specific applications. The most universally applied of these is the so-called *insertion loss* D_{IL}.; it is defined in either of the two following alternative ways:

$$D_{IL}^{v} = L_{v}^{before} - L_{v}^{after} \quad \text{[dB]} \tag{7.32}$$

$$D_{IL}^{F} = L_{F}^{before} - L_{F}^{after} \quad \text{[dB]} \tag{7.33}$$

Where the velocity and force levels L_v and L_F . The insertion loss is, therefore, defined as the difference in level at a given point before and after the vibration isolation is provided; see figure 7.27. With these definitions as a model, it is of course possible to devise other such measures of the isolation effectiveness based on weighting different frequency components and bands, e.g., using A-weighting for instance. The choice of the relevant gauge of effectiveness, is ultimately determined by the specific application.

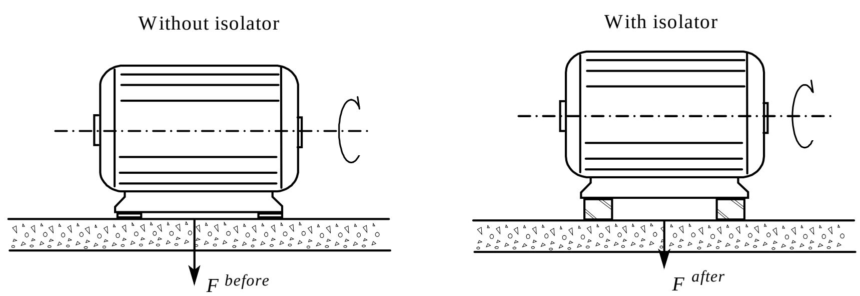

Figure 7.27 The insertion loss can be defined as the difference in the force level acting on the foundation before and after the implementation of isolation. [1]

VIBRATION ISOLATION IN PRACTICE

By way of a number of examples, we have demonstrated that the high insertion losses predicted for high frequencies by the crudest models are not actually realized. That is because, as we have already pointed out, the assumptions that underlie the simple models are not valid at high frequencies. Machines and foundations are not rigid, and isolators do not remain compliant. In practice, the simpler models often give acceptable results up to about 100 Hz. After that, the trend is that the insertion loss varies around a constant value. In typical machine mounting situations, the average attainable insertion loss at high frequencies is about 20 to 30 dB; see figure 7.28.

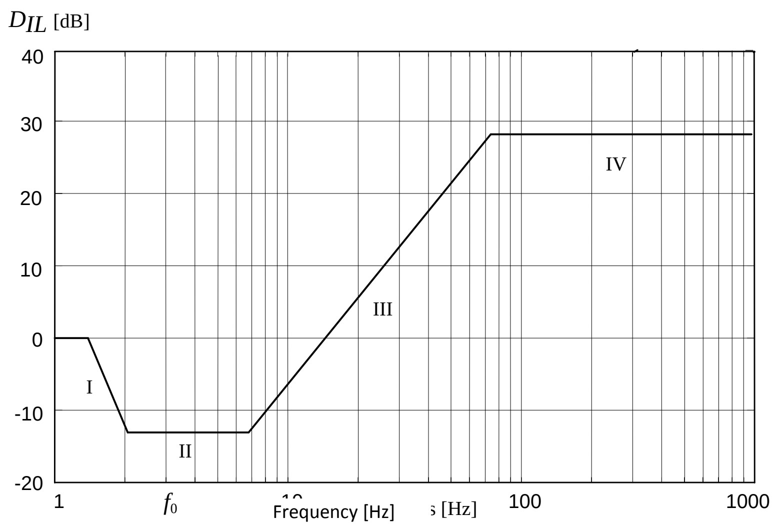

Figure 7.28 schematic sequences for a real insertion loss. I) Frequencies below the lowest mounting resonance II) Mounting resonances. III) Rigid machine-soft isolator-rigid foundation. IV) Internal resonances in the machine, foundation, and isolators. [1]

