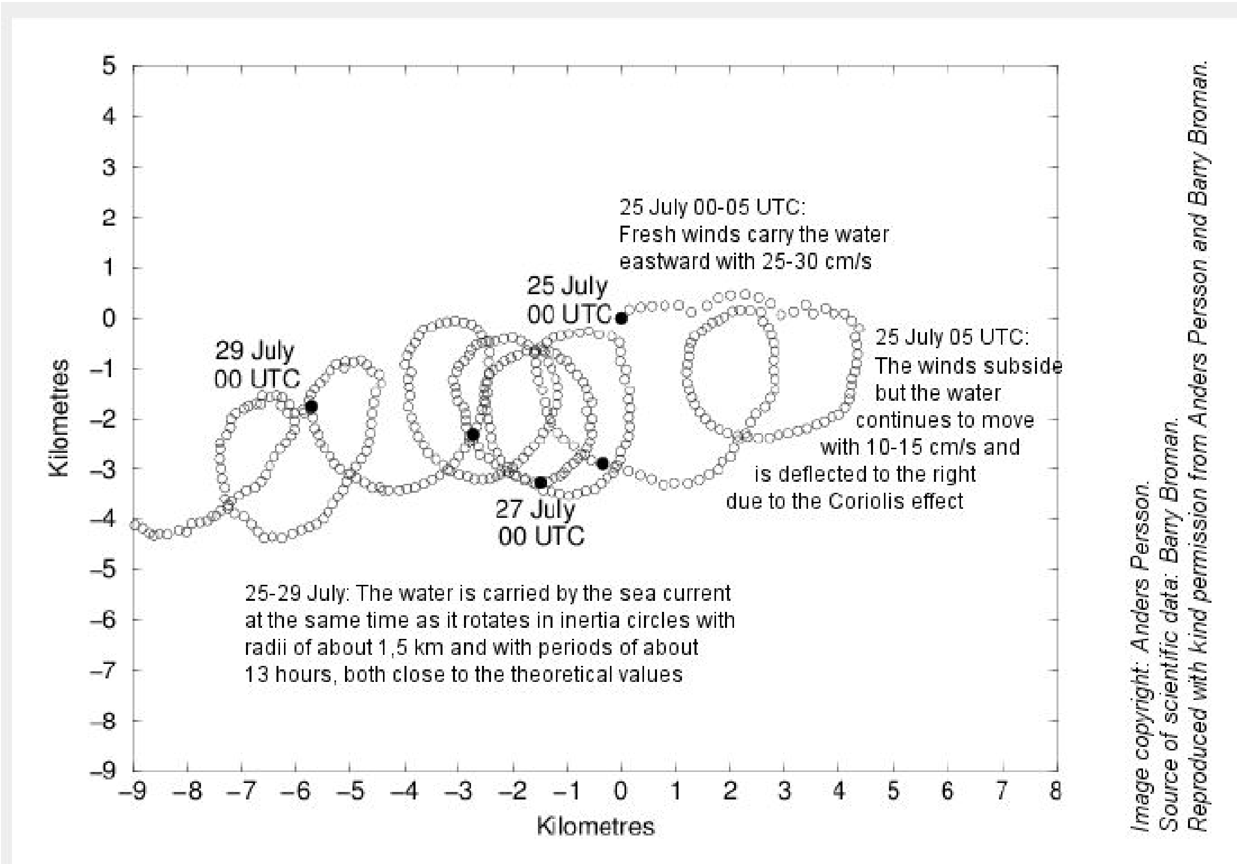
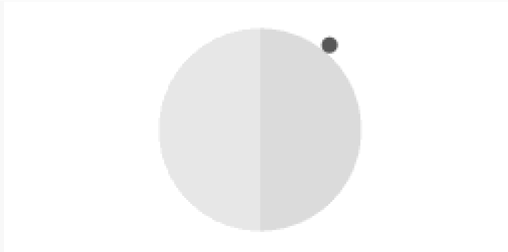
INERTIAL OSCILLATIONS



A drifting buoy set in motion by strong westerly winds in the Baltic Sea in July 1969. When the wind has decreased the uppermost water layers of the oceans tend to follow approximately inertia circles due to the Coriolis effect. This is reflected in the motions of drifting buoys. In the case there are steady ocean currents the trajectories will become cycloides.
The inertia circles are not eddies; a set of buoys close to each other would be co-moving, rather than revolve around each other

Rotating planet



Picture 2. Animation
Schematic representation of inertial oscillation.

