Principles of Passive Vibration Control: Absorber with ideal spring

INTRODUCTION

A tuned vibration absorber is a relatively small spring-mass oscillator that suppresses the response of a relatively large, primary spring-mass oscillator at a particular frequency. Tuning (changing) the natural frequency of a system or component will reduce or eliminate amplification due to resonance. It is a process used to eliminate amplification due to resonance by changing a system or component natural frequency, f_n, so that it is no longer coincident with the frequency of a specific force input. Resonance of industrial equipment will amplify vibration response, in theory up to ∞, depending on system damping characteristics. The Synchronous Amplification Factor (SAF) is a measure of how much 1-X vibration is amplified when the system passes through a resonance. Systems with a high effective damping tend to have a low SAF, and systems with low effective damping have a high SAF.

Rigid body – ideal spring – rigid foundation

At much lower excitation frequencies, considerably simplified models of the components are usable. Assume, for example, that we analyze a machine mounted at four points on a system of concrete joists. Assume, moreover, that the machine has an axle that generates sinusoidal bearing forces at the rotational frequency. At very low disturbance frequencies (i.e., low rotational speeds), the deformations of the machine itself are negligible, i.e., the machine acts as a *rigid body*. Physically, one can regard the force acting on the machine as so slowly changing in time that all parts of it have time to react to small changes in the force magnitude before the next such change occurs. Mathematically, the machine's movements can be described by means of equations from rigid body mechanics. The instantaneous state of the machine is then completely described by six degrees-of-freedom, three translational and three rotational. In practice, the number of degrees-of-freedom can normally be further reduced to one or two, eliminating those which are not relevant.

As the rotational speed of the axle increases, we eventually arrive at a situation in which the force changes so rapidly that not all parts of the machine have time to react before the force changes again at the point of its application. At that stage, we can begin to speak of wave propagation in the machine. If the rotational speed continues to increase further, we will arrive at a certain excitation frequency at which the amplitude of the machine deformations has a strong peak. At that frequency, the deformation waves and their reflections interact constructively to bring about the maximum in the response. That phenomenon is the so-called resonance phenomenon with which we are already familiar from chapters 7 and 8. At these frequencies, we can no longer regard the machine as a rigid body. A commonly used rule of thumb is that the rigid body assumption is useful up to frequencies of 1/3 of the first resonance frequency, i.e., for low Helmholtz numbers.

The rigid body assumption for the machine has an analogue that can be used in the description of the foundation. Consider now the example of the machine described above. At very low excitation frequencies, the joists respond with a (quasi-)static bending due to the slowly varying force acting at the machine mounting points. If the excitation frequency is so low that the deformation of the joists is so small as to be negligible in comparison to the deformation of the isolators, then the joists can be regarded, from the vibrations perspective, as a *rigid foundation.* Note that this doesn't imply that the foundation is not excited into vibration; that would apply no transmission whatsoever. Let the excitation frequency now increase, just as it does when considering the machine. At sufficiently high frequencies, the deformation can no longer be ignored. When the frequency has increased sufficiently, an ever more distinct wave propagation becomes apparent in the foundation. If the geometrical limits of the foundation are far away, then we will eventually reach the first resonance frequency of the foundation. The description of the foundation as rigid can, consequently, only be applied at low frequencies, say up to 1/3 of the first resonance frequency, i.e., once again at low Helmholtz numbers.

Assume now that we would like to reduce the vibrations transmitted from the machine into the system of joists by incorporating soft vibration isolators at the mounting positions between the machine and the joists. Under the influence of forces from the machine, the springs are deformed. At low excitation frequencies, all parts of the isolator itself react to the changing of the force. That implies that the cross-sectional load is uniform along the entire isolator. We have, in other words, no considerable wave propagation. Yet another consequence is that the isolator can be considered massless. In contrast to the joists, the isolator is compliant. We can, therefore, not ignore its deformation under load. In these circumstances, the isolator can be regarded as an *ideal massless spring*. As the frequency increases, the motion in the spring takes on the character of wave propagation more and more. Once again, at a certain point, the situation becomes resonant. In exactly the same way as before, we can adopt the rule of thumb that the spring idealization applies up to about 1/3 of the first resonance frequency.