In some situations, not even that much isolation is attainable at high frequencies. For a relatively compliant foundation, the high frequency isolation is seldom better than 15 dB. Figure 7.29 shows how the choice of mounting positions affects the insertion loss. The figure shows the insertion loss for a machine mounted elastically to a section of an aluminum ship's hull. The machine is regarded as a rigid point mass, and the isolator as an ideal spring. Curve a) shows the insertion loss when the mounting positions are taken to be rigid. In both of the other curves, the measured stiffnesses of two alternate mounting positions, one stiff at the intersection of two ribs, and one softer on a single rib, are used. If the softer mounting position is chosen, the isolation obtained at higher frequencies is never more than 7 - 8 dB.

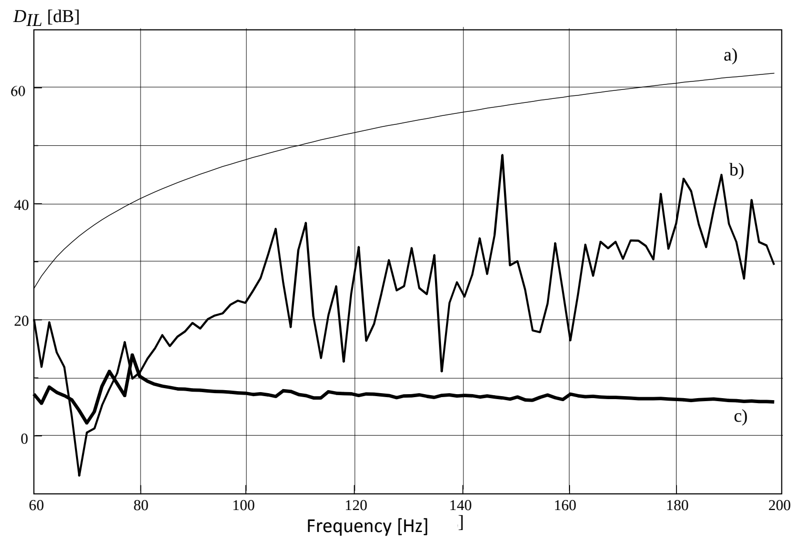

Figure 0 Insertion loss for vibration isolation in a stiffened ship construction. The results clearly show that an inappropriate choice of the mounting position can degrade the vibration isolation. a) Rigid foundation; b) Flexible foundation; mounting at the intersection of two ribs; c) Flexible foundation, mounting to a single rib.

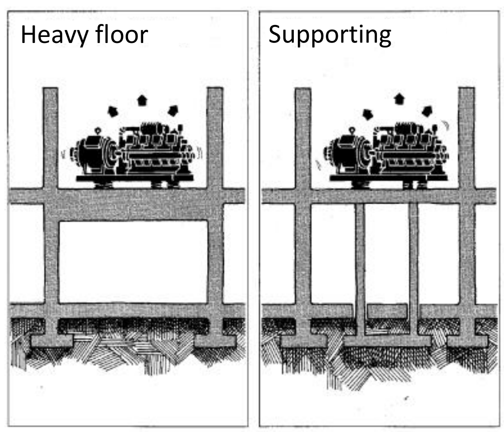

Figure 07.30 Heavy machines are common sources of vibrations and structure-borne sound. If a machine is to be installed in a building, it is important that the machine foundation be designed in an appropriate way. The principle is that the impedance and mobility difference between the isolator system and the foundation be as large as possible. Two alternative approaches can be to provide extra heavy joists in the machine room, or to stiffen them with braces directly supported by the bedrock.

(Picture: Asf, Bullerbekämpning, 1977. Ill: Claes Folkesson.)

There are many different ways to design the foundation such that the mounting points have the desired properties, i.e., low mobility. Most of the methods used in practice are based on the use of added masses and stiffening beams applied in an appropriate way. In all such situations, it is important the plan such solutions right from design stage. Their incorporation at a later stage, to give the desired dynamic properties, is in most cases both expensive and time consuming. Figure 7.30 demonstrates a couple of possible ways to design a system of joists in a building with provisions for the incorporation of vibration isolation.

