Module 3

Process Control

Lesson 11

Introduction to Process Control

Instructional Objectives

At the end of this lesson, the student should be able to

- Distinguish with examples the difference between sequential control and continuous process control.
- Identify three special features of a process.
- Differentiate between manipulating variable and disturbance.
- Distinguish between a SISO system and MIMO system and give at least one example in each case.
- Develop linearised mathematical models of simple systems.
- Give an example of a time delay system.
- Identify the parameters on which the time delay is dependent.
- Sketch the step response of a first order system with time delay.
- State and explain the significance of transfer function matrix.

1. Introduction

We often come across the term *process* indicating a set up or a plant that we want to control. Thus by a process we may mean a unit of chemical plant (say, a distillation column), or a manufacturing system (say, an assembly shop), or a food processing industry and so on. We may want to automate the process; we may also like to control certain parameters of the system output (say, level of a tank, pressure of steam etc.). Broadly speaking, there could be two types of control; we might want to carry out. The first one is called *sequential control*, where the control action is carried out in a sequence. A good example for this type of operation could be in an automated car manufacturing system, where the assembly of parts is carried out in a sequence (on a conveyor line). Here the control action is sequential in nature and works in a preprogrammed open loop fashion (implying that there is no feedback of the output signal to the controller). Programmable Logic Controller (PLC) is often used to carry out these operations.

But there are cases, where the control action needed is continuous in nature and precise control of the output variable is required. Take for example, the drum level control of a boiler. Here, the water level of the drum has to be maintained within a small band, in spite of variations of steam flow rate, steam pressure etc. This type of control is sometimes called *modulating control*, as the control variable is *modulated* to keep the process variable at a constant value. Feedback principle is used for these types of control. Now onwards, we would concentrate on the control of these types of processes. But in order to design a controller effectively, we must have a thorough knowledge about the dynamics of the process. A mathematical model of the process dynamics often helps us to understand the process behaviour under different operational conditions.

In this lesson, we would discuss the basic characteristics of this type of processes where continuous control is used for controlling certain variables at the outputs.

2. Characteristics of a Process

Different processes have different characteristics. But, broadly speaking, there are certain characteristics features those are more or less common to most of the processes. They are:

(i) *The mathematical model of the process is nonlinear in nature.*
(ii) *The process model contains the disturbance input*
(iii) *The process model contains the time delay term.*

In general a process may have several input variables and several output variables. But only one or two (at most few) of the input variables are used to control the process. These inputs, used for manipulating the process are called *manipulating variables*. The other inputs those are left uncontrolled are called *disturbances*. Few outputs are measured and fed back for comparison with the desired set values. The controller operates based on the error values and gives the command for controlling the manipulating variables. The block diagram of such a closed loop process can be drawn as shown in Fig. 1.

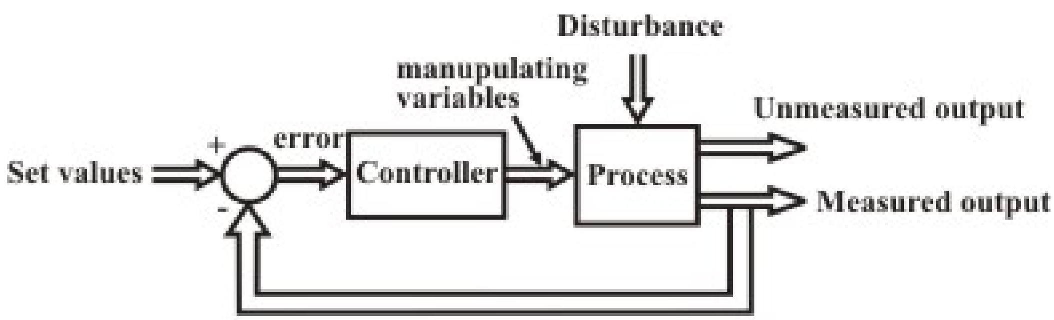

Fig. 1 General Description of a Closed loop process

In order to understand the behavour of a process, let us take up a simple open loop process as shown in Fig. 2. It is a tank containing certain liquid with an inflow line fitted with a valve V_1 and an outflow line fitted with another valve V_2. We want to maintain the level of the liquid in the tank; so the *measured output* variable is the liquid level *h*. It is evident from Fig.2 that there are two variables, which affect the measured output (henceforth we will call it only *output*) - the liquid level. These are the throttling of the valves V_1 and V_2. The valve V_1 is in the inlet line, and it is used to vary the inflow rate, depending on the level of the tank. So we can call the inflow rate as the manipulating variable. The outflow rate (or the throttling of the valve V_2) also affect the level of the tank, but that is decided by the demand, so not in our hand. We call it a *disturbance* (or sometimes as *load*).