Example 7.1

Consider the machine arrangement illustrated in figure 1. An electric motor is elastically mounted, by way of 4 identical isolators, to a 2-mm thick steel plate. When the motor is driven, its rotating parts generate a vertically-oriented, sinusoidal exciting force between the machine and the joists. Calculate the ratio between the total force acting on the foundation with and without the vibration isolators. Carry out the calculations at low frequencies under the assumption that the electric motor, when operating, generates a vertical harmonic exciting force with circular frequency □ and amplitude $\hat{F}$. The mass of the motor is 100 kg, and each isolator's complex stiffness (see chapter 5, section 5.2.5) is $(1.0 + 0.01i)\cdot 10^4$ N/m.

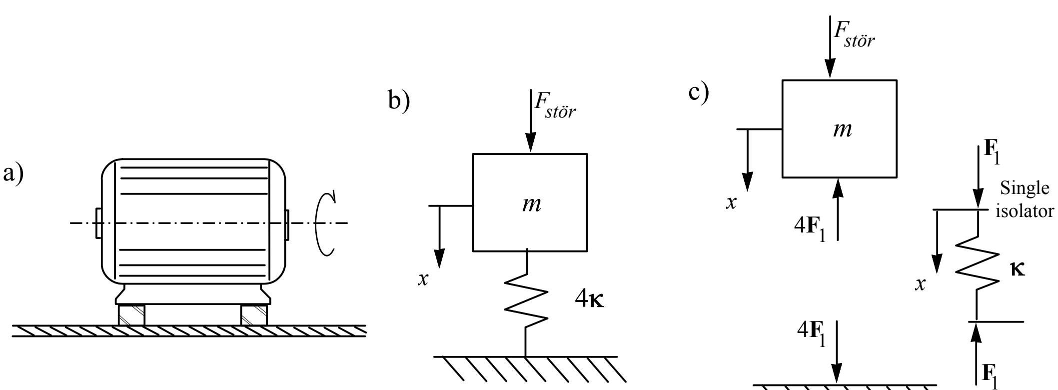

Figure 7.14 a) Electric motor elastically mounted to a large steel plate via four vibration isolators. b) Simplified model of the system in a. c) The system in b represented by its separated subsystems. [1]

Solution

Assume that the excitation frequency is so low that: (*i*) the motor can be considered a rigid body; (*ii*) the foundation can be regarded as rigid; and, (*iii*) each isolator can be described as an ideal massless spring. Assume, additionally, that the motors motions are strongly dominated by small-amplitude vertical translations. In these circumstances, the single degree-of-freedom system is a useful model to describe the problem.

<u>*With isolators.*</u>

Starting with the system in figure 7.14 c, the equation of motion can be constructed for the mass *m*, as well as Hooke's law for spring $\boldsymbol{\kappa}$ Thus,

$$m\frac{d^2\mathbf{x}}{dt^2} = F_{exc} - 4\mathbf{F}_1$$

where $4\mathbf{F}_1$ is the total force acting on the foundation, i.e.,. the force transmitted through all four isolators, and

$$\mathbf{F}_1 = \boldsymbol{\kappa}(\mathbf{x} - 0)\ .$$

Assume a sinusoidal, complex-valued displacement $\mathbf{x} = \hat{\mathbf{x}}e^{i\omega t}$ and eliminate $\mathbf{x}$ using both of the relations given above. Then, the force on the foundation, normalized by the exciting force, is

$$\frac{4\mathbf{F}_1}{F_{exc}} = \left(1 - \frac{\omega^2}{4\boldsymbol{\kappa}/m}\right)^{-1} = \left(1 - \frac{\omega^2}{\boldsymbol{\omega}_0^2}\right)^{-1},$$

where ω_0 is the machine's so-called *mounting resonance*, i.e., the resonance frequency of the machine mass on the compliance of the isolators. Note that the first term in the equation only applies to machines with four mounting points. For machines mounted at *n* points, the term $4\kappa/m$ should be replaced by $n\,\kappa/m$.

