GEOMETRICAL METHODS TO LOCATE SECONDARY INSTANTANEOUS POLES OF SINGLE-DOF INDETERMINATE SPHERICAL MECHANISMS

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Abstract: *A single-degree-of-freedom (DOF) indeterminate spherical mechanism is defined as a mechanism for which it is not possible to find all the instantaneous poles by direct application of the Aronhold-Kennedy theorem. This paper shows that a secondary instantaneous pole of a two DOFs spherical mechanism lies on a unique great circle instantaneously. Using this property, two geometric methods are presented to locate secondary instantaneous poles of indeterminate single DOF spherical mechanisms. Common approach of the methods is to convert a single DOF indeterminate spherical mechanism into a two DOFs mechanism and then to find two great circles that the unknown instantaneous pole lies on the point of intersection of them. The presented methods are directly deduced from a work done for indeterminate single DOF planar mechanisms.*

Key Words: *instantaneous poles*, *geometrical techniques*, *indeterminate single-dof spherical mechanisms*

Nomenclature

PGC	Primary great circle
IGC	Infinity great circle
DGC	Declination great circle
P_{ij}	Instant pole between links *i* and *j*

INTRODUCTION

For two co-spherically moving shells, there exist two instantaneously coincident points, each belonging to the respective shell, the linear velocities of which are identical. The place of these common points is called Instantaneous Pole, henceforth referred to as Instant Pole, of the two shells[1]. Instant poles are spherical counterpart of instant centers in planar mechanisms; however they are not fully exploited to study kinematic behavior of spherical mechanisms as the instant centers are for planar ones, see for instance [2-6].
An instant pole which can be found by direct inspection will be referred to as a primary instant pole and an instant pole which cannot be found by direct inspection will be referred to as a secondary instant pole. Planar counterparts of these poles are primary instant center and secondary instant center, respectively. Some scholars dealt with determining secondary instant centers in planar mechanisms [e.g. 7-9]. Foster and Pennock [7, 8] presented some graphical methods to determine the secondary instant centers of any single DOF planar mechanism with kinematic indeterminacy. Gregorio[9] presented an algorithm that analytically computes the instant centers in single DOF planar mechanisms.
All of the above mentioned works can be done for spherical mechanisms using the concept of instant poles. For instance, deducing from his work[9], Gregorio presented an exhaustive algorithm[10] to determine the instant poles' positions of single DOF spherical mechanisms. Here, the author extends the methodology presented by Foster and Pennock [8] to locate secondary instant poles of indeterminate single DOF spherical mechanisms. The approach adopted in this paper to locate a secondary instant pole of a single DOF indeterminate mechanism is to convert the mechanism into a two DOFs mechanism[7]. There may be several strategies to accomplish this goal, this paper presents two; namely, (i) remove a binary link, or (ii) replace a single link with a pair of connected links by adding a revolute joint. By following a sequence of geometric constructions on the obtained two DOFs mechanism, the secondary instant poles of the original single DOF indeterminate mechanism can be located at the intersection of two unique two great circles. After using this method to find a secondary instant pole, some, or all, of the remaining secondary instant poles can be located by use of Aronhold-Kennedy theorem.
This paper is organized as follows: in next section, it is shown that a secondary instant pole of a two DOFs spherical mechanism locates on a specific great circle, instantaneously; in section 3, two geometric methods are presented to show how to apply this concept to determine secondary instant pole of single DOF indeterminate spherical mechanisms; section 4 presents two illustrative examples to show the methods; finally, section 5 presents some conclusions of this research activity.

SECONDARY INSTANT POLES OF A TWO DOFS SPHERICAL MECHANISM

Spherical mechanisms can be studied by projecting them through the spherical motion center onto a reference sphere with center at the spherical motion center, Fig. 1. So doing, the instant poles become points of the reference sphere and the links become spherical sells that move on the reference sphere.

With reference to a Cartesian system (Fig. 1) whose origin is at the spherical motion center, intersections between the reference sphere and the xy plane, the yz plane, and zx plane will be called primary great circle (PGC), infinity great circle (IGC), and declination great circle (DGC), respectively[10].