The pattern of motion of the buoy that is being tracked is due to a pattern of dynamics that is present only in the case of a rotating planet. Animation 2 shows a preview. I will first discuss a simpler case, and then I will proceed to how it works for the Earth as a whole.

Also available: the java applet inertial oscillation, a 3D simulation of the physics of inertial oscillation.

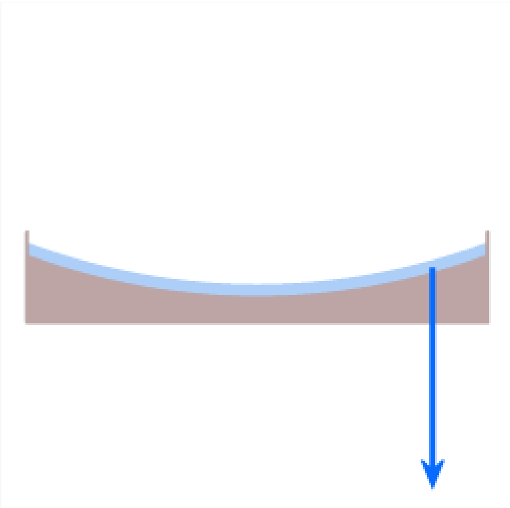
Understanding of the physics of inertial oscillation is the key to understanding how the rotation of the Earth affects the oceanic and atmospheric dynamics. Inertial oscillation is the simplest, purest case. Usually the motion of air mass is affected by both pressure gradient and Coriolis effect. Inertial oscillation shows how water mass and air mass move when there is no pressure gradient to begin with, and no buildup of pressure gradient in the course of the motion.

Simpler case as physical model: parabolic dish

The dynamics of terrestrial inertial oscillations can be understood by noting the parallels with a simpler case: motion over the surface of a dish with a parabolic cross section.

In the fluid dynamics lab of the MIT Earth, Atmosphere and Planetary sciences faculty students can set up various demonstrations, and one of them is the construction of a parabolic turntable. A platform with a diameter of one meter, with a rim of a couple of centimeters, rotating at a constant angular velocity of about 30 revolutions per minute, is filled with a resin that takes several hours to set. Because the platform is rotating, the resin gets redistributed in such a way that in the final state the surface has a parabolic shape; the center is a centimeter or so deeper than the perimeter. The equilibrium state of the rotating fluid is called 'solid body rotation'. The formal name of the solid that is formed is 'paraboloid of revolution'. After the resin has set the surface is sanded to a smooth finish.

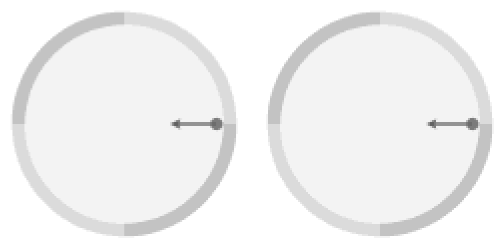
For demonstrations a small disk of dry ice is placed on the parabolic dish. The evaporating carbondioxide formes an air cushion, resulting in very low friction. Also, in demonstrations the parabolic dish must be rotating at exactly the same angular velocity as when it was manufactured.



Picture 3. Image
The blue arrow represents the force of gravity. The red arrow represents the normal force. The green arrow represents the resultant force.

When it was still liquid the resin was in solid body rotation, hence at every distance to the center of rotation the inclination of the surface is such that the resultant force of gravity and the normal force provides the amount of centripetal force that is necessary to co-rotate with the dish. The strength of this centripetal force is exactly proportional to the distance to the axis of rotation.

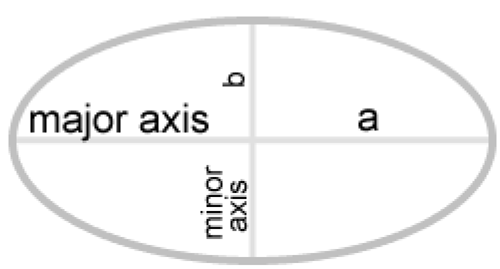
If the puck is released in such a way that it has a small velocity relative to the dish then it will follow (to a first approximation) the trajectory that is shown on the left side of animation 4. The shape of the trajectory is an ellipse, and the motion is rather like an orbit. The right side depicts the motion as seen from a point of view that is co-rotating with the dish.