Without isolators.

For the case of no isolators, it becomes evident upon reflection that the force on the foundation is equal to F_{exc}. The desired ratio between the force with and without isolators is therefore

$$\frac{\mathbf{F}_u}{\mathbf{F}_m} = 1 - \frac{\omega^2}{\boldsymbol{\omega}_0^2} \ .$$

A very important conclusion from example 1 is that the vibration isolators must be designed to prevent the coincidence of the machine's mounting frequency with any important excitation frequency. Moreover, it is clear that a positive effect is obtained from the isolators at frequencies above the mounting frequency. The implication is that as low as possible a mounting resonance frequency must be sought. In practice, machine mounting is often designed so that the *mounting resonance frequency* falls in the 2-10 Hz band.

Flexible foundation

As the excitation frequency increases, the deformation of the foundation due to the excitation force soon becomes too large to ignore. A model in which the *foundation* is *flexible* must then be used. A number of different models with differing characteristics are available for this situation. If, for example, the foundation is a system of joists with considerable dimensions, an infinite plate model might be used to describe the motions of the foundation. If the foundation exhibits a resonance, then a mass-damper system can be used as a first approximation to describe its behavior.

Example 7.1

Consider the machine mounting situation of example 7.1. Assume that an infinite plate would be a valid model of the foundation response. Calculate the ratio between the total force on the foundation with and without isolators.

Solution

Assume that the deformation of the foundation is the same at all four machine feet. Additionally, conditions (*i*) and (*iii*) from example 1 hold,

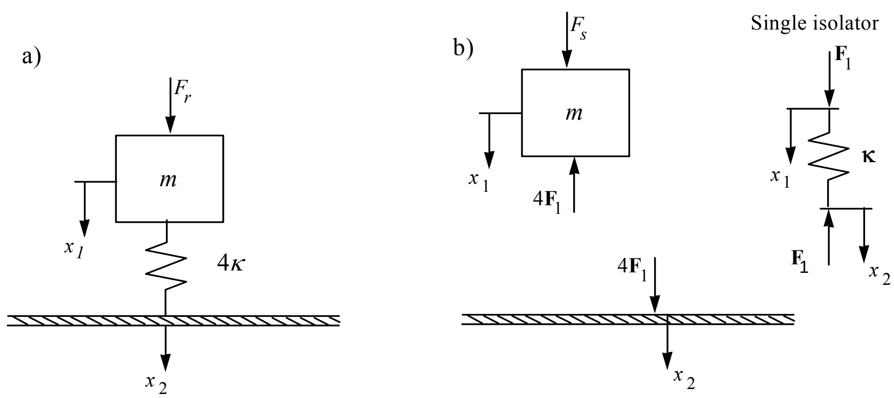

Figure 7.15 Simple model of a machine mounted to a flexible foundation.

The equation of motion, Hooke's law, and the mobility of a plate yield the following system of equations:

$$m\frac{d^2\mathbf{x}_1}{dt^2} = F_{exc} - 4\mathbf{F}_1,$$

$$\mathbf{F}_1 = \kappa(\mathbf{x}_1 - \mathbf{x}_2),$$

$$\mathbf{x}_2 = (i\omega)^{-1}\mathbf{Y}_{plate}4\mathbf{F}_1.$$

Eliminate $\mathbf{x}_1$ and $\mathbf{x}_2$,

$$\frac{4\mathbf{F}_1^{with}}{F_{exc}} = \frac{-4\boldsymbol{\kappa}/m\omega^2}{1-4\boldsymbol{\kappa}/m\omega^2 + 4\boldsymbol{\kappa}/(i\omega)\mathbf{Y}_{plate}} =$$

$$= \begin{Bmatrix} 1/i\omega m = \mathbf{Y}_m \\ i\omega/4\boldsymbol{\kappa} = \mathbf{Y}_I \end{Bmatrix} = \frac{\mathbf{Y}_m}{\mathbf{Y}_m + \mathbf{Y}_I + \mathbf{Y}_{plate}} .$$