The major feature of this process is that it has a single input (manipulating variable) and a single output (liquid level). So we call it a *Single-Input-Single-Output* (SISO) process. We would see afterwards that there are *Multiple-Input-Multiple-Output* (MIMO) processes also.

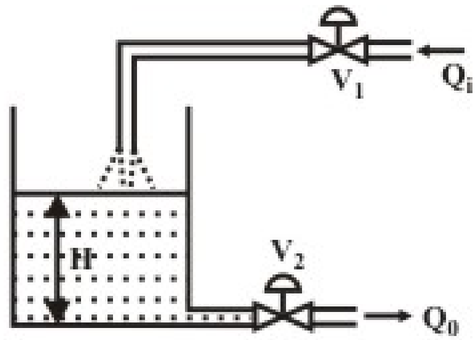

Fig. 2 Example of a Physical Process

3. Mathematical Modeling

In order to understand the behaviour of a process, a mathematical description of the dynamic behaviour of the process has to be developed. But unfortunately, the mathematical model of most of the physical processes is nonlinear in nature. On the other hand, most of the tools for analysis, simulation and design of the controllers, assumes, the process dynamics is linear in nature. In order to bridge this gap, the linearization of the nonlinear model is often needed. This linearization is with respect to a particular operating point of the system. In this section we will illustrate the nonlinear mathematical behaviour of a process and the linearization of the model. We will take up the specific example of a simple process described in Fig.2.

Let Q_i and Q_o are the inflow rate and outflow rate (in m^3/sec) of the tank, and H is the height of the liquid level at any time instant. We assume that the cross sectional area of the tank be A. In a steady state, both Q_i and Q_o are same, and the height H of the tank will be constant. But when they are unequal, we can write,

$$Q_i - Q_o = A\frac{dH}{dt} \qquad (1)$$

But the outflow rate Q_o is dependent on the height of the tank. Considering the Valve V_2 as an orifice, we can write, (please refer eqn.(4) in Lesson 7 for details)

$$Q_o = \frac{C_d A_2}{\sqrt{1-\beta^4}}\sqrt{\frac{2g}{\gamma}(P_1 - P_2)} \qquad (2)$$

We can also assume that the outlet pressure P_2=0 (atmospheric pressure) and

$$P_1 = \rho g H \qquad (3)$$

Considering that the opening of the orifice (valve V_2 position) remains same throughout the operation, equation (2) can be simplified as:

$$Q_o = C\sqrt{H} \qquad (4)$$

Where, C is a constant. So from equation (1) we can write that,

$$Q_i - C\sqrt{H} = A\frac{dH}{dt} \qquad (5)$$

The nonlinear nature of the process dynamics is evident from eqn.(5), due to the presence of the term $\sqrt{H}$.

In order to linearise the model and obtain a transfer function between the input and output, let us assume that initially $Q_i = Q_o = Q_s$; and the liquid level has attained a steady state value H_s. Now suppose the inflow rate has slightly changed, then how the height will change?

Now expanding Q_o in Taylor's series, we can have:

$$Q_o = Q_o(H_s) + \dot{Q}_o(H_s)(H - H_s) + ... \tag{6}$$

Taking the first order approximation, from eqn.(4),

$$Q_o(H_s) = C\sqrt{H_s} = Q_s$$

$$\dot{Q}_o(H_s) = \frac{C}{2\sqrt{H_s}}$$

Then from (1) and (6), we can write,

$$Q_i - Q_s - \frac{C}{2\sqrt{H_s}}(H - H_s) = A\frac{dH}{dt} = A\frac{d(H - H_s)}{dt} \tag{7}$$

Now, we define the variables q and h, as the deviations from the steady state values,

$$\begin{aligned} q &= Q_i - Q_s \\ h &= H - H_s \end{aligned} \tag{8}$$

We can write from (7),

$$q = A\frac{dh}{dt} + \frac{1}{R}h \tag{9}$$

Where, $R = \frac{2\sqrt{H_s}}{C}$ (10)

It can be easily seen, that eqn.(9) is a linear differential equation. So the transfer function of the process can easily be obtained as:

$$\frac{h(s)}{q(s)} = \frac{R}{\tau s + 1} \tag{11}$$

Where, $\tau = RA$.