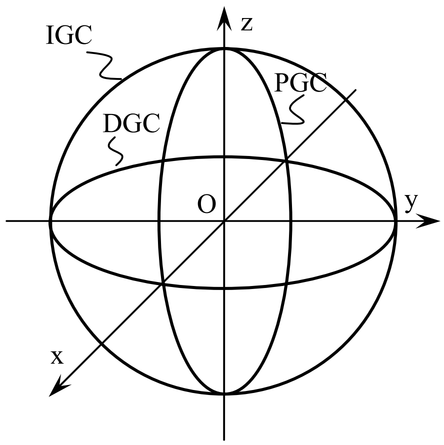

Figure 1. Reference sphere and Cartesian reference system fixed to the frame: IGC=infinity great circle, PGC=primary great circle, DGC=declination great circle.

The IGC cuts the reference sphere into two hemispheres: the one (positive hemisphere) whose points have a positive x coordinate and the other (negative hemisphere) whose points have a negative x coordinate. Two great circles with different slopes have only one intersection in the positive (negative) hemisphere, whereas two great circles with the same slope (i.e. that belong to the same pencil of meridians) do not intersect each other in the positive (negative) hemisphere. An Instantaneous pole Axis that does not lie on the yz plane intersects the reference sphere at two diametrically opposite points: one lying on the positive hemisphere and the other lying on the negative hemisphere. An Instantaneous pole Axis that lies on the yz plane cuts the IGC into two diametrically opposite points: one either with positive y coordinate, or with zero y coordinate and positive z coordinate, and the other either with negative y coordinate, or with zero y coordinate and negative z coordinate. Therefore, the points of the positive (negative) hemisphere plus the points of the IGC either with positive (negative) y coordinate or with zero y coordinate and positive (negative) z coordinate are sufficient to identify all the possible instant pole. This set of points, which is a subset of the RS, will be called positive (negative) shell.

Since instant poles' positions are sufficient to fully describe first-order kinematics of the spherical mechanisms, the first-order kinematics of spherical mechanisms can be studied by using only one (either positive or negative) shell of the reference sphere[10]. Hereafter, the positive shell will be used.

A bijective mapping can be defined[10] between points of the positive hemisphere and those of the reference plane x=1, which is tangent to reference sphere at point (1, 0, 0). Moreover, the slopes of the reference plane's straight lines, passing through the point of tangency between reference plane and reference sphere, uniquely locate IGC points of the positive shell. Therefore, the first-order kinematics of the spherical mechanisms can be fully described by using only points of the reference plane.

As a consequence of the above discussion, any theorem, considering the instantaneous kinematics of planar mechanisms, can have a spherical counterpart. Two of the theorems, which are useful in this study, are the Aronhold-Kennedy theorem and the one presented by Foster and Pennock[6]. Spherical counterparts of these theorems are stated as follows.

Theorem 1 (Aronhold-Kennedy theorem). Three instant poles of the three co-spherically moving links lie on a unique great circle.

Theorem 2. Secondary instant pole of a two degrees of freedom spherical mechanism lies on a unique great circle for a specific configuration of the mechanism.

In the following section, two methods are presented to find the secondary instant pole of an indeterminate single DOF spherical mechanism. In both methods, the indeterminate mechanism is converted into a two DOFs mechanism and for this obtained two DOFs mechanism, the great circle, on which the secondary instant pole must lie, is found by locating two points of it as follows: First, a constraint is introduced to convert the obtained two DOFs spherical mechanism into a single DOF one, in such a manner that the resulting mechanism does not have kinematic indeterminacy and the instant pole can be located by Aronhold-Kennedy theorem. The place of this instant pole identifies one of the points of the great circle. Then the procedure is repeated to find the second point of the great circle but using a different constraint.

TWO GEOMETRIC METHODS

In this section, two geometric methods are presented to locate a secondary instant pole, say P_{ij}, of the two links, say link *i* and link *j*, in an indeterminate single DOF spherical mechanism.