Without isolators, the force on the foundation can be determined by excluding the second of the equations from the system given above, and setting $\mathbf{x}_1$ equal to $\mathbf{x}_2$. The system then has the solution

$$\frac{4\mathbf{F}_1^{without}}{F_{exc}} = \frac{1/i\omega m}{1/i\omega m + \mathbf{Y}_{plate}} = \frac{\mathbf{Y}_m}{\mathbf{Y}_m + \mathbf{Y}_{plate}} .$$

A couple of methods to improve vibration isolation

In some situations, it is possible to significantly improve the isolation performance with relatively modest additional effort. If very good isolation is a requirement, a so-called *double layered isolation* can be used. That can be regarded as a combination of *elastic elements* and a *blocking mass*; see figure 7.16. In practice, a double layered isolation is realized by interposing a large mass between the machine and the foundation. The blocking mass should behave as a rigid body up to frequencies that are as high as possible.

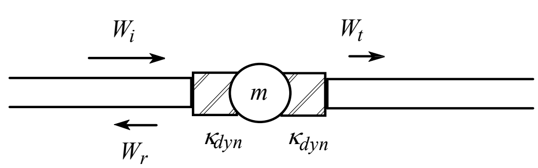

Figure 7.16 Schematic illustration of double layered isolation with two compliant elements and one stiff element. [1]

Passenger railway wagons are an example of double elastic mounting. The vibration source, i.e., the wheel-rail contact zone, is isolated first by a primary suspension between the bearings and the frame of the bogies; see figure 8.4. To further improve passenger comfort and obtain smooth ride characteristics, a secondary suspension, or comfort suspension, is interposed between the bogies and the body of the wagon.

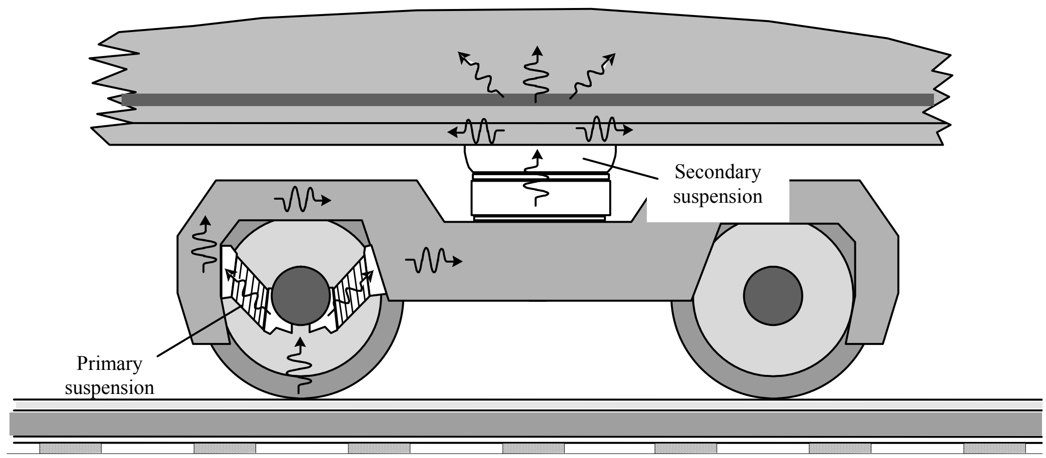

Figure 07.17 An example of double elastic mounting is the attachment of a railway wagon chassis to a bogie. The so-called primary suspension between the bearings and the and the frame is commonly built up of stiff, so-called chevron elements, of rubber. The secondary suspension, or comfort suspension, which connects the bogie to the chassis of the rail car consists of very compliant air springs or spiral springs in steel. [1]