Picture 4. Animation
Frictionless motion along the surface of a parabolic dish. The arrow represents the centripetal force due to the inclination of the surface.

Orbit

Normally, the word 'orbit' is used for the motion of a satellite in space, being subject to the inverse square law of gravity. There are in fact two force laws that give rise to periodic orbits. One is the inverse square law of gravity, and the other one is this case: a proportional force. The shallower the parabolic dish the better the validity of approximating the trajectories as ellipses.
The MIT rotating dish demonstration shows a picture.



Picture 5. Image

This motion allong an ellipse-shaped orbit can be thought of as a linear combination of two (perpendicular) harmonic oscillations. The following parametric equation of the position as a function of time describes the motion of the puck over the surface of the dish. The parametric equation provides a complete description: it describes the shape of the orbit and the velocity at each point in time.

$$x = a\cos(\Omega t) \quad (1)$$

$$y = b\sin(\Omega t) \quad (2)$$

a	half the length of the major axis
b	half the length of the minor axis
Ω	360° divided by the duration of one revolution

In the articles on this site I use the greek capital Ω (Omega) to refer to a constant factor; when the small greek letter ω (omega) is used it refers to the instantaneous angular velocity of some object. The instantaneous angular velocity of some object may fluctuate, depending on the circumstances. In the case of a harmonic force the trajectory of the object can be expressed as a function of a *constant*factor Ω.

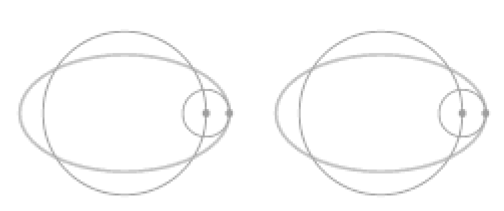
Usually the orbit will be an eccentric orbit, and then the actual angular velocity oscillates, together with an oscillation in distance to the center of rotation.

Circle and epi-circle

Interestingly, the parametric equation can be rearranged into the following two components:

$$x = \left(\frac{a+b}{2}\right)\cos(\Omega t) + \left(\frac{a-b}{2}\right)\cos(\Omega t)$$

$$y = \left(\frac{a+b}{2}\right)\sin(\Omega t) - \left(\frac{a-b}{2}\right)\sin(\Omega t)$$



Picture 6. Animation

Animation 6 represents this rearrangement geometrically. It shows that the ellipse-shaped orbit has some remarkable symmetries: it can be thought of as a combination of two circular motions, both with *uniform*angular velocity Ω. The epi-circle corresponds with the degree of*eccentricity* of the overall orbit as compared to perfectly circular motion. When motion along this particular ellipse-shaped orbit is mapped in a rotating coordinate system the eccentricity of the ellipse-shaped trajectory shows up as motion along a small circle. In the case of modeling atmospheric dynamics on a parabolic dish this small circle is called an **inertia circle**.

$$\begin{aligned} x &= \left(\frac{a-b}{2}\right)\cos(2\Omega t) \\ y &= -\left(\frac{a-b}{2}\right)\sin(2\Omega t) \end{aligned}$$

The factor 2 in the above parametric equation is especially noteworthy. The frequency of the motion along the inertia circle is *twice* the frequency of the overall rotation.

Separating the coordinate transformation

In this discussion I separate the task of taking coordinate transformation into account from the discussion of the physics. In this article, the coordinate transformations are taken care of by the animations, allowing me to focus on the physics taking place.

Java applet

When motion is mapped in a rotating coordinate system the rotation is taken into account by adding the centrifugal term and the coriolis term. The centrifugal term and the Coriolis term are depicted in the following interactive animation: Coriolis effect.

Careful arrangement

When the parabolic dish was manufactured the dish was rotating at a certain angular velocity, and the resin assumed a shape that matched the rotation rate. In other words, the manufacturing process *tuned* the parabolic dish to a certain rotation rate. The period of an orbit over the surface of that parabolic dish coincides with the dish's rotation rate. In demonstrations the instruction is to use precisely the same rate of rotation as when the parabolic dish was manufactured, and to use that angular velocity for the rotating coordinate system. The outcome of that careful arrangement is that the *Coriolis term* in the equation of motion matches the motion along the epi-circle precisely. This correspondence is visualized in the above mentioned interactive animation: Coriolis effect

Source : http://www.cleonis.nl/physics/phys256/inertial_oscillations.php