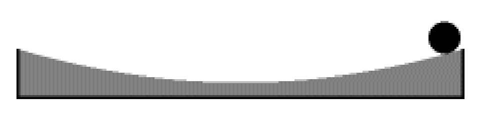
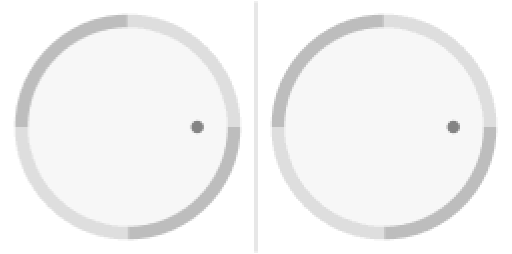
INERTIAL OSCILLATION – DEFINITION

Energy conversions: doing work



Picture 7. Animation
Harmonic oscillation; the restoring force is proportional to the distance to the center.

The case of inertial oscillation on a shallow parabolic dish and the rotational-vibrational coupling discussed in the previous article are perfectly analogous. In the case of inertial oscillations the potential energy reaches its maximum at the extremal points. As the object is being drawn closer to the center of rotation the centripetal force is doing work, converting potential energy to kinetic energy. The kinetic energy reaches its maximum at the points where the object is at its closest to the center of rotation. When the object moves away from the center of rotation again the centripetal force is doing negative work, decreasing the kinetic energy of the object.



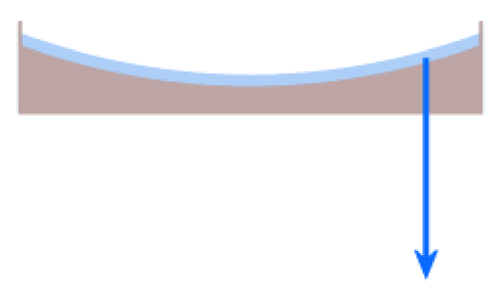
Picture 8. Animation
Final situation after friction has dissipated the energy that was associated with the *eccentricity* of the trajectory.

Dissipation of energy

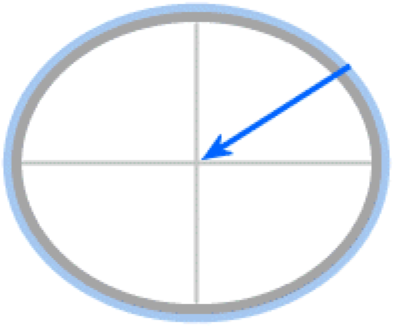
I will discuss dissipation of energy now, for that will reveal some interesting aspects. The natural process is: as long as a system can dissipate energy, it will. Consider a situation where the only way to dissipate energy is friction between the surface and the object that is supported by the surface. In the case of inertial oscillation there is dissipation of energy as long as there is a velocity with respect to the rotating parabolic dish. Friction reduces the eccentricity of the orbit. When the orbit has become circular there is still a lot of potential and kinetic energy, but as there is no friction there is no opportunity for energy dissipation.

Animation 8 depicts the state that the system will evolve to. Interestingly, it is an example of equipartition of energy. It is a common property of many kinds of dynamical systems that when the system has reached a final state (no more opportunity for energy dissipation) then the system's total energy will be divided evenly over the available degrees of freedom. Here, the available degrees of freedom are gravitational potential energy and rotational kinetic energy. The final state can be understood as energized degrees of freedom in a state of equilibrium with each other. In the final state the ratio of rotational kinetic energy to gravitational potential energy is 1:1 at every distance to the center of rotation. During manufacture of the parabolic dish, this is the state of equilibrium that the still liquid resin ended in when it reached a state of solid body rotation.

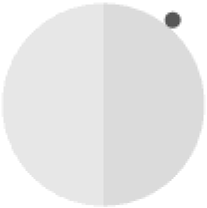
Analogy between parabolic dish and oblate spheroid



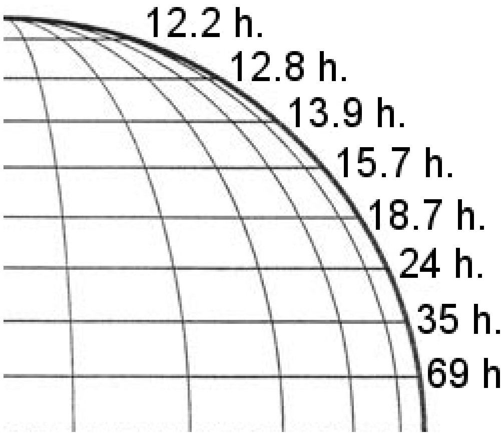
Picture 9. Image
The blue arrow represents the force of gravity. The red arrow represents the normal force. The green arrow represents the resultant force.