If the double layer elastic mounting is well-constructed, the insertion loss can be improved. Because a rigid body has been added to the vibration isolation system, it now has six internal rigid body resonances. These cause another set of insertion loss minima at frequencies above the mounting resonance frequency. The isolation should therefore be designed such that those specific frequencies fall below the lowest important excitation frequency. If the added structure is designed to have mass and inertias of the same order of magnitude as those of the machine, then the internal resonances of the isolation system fall in the same range as the mounting resonance.

In some types of mechanical constructions, machines must be mounted to relatively compliant points. Examples are vehicles of various types. Motors on small boats, such as pleasure craft, are often mounted via vibration isolators directly to a thin hull. In some cases, the vibration isolation becomes completely ineffective as a result. The reason is that the impedance difference between the isolators and the mounting positions is too small. A way to increase that impedance difference is to add so-called *added masses* at the mounting points. If those are sufficiently large, the insertion loss can be considerably enhanced.

Commercially available vibration isolators

The market for vibration isolators is large. Commercially-available vibration isolators can be divided into several important types, including, among others, steel *coil springs*, *rubber isolators*, and gas springs; see figure 7.18. The two fundamental properties of an isolator are its dynamic stiffness and loss factor. The stiffness is, as we have seen, the property that largely determines the suitability of an isolator. The loss factor is significant as an amplitude-limiting parameter at resonances. Both of these parameters are dependent on, among other things, the frequency, and are usually experimentally determined.

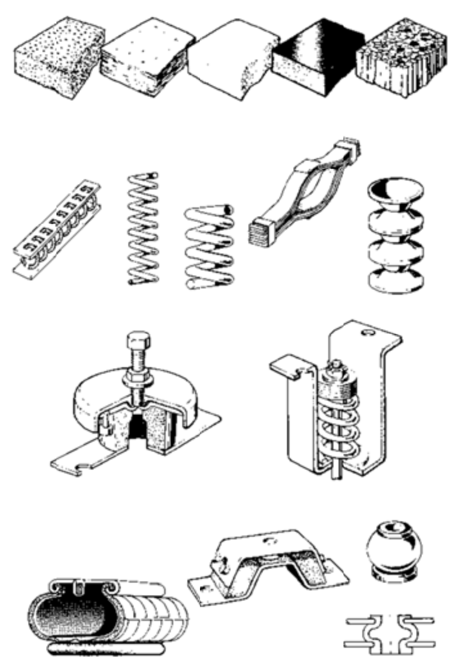

Figure 7.18 Examples of commercially-available vibration isolators. In practice, vibration isolators are usually either metallic coil springs or rubber blocks of various forms. Coil springs can be made very soft, but provide little damping. Rubber blocks

are relatively stiffer, but provide a significant amount of damping. (Picture: Brüel & Kjær.) [1]

Steel coil springs can be designed with very small stiffness values. If the lower frequency bound for isolation must be very low, say 2 - 3 Hz, then coil springs may be appropriate. A disadvantage, however, is that coil springs have a very small loss factor. Rubber isolators are the most commonly occurring type of isolator. They can be designed for either shear or compressive loading. In shear, they can be used down to about 3 Hz, and in compression down to about 5 Hz. A typical problem, however, is that the dynamic properties can vary considerably from one sample to the next; a variation of 30 - 40 % in the static stiffness of a certain type of isolator can occur. In critical cases, it can therefore be necessary to measure the actual, individual isolators to be used. *Gas springs* can be appropriate in situations where especially low resonance frequencies are desirable. Railway wagons and buses sometimes have gas springs that isolate the wagon from the bogie; see figure 7.17.

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