It is to be noted that all the input and output variables in the transfer function model represent, the deviations from the steady state values. If the operating point (the steady state level H_s in the present case) changes, the parameters of the process (R and τ) will also change.

The importance of linearisation needs to be emphasized at this juncture. The mathematical models of most of the physical processes are nonlinear in nature; but most of the tools for design and analysis are for linear systems only. As a result, it is easier to design and evaluate the performance of a system if its mathematical model is available in linear form. Linearised model is an approximation of the actual model of the system, but it is preferred in order to have a physical insight of the system behaviour. It is to be kept in mind that this model is valid as long as the variation of the variables around the operating point is small. There are few systems whose dynamic behaviour is highly nonlinear and it is almost impossible to have a linear model of a system. For example, it is possible to develop the linearised transfer function model of an a.c. servomotor, but it is not possible for a step motor.

Referring to Fig. 2, if the valve V_1 is motorized and operated by electrical signal, we can also develop the model relating the electrical input signal and the output. Again, we have so far assumed that the opening of the valve V_2 to be constant, during the operation. But if we also consider its variation, that would also affect the dynamics of the tank model. So, the effect of disturbance can be incorporated in the overall plant model, as shown in Fig.3, by introducing a *disturbance transfer function D(s)*. *D(s)* can be easily by using the same methodology as described earlier in this section.

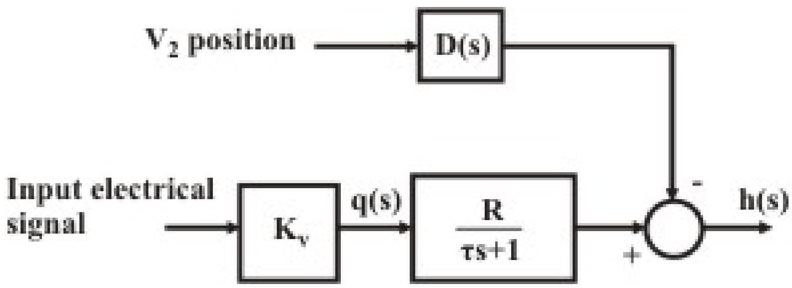

Fig. 3 Block diagram representation of the process shown in Fig. 2.

4. Higher Order System Model

We have considered a single tank and developed the linearised model of it. So it has a single time constant τ . But there are more complex processes. If there are two tanks coupled together, as shown in Fig.4, then we would have two time constants τ_1 and τ_2 . But it is evident that the dynamics of two tanks are coupled. Considering the change in the inflow *q(t)* as the input and the change in the level of the second tank $h_2(t)$ as the output variable, with a little bit of calculation, it can be shown that the transfer function of this coupled tank system is,

$$\frac{h_2(s)}{q(s)} = \frac{R_2}{\tau_1\tau_1 s^2 + (\tau_1 + \tau_2 + A_1 R_2)s + 1} \qquad (12)$$

The constants are similar to the earlier section with added suffixes corresponding to tank 1 and tank 2 respectively. In this case we have neglected the effects of the disturbances.

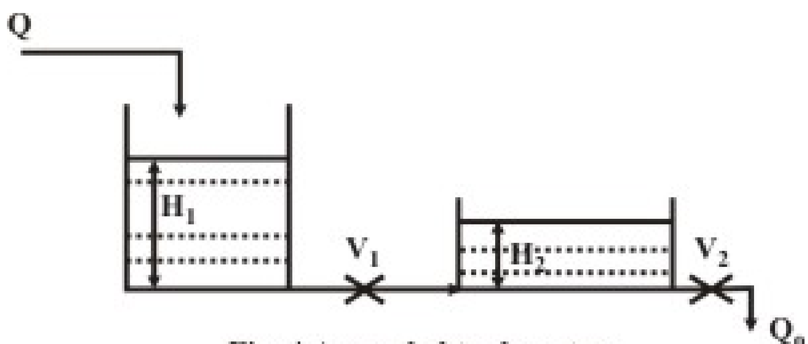

Fig. 4 A coupled tank system

5. Time delay

It has been mentioned earlier that one of the major characteristics of a process is the presence of time delay. This time delay term is often referred as “transportation lag”, since it is generated due to the delay in transportation of the output to the measuring point. The presence and effect of time delay can be easily explained with an example of a simple heat exchanger, as shown in Fig. 5.