Picture 10. Image
Forces in the case of an oblate spheroid. The resultant force provides the required centripetal force.



Picture 11. Animation
Inertial oscillation over the surface of an oblate speroid.



Picture 12. Diagram
The theoretically predicted period of inertial oscillations at various latitudes.

The shape of the Earth and the shape of a parabolic dish manufactured as described above are analogous cases. The solid Earth is ductile; while the Earth's rock is brittle under impact, over periods of millions of years it behaves as a very viscous fluid. (For a demonstration of a very viscous fluid see the Pitch drop experiment.) The Earth's shape develops automatically towards an equilibrium shape. The equilibrium state is one of hydrostatic equilibrium. When the solar system was formed the Earth started out as a protoplanetary disk, and over time this distribution of space debris contracted and compacted to a final rotation rate and shape: an oblate spheroid. The current oblateness is tuned to the current rotation rate, just as the parabolic dish's manufacturing process tuned it to a particular rotation rate.

On the poles and on the equator the force of gravity and the normal force are exactly opposite in direction; on all other latitudes the two are not exactly opposite, giving rise to a resultant force, that provides the centripetal force that is necessary for buoyant objects to remain on the same latitude. Without this centripetal force objects would slide towards the equator.

The centripetal force that is depicted in Image 10 can be decomposed in a component parallel to the local surface and a component perpendicular to the local surface. The component that is parallel to the local surface is called the **poleward force**.

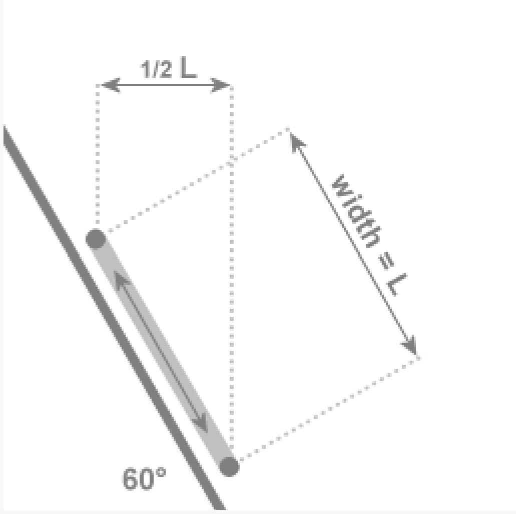
Inertial oscillation over the surface of an oblate spheroid

In effect each hemisphere is a bowl, with the *poles* as points of *lowest gravitational potential*. Animation 11 represents the motion of a buoy that is co-moving with a mass of seawater that is in inertial oscillation as depicted in the image at the start of the article. The motion pattern is the same as the motion pattern depicted in animation 4. If you would be far in space, exactly above the north pole, but thousends of kilometers above it, then the ellipse-shape of the trajectory would be plain to see.

The depiction in animation 11 is highly schematized. Actual inertial oscillations of surface level seawater have amplitudes in the order of several kilometers, in the animation the oscillations have an amplitude that is a considerable fraction of the Earth's diameter.

As in the analogous case of motion over the surface of parabolic dish the motion pattern is determined by a centripetal force. There is an oscillation between the poleward force doing positive work and doing negative work. When the buoy is furthest away from the poles it is circumnavigating the Earth's axis slower than the Earth, and consequently the poleward force pulls it closer to the Earth's axis, and in pulling it closer the angular velocity of the buoy increases again.

There is a very interesting inertial oscillation visualizer available on the oceanmotion.org website. Try for example the following settings:
· Starting direction: Southward.
· Starting velocity 20 m/s.
· Latitude 15 degrees.
The Oceanmotion.org inertial oscillation visualizer also incorporates the beta-effect. Closer to the equator the Coriolis effect is weaker, so closer to the equator the deflection is less strong. As a consequence the inertion oscillation motion as a whole will travel westward.