METHOD 1. This method should be used when links *i* and *j* are connected by a binary link. Let P_{ik} and P_{jk} be the instant poles which are coincident with two joints of link k. instant pole P_{ij} must lie on the great circle, denoted as the great circle m, passing through instant poles P_{ik} and P_{jk}. On the other hand, if link k is removed then degree of freedom of the mechanism becomes two. The Kutzbach mobility criterion[5] can be used to show that the new mechanism has gained a degree of freedom. For this new mechanism, the great circle, denoted as the great circle n, on which P_{ij} must lie, can be found using theorem 2. Finally, instant pole P_{ij} locates at the intersection of the great circles m and n. some, or all, of the remaining secondary instant poles can be found using the Aronhold-Kennedy theorem.

METHOD 2. Advantage of this method with respect to the previous method is that it places no restriction on the two links (i.e. links *i* and *j*). However this method needs to the more geometric constructions. The procedure is to replace one of the links, say link *j*, by a pair of links, say links k and k′, connected to each other through a revolute joint. The Kutzbach mobility criterion can again be used to show that this new mechanism has gained a degree of freedom. This new joint may, or may not, be coincident with an existing joint (See example 2). Note that relative motion of links k and k′ with respect to link *i*, must depend on both degrees of freedom of the new mechanism. If this is not the case, a different choice of links i and j must be made. Now, the great circles, on which instant poles P_{ik} and $P_{ik'}$ must lie respectively, can be found using the theorem 2. Links k and k′ can be considered as a single link when they have the same relative velocity with respect to any link of the mechanism, specially link *i*; in this case, instant poles P_{ik} and $P_{ik'}$ are coincident and the new mechanism is equivalent to the original indeterminate spherical mechanism. Therefore, the secondary instant pole P_{ij} locates at the intersection of the two great circles passing through P_{ik} and $P_{ik'}$, respectively. Now, the Aronhold-Kennedy theorem can be used to locate some, or all, of the remaining secondary instant poles.
After using method 1 or method 2, if all the remaining secondary instant poles can not be located by the Aronhold-Kennedy theorem, then make a different choice of links *i* and *j* and apply either method again. Also, additional constraint can be chosen to minimum the number of steps required to locate the instant pole.

ILLUSTRATIVE EXAMPLES

In this section, the secondary instant poles of two indeterminate single DOF spherical mechanisms are located to show the usefulness of the methods presented here.

EXAMPLE. 1. A TEN BAR INDETERMINATE SINGLE DOF SPHERICAL MECHANISM

Figure 2 shows a ten bar indeterminate single DOF spherical mechanism. Links 2 and 9 slide on the surface of reference sphere; also links 4 and 6 are connected to each other by a sliding joint. The focus of this example is to locate the secondary instant pole of the link 3, i.e. P_{13}. Note that links 1 and 3 are connected to each other through a binary link, so first method is used to locate the unknown instant pole. First, link 2 is removed which results in a two DOFs nine bar mechanism. This new mechanism along with 12 primary instant poles is shown in Fig. 3. Note that this mechanism has no four bar loops; therefore, there are no secondary instant poles that can be found using the Aronhold-Kennedy Theorem. Also note that primary instant poles P_{19}, P_{46} and P_{12} are located at the intersection of great circles normal to both spherical curves, slipping on each other, of the correspondent slipping contact.

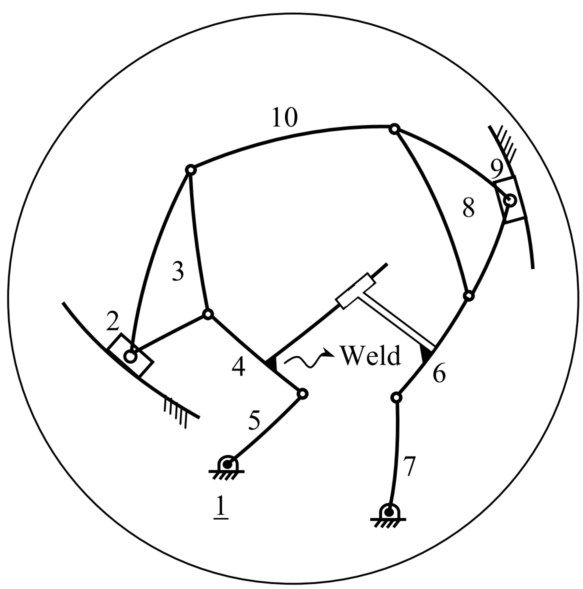

Figure 2. A Ten-Bar spherical mechanism with revolute and sliding joints.