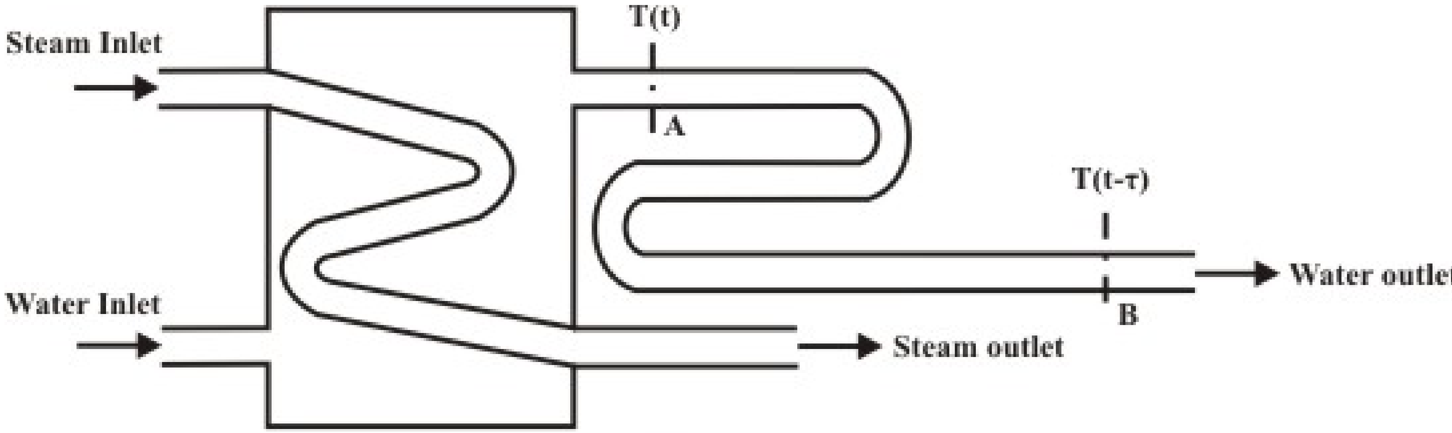

Fig. 5 Example of a time-delay system: a heat exchanger

In this case the transfer of heat takes place between the steam in the jacket and water in the tank. The measured output is the water temperature at the outlet T(t). For controlling this temperature, we may vary the steam flow rate at its inlet. So the manipulating variable is the steam flow rate. We can also identify a number of input variables those act as the disturbance, thus affecting the temperature at the water outlet; for example, inlet steam temperature, inlet water temperature and the water flow rate.

The temperature transducer should be placed at a location in the water outlet line just after the tank (location A in Fig. 5). But suppose, due to the space constraint, the transducer was placed at location B, at a distance L from the tank. In that case, there would be a delay sensing this temperature. If *T(t)* is the temperature measured at location A, then the temperature measured at location B would be $T(t-\tau_d)$. The time delay term τ_d can be expressed in terms of the physical parameters as:

$$\tau_d = L/v \qquad (13)$$

where *L* is the distance of the pipeline between locations A and B;

and *v* is the velocity of water through the pipeline.

Noting from the Laplace Transformation table,

$$\mathcal{L}\ f(t-\tau_d) = e^{-s\tau_d} F(s) \qquad (14)$$

we can conclude that an additional term of $e^{-s\tau_d}$ would be introduced in the transfer function of the system due to the time delay factor. Thus the transfer function of an ordinary first order plant with time delay is

$$G(s) = \frac{Ke^{-s\tau_d}}{1+s\tau}$$

and its step response to a unit step input is as shown in Fig. 6.