Picture 13. Image
At 30 degrees latitude, when the inertial oscillation has an amplitude of L, the motion towards and away from the Earth's axis is 1/2 L

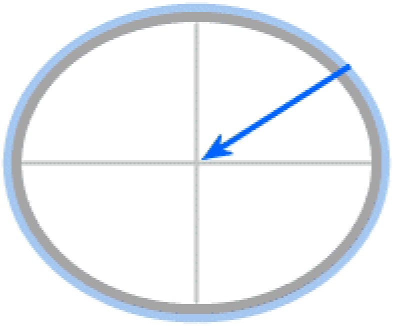
Dependency on latitude

The period of revolution of inertial oscillation is different at different latitudes. Diagram 12 shows the theoretically predicted period of inertial oscillation at various latitudes. The closer to the equator, the longer the period.

Work

Image 13 illustrates why the inertial oscillations have longer periods the further away from the poles. The amount of work that the centripetal force does is proportional to how much distance is travelled towards and away from the central axis of rotation. The inertial mass of the object in inertial oscillation is always the same, but the amount of work that the centripetal force does is not the same at every latitude; the closer to the equator, the smaller the amount of work that the centripetal force can do.

Buoyancy and the poleward force



Picture 14 . Image
The poleward force.

In Image 14 the blue arrow represents the poleward force. It's important to note that the poleward force applies if and only if the object is *supported*; it applies only when a normal force sustains the object's height with respect to the Earth's surface. Without the normal force as shown in the image there is no poleward force.

An airship (also known as a zeppelin) is an interesting example of a buoyant object. It is tempting to think of an airship as weightless, having released itself from effects of gravity altogether, and accordingly it's tempting to expect an airship to move in the same way as a satellite does, only slower. In fact an airship is subject to a normal force, as the airship displaces a large volume of air. Gravity and the normal force do not cancel each other, there is a small angle between them, that provides required centripetal force. A satellite on the other hand is *does not experience a normal force*; a satellite is subject to gravity only.

To denote the sense of an object's weight being supported, with little to no friction to move parallel to the supporting surface, I use the expression 'buoyant'. All of the air mass of the atmosphere is buoyant, and hence subject to the poleward force.

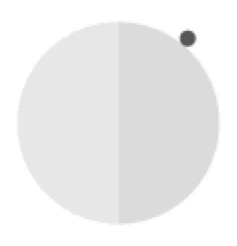
In the next article Comparison of ballistics and inertial oscillations I discuss the contrast between inertial oscillation and ballistic trajectories.

Presence of other influences

Inertial oscillation is the case where any pressure gradient is negligably small, the only influence is the terrestrial Coriolis effect.

In the actual atmosphere pressure gradients are affecting the direction of air mass all the time. In the articleformation of cyclonic flow. I discuss how the terrestrial Coriolis effect and pressure gradient force are both involved.

Why the oscillations are called inertial oscillations



Picture 15. Animation
Inertial oscillation over the surface of an oblate speroid.

Generally, inertial motion is asscociated with uniform motion along a straight line, and the motion depicted in the animation is far from uniform: it's not along a straight line and the velocity isn't constant. But there is another sense of 'inertial motion' that does apply in the case of inertial oscillation. In general, motion is referred to as 'inertial motion' when an accelerometer doesn't register any acceleration. Accelerometers onboard a satelite in orbit do not register acceleration since both the accelerometer casing and the movable parts inside are*identically influenced* by the Earth's gravitation. Orbital motion is free fall motion, the satellites are falling towards the Earth all the time - but they don't get there because they also have sufficient tangential velocity to overshoot the Earth all the time.

In the case of a buoy or a weather balloon that is moving along with inertial oscillation an onboard accelerometer would not register acceleration in the direction parallel to the local surface. Of course the accelerometer will register the gravitational acceleration that is associated with the local vertical component of the gravitational force; the buoy is supported by its buoyancy, the buoyancy support prevents it from "going with the flow" of the local vertical component of the gravitational acceleration. But the accelerometer won't register an acceleration *parallel* to the surface. The motion of the object parallel to the surface is motion that is "going with the flow" of gravitational acceleration, and then an accelerometer doesn't register acceleration. This illustrates again that it's useful to think of inertial oscillation as a form of orbital motion.

Source : http://www.cleonis.nl/physics/phys256/inertial_oscillations.php