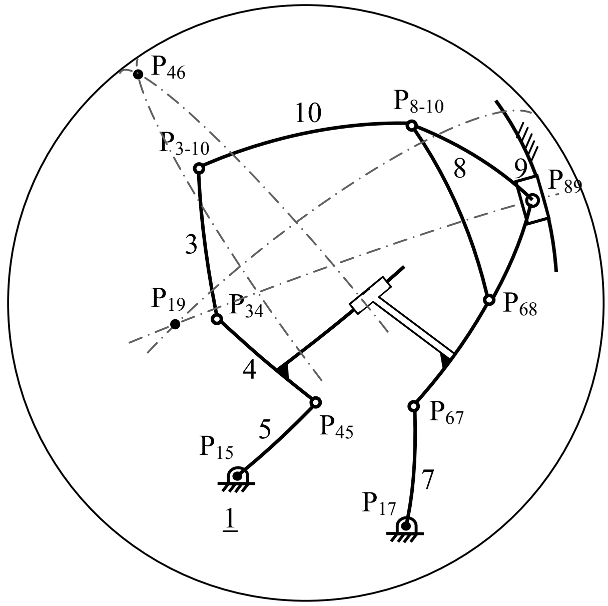

Figure 3. A two DOFs nine-bar spherical mechanism.

Instant pole P_{14} must lie on the great circle passing through P_{15} and P_{45}. An arbitrary point on this great circle is selected as the first choice for P_{14}, denoted as P^1_{14}, as shown in Fig. 4; then location of instant pole P^1_{13} can be obtained after locating instant poles P^1_{16}, P^1_{18}, P^1_{48} and P^1_{38}, respectively by use of the Aronhold-Kennedy Theorem, Fig. 4. Now a different point is selected on the locus of P_{14} as the second choice of the instant pole P_{14}, denoted as P^2_{14}, see Fig. 5. Doing a procedure similar to the previous step leads to the location of instant pole P^2_{13}, Fig. 5. With reference to theorem 2, P_{13} must lie on the great circle passing through P^1_{13} and P^2_{13}, as shown in Fig. 5. Finally, link 2 is replaced to restore the original mechanism. In the original mechanism, P_{14} must lie on the great circle passing through P_{12} and P_{23}. Point of intersection of the two aforementioned great circles determines the location of instant pole P_{13}, Fig. 6. The remaining instant poles can be located by the Aronhold-Kennedy Theorem.

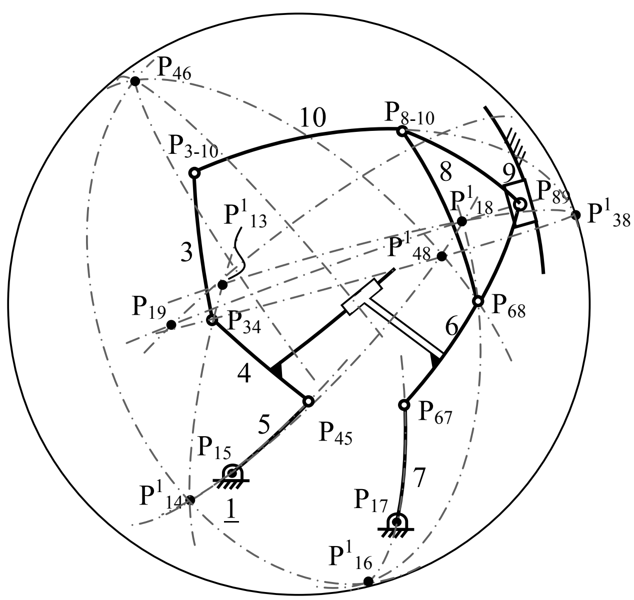

Figure 4. The first arbitrary choice of instant pole P_{14}.