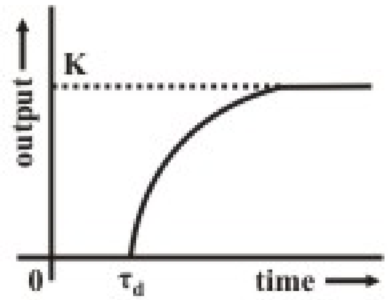

Fig. 6 Unit step response of a 1st order system with time delay

It can be seen that though the input has been at t = 0, the output remains zero till t = τ_d. This time delay present in the system may often be the main cause for instability of a closed loop system operation.

6. Multiple Input Multiple Output Systems

So far we have considered the behviour of single input single output (SISO) systems only. In these cases, we had a single manipulating variable to control a single output variable. But in many cases, we have a number of inputs to control a number of outputs simultaneously, and the input-outputs are not decoupled. This will be evident if we consider a system, slightly modified system from that one shown in Fig. 4. In the modified system, we have added another inlet flow line in tank 2, as shown in Fig. 7.

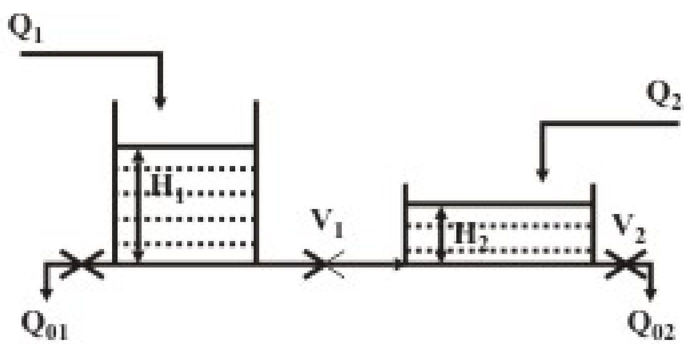

Fig. 7 A two-input two-output coupled tank system

If we consider the changes in inflow rates q_1 and q_2 are in inputs and the changes in the liquid levels of the two tanks h_1 and h_2 as the outputs, then the complete input-output behaviour can be modeled using the transfer function matrix, as shown below:

$$\begin{bmatrix} h_1(s) \\ h_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} q_1(s) \\ q_2(s) \end{bmatrix} \qquad (15)$$

We define $G(s)$ as the transfer function matrix and

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

In general, if there are *m* inputs and *p* outputs, then the order of the transfer function matrix is *p X m*. The MIMO system can also be further classified depending on the number of inputs and outputs. If the number of inputs is more than the number of outputs ($m>p$), then the system is called an *overactuated system*. If the number of inputs is less than the number of outputs ($m<p$), then the system is an *underactuated system*; while they are equal then the system is *square* (implying the *G(s)* is a square matrix).

A Multi-input-multi-output nonlinear system can be described in its state variable form as:

$$\dot{x} = f(x,u)$$
$$y = g(x,u)$$

where *x* is the state vector, *u* is the input vector and *y* is the output vector. *f* and *g* are nonlinear functions of *x* and *u*.
The above nonlinear system can also be linearised over its operating point and can be described in the state-space form as:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{16}$$

where *u* is the input vector of dimension *m*; *y* is the output vector of dimension *p* and *x* is an *n*-dimensional vector representing the states. The transfer function matrix $G(s)$ can be obtained as:

$$G(s) = C(sI - A)^{-1} B + D \tag{17}$$

For more details, please refer any book on Control Systems.

Example -1

A typical example of a MIMO process and development of its model has been taken up in this example. This is related to the control of temperature and humidity in order to maintain an artificial tropical climate inside a room in winter. Fresh cold air is forced inside the room through a fan. Steam is added to the air in order to maintain the humidity. An electric heater is used to increase the temperature of the room. So the control inputs are the steam valve position $\theta(t)$ and the voltage applied to the heater *v(t)*. The measured outputs are room temperature *c(t)* and the relative humidity *h(t)*. The detail process description is shown in Fig. 8.