Isolator Selection

Isolators are usually specified by their static deflection D, or how much they deflect when the weight of the machine is placed on them. This is equivalent to specifying their stiffness and has the additional benefit of making it easy to calculate the system natural frequency. Coil spring isolators are available in up to 3” static deflection. If more flexibility is needed, air springs are used. The natural frequency of the system (assuming a single degree of freedom) can be calculated by:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{kg}{W}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} = 3.13\sqrt{\frac{1}{\Delta}} \quad \text{Hz} \tag{7.34}$$

<u>Important Considerations with Vibration Isolator Selection</u>

1) Machine Location

- As far away from sensitive areas as possible
- And on as rigid a foundation as possible (on grade is best)

2) Proper sizing of isolator units

- Correct stiffness (specified by the static deflection, more flexible is generally better)
- Sufficient travel to prevent bottoming out during shock loads, or during system startup and shutdown

3) Location of isolators – isolators should be equally loaded, and the machine should be level.

4) Stability – sideways motion should be restrained with snubbers. The diameter of the spring should also be greater than its compressed height. Isolator springs should occupy a wide footprint for stability.

5) Adjustment – springs should have free travel, should not be fully compressed, nor hitting a mechanical stop.

6) Eliminate vibration short circuits – any mechanical connection between machine and foundation which bypasses the isolators, such as pipes, conduits, binding springs, poorly adjusted snubbers or mechanical stops

7) Fail safe operation – should a spring break or become deflated, you must have mechanical supports on which the machine can rest without tipping.

Design of absorber for continuous system

A general beam as the primary system with absorber attached to it and subjected to a harmonic force excitation is considered, as shown in Figure 7.31.The point excitation is located at b; and the absorber is placed at a: A uniform cross section is considered for the beam and Euler–Bernoulli assumptions are made. The beam parameters are all assumed to be constant and uniform. The elastic deformation from the undeformed natural axis of the beam is denoted by γ(x, t) and, in the derivations that follow, the dot (·) and prime (‘) symbols indicate a partial derivative with respect to the time variable, t; and position variable x; respectively. Under these assumptions, the kinetic energy of the system can be written as

$$T = \frac{1}{2}\rho\int_0^L \left(\frac{\partial y}{\partial t}\right)^2 \mathrm{d}x + \frac{1}{2}m_a\dot{q}_a^2 + \frac{1}{2}m_e\dot{q}_e^2 \tag{7.35}$$

The potential energy of this system using linear strain is given by

$$U = \frac{1}{2}\mathrm{EI}\int_0^L \left(\frac{\partial^2 y}{\partial x^2}\right)^2 \mathrm{d}x + \frac{1}{2}k_a\{y(a,t) - q_a\}^2 + \frac{1}{2}k_e\{y(b,t) - q_e\}^2 \tag{7.36}$$

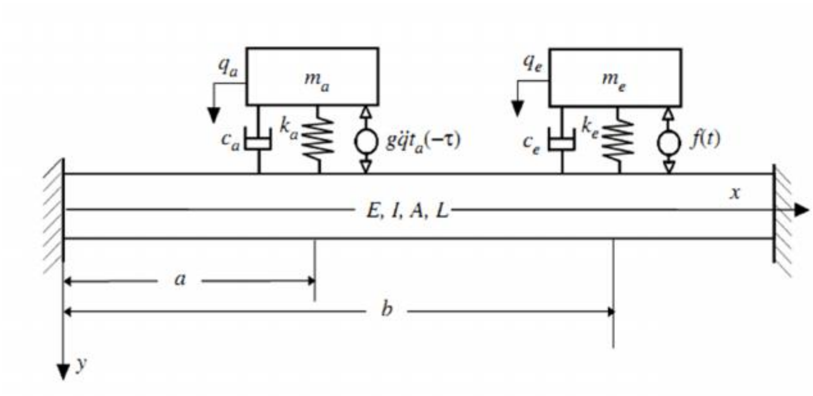

Figure 7.31 Beam under harmonic excitation [Meirovitch, 1986]