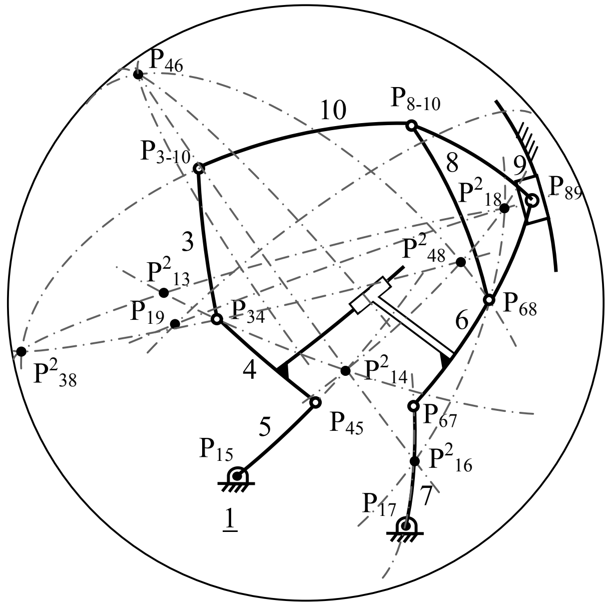

Figure 5. The second arbitrary choice of instant pole P_{14}.

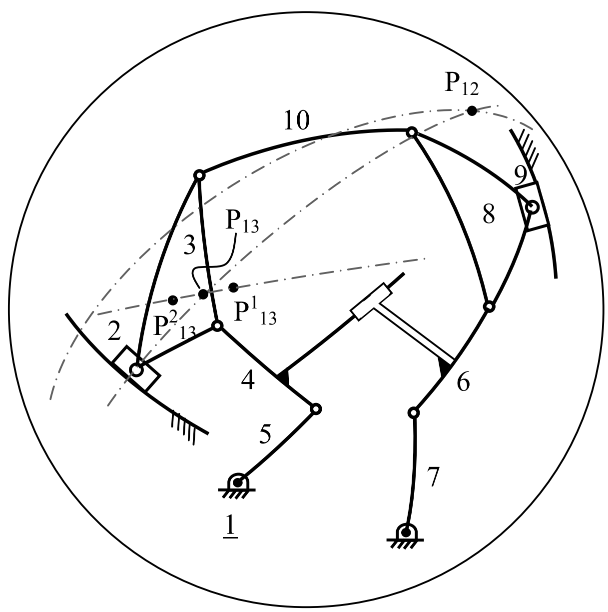

Figure 6. The secondary instant pole P_{13}.

EXAMPLE. 2. A INDETERMINATE SINGLE FLIER EIGHT-BAR SPHERICAL MECHANISM

Figure 7 shows a single flier eight-bar spherical mechanism. The purpose of this example is to locate the secondary instant pole P_{48}. The second method is chosen to locate the instant pole. It is worth noting that links 4 and 8 are not connected to each other by a binary link, so the method 1 can not be used here.

First, link 8 is replaced by two links, denoted as 8 and 8′, which results in a new two DOFs spherical mechanism with nine links, Fig. 8. Note that secondary instant poles P_{13} and P_{24} can be located by Aronhold-Kennedy theorem.

Connecting joint of links 8 and 8′ is considered to be coincident with P_{68} of the original single DOF mechanism. In this step, the two great circles, on which the instant poles P_{48} and $P_{48'}$ must lie, are found. Instant pole P_{27} must lie on the great circle passing through P_{24} and P_{47}; an arbitrary point on this great circle is chosen as the first choice for the instant pole P_{27} (denoted here as P^1_{27}). This action reduces the two DOFs mechanism to a single DOF mechanism in which the location of instant poles P^1_{48} and $P^1_{48'}$ can be obtained after finding the location of instant poles P^1_{37}, $P^1_{38'}$, $P^1_{28'}$ and P^1_{28}, respectively, using the Aronhold-Kennedy theorem, see Fig. 9.

Now, a second arbitrary point is chosen on the great circle as the location of instant pole P_{27} (denoted here as P^2_{27}). In this case, repeating the above procedure, using P^2_{27}, leads to the location of instant poles P^2_{48} and $P^2_{48'}$, Fig. 10.