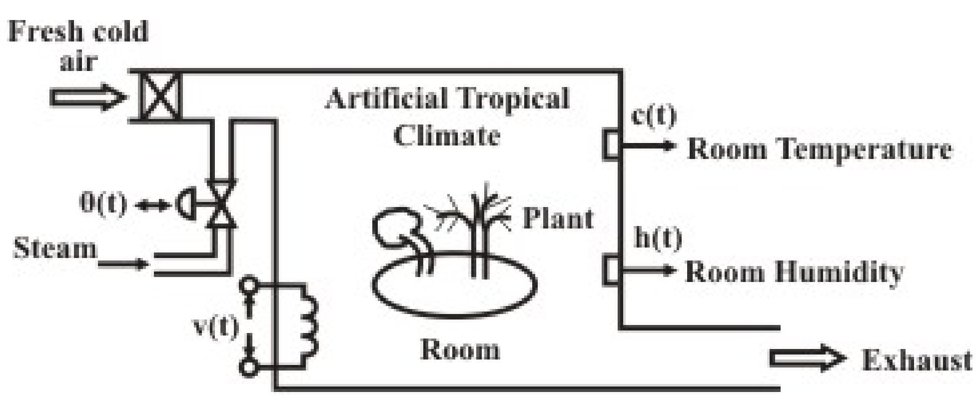

Fig. 8 Temperature and humidity control for maintaining artificial tropical climate

The room humidity is controlled by manipulating the valve position in steam input. The dynamics of the humidity inside the room can be expressed as:

$$\tau_\theta \frac{dh}{dt} + h(t) = K_h \theta(t) \tag{18}$$

On the other hand, both the steam flow rate and the voltage to the heater contribute in deciding the temperature of the room. The dynamics of the room temperature can be expressed by the equations as:

$$\begin{aligned} &\tau_\theta \frac{dc_\theta}{dt} + c_\theta(t) = K_\theta \theta(t) \\ &\tau_v \frac{dc_v}{dt} + c_v(t) = K_v v(t) \\ &c(t) = c_\theta(t) + c_v(t) \end{aligned} \tag{19}$$

Taking the Laplace transformation, the overall system dynamics can be written in form of the transfer function matrix as:

$$\begin{bmatrix} h(s) \\ c(s) \end{bmatrix} = \begin{bmatrix} \dfrac{k_h}{s\tau_\theta + 1} & 0 \\ \dfrac{k_\theta}{s\tau_\theta + 1} & \dfrac{k_v}{s\tau_v + 1} \end{bmatrix} \begin{bmatrix} \theta(s) \\ v(s) \end{bmatrix} \tag{20}$$

7. Conclusion

In this lesson, a brief introduction about several aspects of process control has been provided. By process control, we mean continuous control of one or few parameters of the process output, and feedback control is used for maintaining these values. This type of operation is distinctly different from “sequential control” that is basically discrete in operation, and open loop in nature. In order to effectively control the process, a thorough knowledge about the characteristics of the process is needed. We have learnt about few terms like, disturbance, time delay etc., whose behaviours affect the performance and stability to a great extent. Again, the mathematical model of the process is often nonlinear in nature, but most of the design and analysis tools are suitable for linear systems only. As a result the system model needs to be linearised over an operating point. The basic techniques for developing the mathematical model and its linarisation are elaborated in this lesson. Many of the process control systems are Single-Input-Single-Output (SISO) type. But there are cases where, a number of outputs are to be simultaneously controlled by manipulating a number of inputs. This requirement leads to Multiple-Input-Multiple-Output (MIMO) systems. Typical examples of MIMO processes and their mathematical models have been discussed in this lesson.

So far, we have refrained from discussing about the controllers. Different types of controllers, their performances and tuning of the controller parameters would be taken up in the subsequent lessons.

References

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Review Questions

1. Take the example of a simple liquid level control system for a vessel. Draw the block diagram for the closed loop control. Identify, the input, output, manipulating variable and disturbance for this case.
2. Explain the physical reason behind generation of time delay. Why time delay is not so prevalent in electrical systems? Justify.
3. The transfer function of a process is given by:
   $G(s) = \frac{5e^{-0.5s}}{1+s}$; where time is expressed in minutes. Sketch the open loop response for a unit step input to the process.
4. What are the factors the time delay is dependent on?
5. Give an example of a multiple-input-multiple-output process.
6. What do you mean by transfer function matrix? What is its relation with the state space description of a system?
7. The dynamic equation of a system is given by:
   $$3\frac{dx}{dt} + x^2 = 4$$
   Is it a linear system, or a nonlinear system? Justify. If it is a nonlinear system, then linearise the equation for a steady state operating condition x_s=2.