The equations of motion may now be derived by applying Hamilton's Principle. However, to facilitate the stability analysis, we resort to an assumed-mode expansion and Lagrange's equations. Specifically, γ is written as a finite sum "Galerkin approximation"

$$y(x,t) = \sum_{i=1}^{n} \Phi_i(x) q_{bi}(t) \tag{7.37}$$

The orthogonality conditions between these mode shapes can also be derived as (Meirovitch, 1986)

$$\int_0^L \rho \Phi_i(x)\Phi_j(x)\mathrm{d}x = N_i\delta_{ij}, \quad \int_0^L \mathrm{EI}\Phi_i''(x)\Phi_j''(x)\mathrm{d}x = S_i\delta_{ij} \tag{7.38}$$

where i, j = 1, 2, …,n, δ_{ij} is the Kronecker delta, and N_i and S_i are defined by setting i = j in above equation.

The feedback of the absorber, the actuator excitation force, and the damping dissipating forces in both the absorber and the exciter are considered as non-conservative forces in Lagrange's formulation. Consequently, the equations of motion are derived.

Absorber dynamics is governed by

$$m_a\ddot{q}_a(t) + c_a\left\{\dot{q}_a(t) - \sum_{i=1}^{n}\Phi_i(a)\dot{q}_{bi}(t)\right\} + k_a\left\{q_a(t) - \sum_{i=1}^{n}\Phi_i(a)q_{bi}(t)\right\} - g\ddot{q}_a(t-\tau) = 0 \quad (7.39)$$

The exciter is given by

$$m_e\ddot{q}_e(t) + c_e\left\{\dot{q}_e(t) - \sum_{i=1}^{n}\Phi_i(b)\dot{q}_{bi}(t)\right\} + k_e\left\{q_e(t) - \sum_{i=1}^{n}\Phi_i(b)q_{bi}(t)\right\} = -f(t) \quad (7.40)$$

Finally, the beam is represented by

$$\begin{aligned} &N_i\ddot{q}_{bi}(t) + S_iq_{bi}(t) + c_a\left\{\sum_{i=1}^{n}\Phi_i(a)\dot{q}_{bi}(t) - \dot{q}_a(t)\right\}\Phi_i(a) + c_e\left\{\sum_{i=1}^{n}\Phi_i(b)\dot{q}_{bi}(t) - \dot{q}_e(t)\right\}\Phi_i(b) \\ &\quad + k_a\left\{\sum_{i=1}^{n}\Phi_i(a)q_{bi}(t) - q_a(t)\right\}\Phi_i(a) + k_e\left\{\sum_{i=1}^{n}\Phi_i(b)q_{bi}(t) - q_e(t)\right\}\Phi_i(b) + g\Phi_i(a)\ddot{q}_a(t-\tau) \\ &\quad = f(t)\Phi_i(b), \quad i = 1, 2, \ldots, n \end{aligned} \quad (7.41)$$

By proper selection of the feedback gain, the absorber can be tuned to the desired resonant frequency, ω_c. This condition, in turn, forces the beam to be motionless at a; when the beam is excited by atonal force at frequency ω_c. This conclusion is reached by taking the Laplace transform of above equation and using feedback control law for the absorber. In short,

$$Y(a,s) = \sum_{i=1}^{n}\Phi_i(a)Q_{bi}(s) = 0 \quad (7.42)$$

Where,

$$Y(a,s) = \Im\{y(a,t)\},\ Q_a(s) = \Im\{q_a(t)\} \text{ and } Q_{bi}(s) = \Im\{q_{bi}(t)\} \tag{7.43}$$

and

$$y(a,t) = \sum_{i=1}^{n} \Phi_i(a) q_{bi}(t) = 0 \tag{7.44}$$

That indicates that, the steady-state vibration of the point of attachment of the absorber is eliminated. Hence, the absorber mimics a resonator at the frequency of excitation and absorbs all the vibratory energy at the point of attachment.

Source: http://nptel.ac.in/courses/112107088/26