Links 8 and 8′ act as a single link instantaneously when P_{48} locates at the intersection of two great circles P^1_{48} P^2_{48} and $P^1_{48'}$ $P^2_{48'}$, as shown in Fig. 11. All of the remaining instant pole can be obtained using the Aronhold-Kennedy theorem.

Note that this method requires a greater number of steps indicating that the first method should be used whenever it is a valid method of solution.

For some indeterminate mechanisms, locating all the secondary instant poles from the methods presented in this paper depends on the links chosen as links *i* and *j*. For instance, in the first example presented above, the choice of links 1 and 3 as links *i* and *j* results in successfully locating all the secondary instant poles. However, if links 3 and 8 are chosen instead and therefore link 10 removed, then the only other secondary instant poles that can be located by the Aronhold-Kennedy theorem are P_{36}, P_{48}, $P_{4\text{-}10}$ and $P_{6\text{-}10}$. Therefore the choice of links 3 and 8 does not lead to the locating all the secondary instant poles.

CONCLUSION

This paper presents two geometrical methods to locate the secondary instant poles of single DOF spherical mechanisms with kinematic indeterminacy. The paper shows that a secondary instant pole of a two DOFs spherical mechanism lies somewhere on a unique great circle for a specific configuration of the mechanism. This fact is used to locate the secondary instant pole at the intersection of two great circles; one of them is found in the original single DOF mechanism and another is found after converting the mechanism into a two DOFs mechanism. Two techniques are presented to convert a single DOF spherical mechanism to two DOFs one; First technique is to remove a binary link and the second technique is to replace a single link with a dyad. The two methods are illustrated by two examples of indeterminate single DOF spherical mechanisms. The methods are purely graphical and can locate the secondary instant pole in indeterminate spherical mechanisms with little geometric constructions.

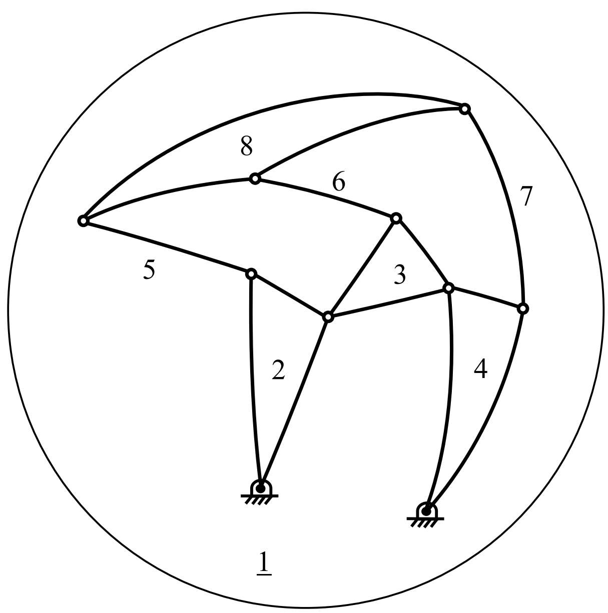

Figure 7. The single flier eight-bar spherical mechanism.

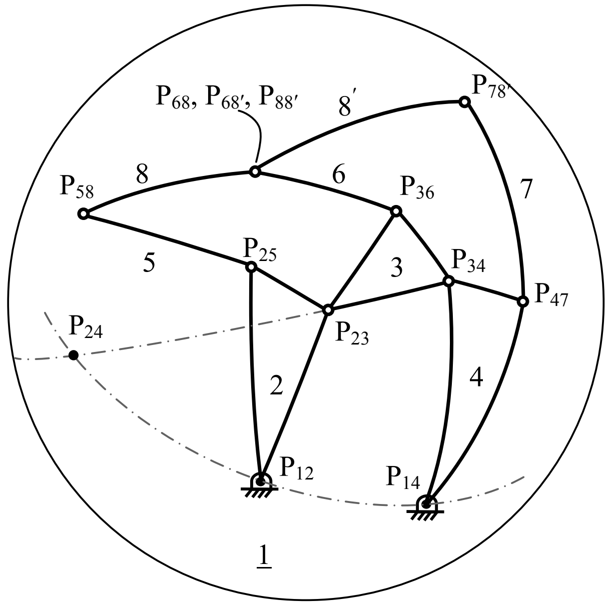

Figure 8. A two degrees of freedom nine-bar spherical mechanism.

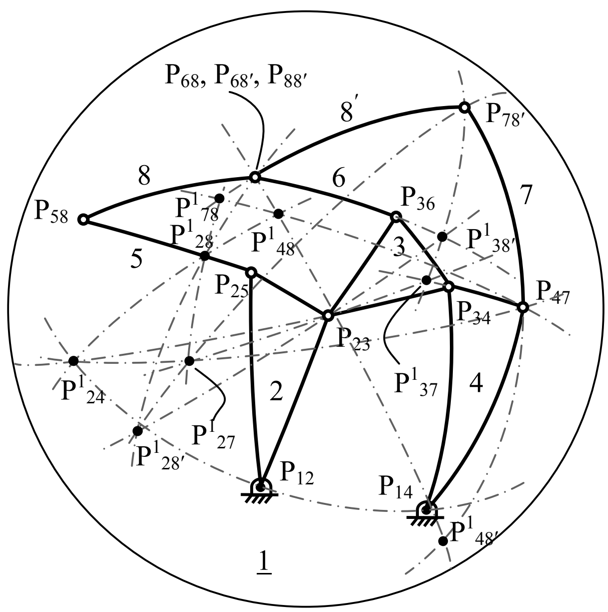

Figure 9. The first arbitrary choice of instant pole P_{27}.

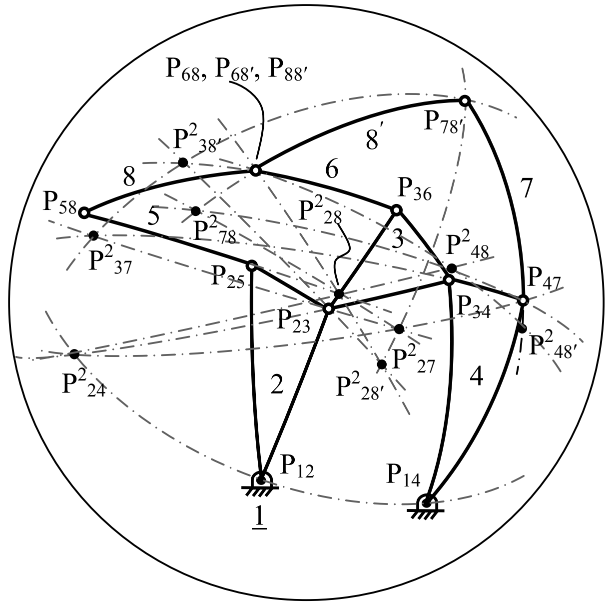

Figure 10. The second arbitrary choice of instant pole P_{27}.

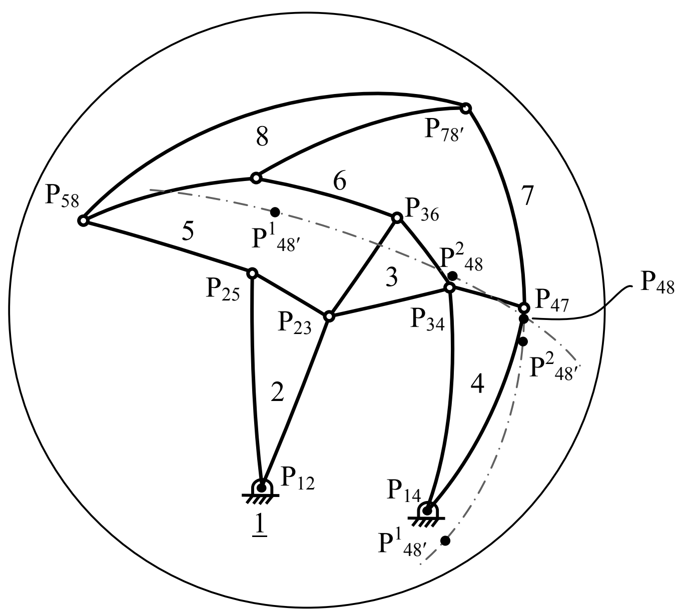

Figure 11. The secondary instant pole P_{28}.

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