Fuzzy Hungarian Approach for Transportation Model

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Abstract - In this paper, a method is proposed to find the fuzzy optimal solution of fuzzy transportation model by representing all the parameters as trapezoidal fuzzy numbers. To illustrate the proposed method a fuzzy transportation problem is solved by using the proposed method and the results are obtained. The proposed method is easy to understand, and to apply for finding the fuzzy optimal solution of fuzzy transportation models in real life situations. However, we propose the method of fuzzy modified distribution for finding out the optimal solution for minimizing the cost of total fuzzy transportation. The advantages of the proposed method are also discussed.

Keywords - *Fuzzy transportation; Trapezoidal fuzzy numbers; Optimal solution.*

I. INTRODUCTION

The transportation problem which, is one of network integer programming problems is a problem that deals with distributing any commodity from any group of 'sources' to any group of destinations or 'sinks' in the most cost effective way with a given 'supply' and 'demand' constraints . Depending on the nature of the cost function, the transportation problem can be categorized into linear and nonlinear transportation problem.

Transportation problem is a linear programming problem stemmed from a network structure consisting of a finite number of nodes and arcs attached to them. In a typical problem a production is to be transported from m sources to n destinations and their capacities are a1, a2,...am and b1 ,b2 ...bn, respectively. There is a penalty Cij & Variable Xij associated with transporting unit of production & unknown quantity to be shipped from source i to destination j.

Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. However, there are cases that these parameters may not be presented in a precise manner. For example, the unit shipping cost may vary in a time frame. The supplies and demands may be uncertain due to some uncontrollable factors.

Bellman and Zadeh [1] proposed the concept of decision making in fuzzy environment. Lai and Hwang [2] others considered the situation where all parameters are fuzzy. In 1979, Isermann [3] introduced algorithm for solving this problem which provides effective solutions. The Ringuest and Rinks [4] proposed two iterative algorithms for solving linear, multi criteria transportation problem. S.Chanas and D.Kuchta [6] the approach based on interval and fuzzy coefficients had been elaborated. In this work, the fuzzy transportation problems using trapezoidal fuzzy numbers are discussed. Here after, we have to propose the method of fuzzy modified distribution to be finding out the optimal solution for the total fuzzy transportation minimum cost. There are also studies discussing the fuzzy transportation problem. Chanas et al. [6] investigate the transportation problem with fuzzy supplies and demands and solve them via the parametric programming technique in terms of the Bellman and Zadeh criterion. Their method is to derive the solution which simultaneously satisfies the constraints and the goal to a maximal degree. In this paper fuzzy transportation problem is discussed with constraints where the supply and demand are trapezoidal fuzzy numbers. This paper aims to find out the best compromise solution among the set of feasible solutions for Fuzzy transportation problem.

II. FUZZY TRANSPORTATION MODEL FORMULATION

We deal with the production and transportation planning of a certain manufacturer that has production facilities and central stores for resellers in several sites in Pune. Each store can receive products from all production plants and it is not necessary that all products are produced in all production units.

Assume that a logistics center seeks to determine the transportation plan of a homogeneous commodity from m sources to n destinations. Each source has an available supply of the commodity to distribute tovarious destinations, and each destination has a forecast demand of the commodity to be received from various sources. This work focuses on developing an FLP method for optimizing the transportation plan in fuzzy environments.

2.1 Index sets

i index for source, for all $i = 1, 2, \ldots, m$

j index for destination, for all $j = 1, 2, \ldots, n$

g index for objectives, for all $g = 1, 2, \ldots, k$

2.2 Decision variables

Q_{ij} units transported from source *i* to destination *j* (units)

2.3 Objective functions

Z_1 transportation costs (Rs.)

2.4 Parameters

C_{ij} transportation cost per unit delivered from source *i* to destination *j* (Rs/unit)

S_i total available supply at each source *i* (units)

D_j total forecast demand at each destination *j* (units)

2.5 Objective functions

Minimize total transportation costs

$$\text{Min } Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} (C_{ij})\, Q_{ij}$$

Constraints on total available supply for each source *i*

$$\sum_{j=1}^{n} Q_{ij} \le S_i,$$

Constraints on total forecast demand for each destination *j*

$$\sum_{j=1}^{m} Q_{ij} \ge D_j,$$

If any of the parameters Q_{ij}, S_i, or D_j is fuzzy, the total transportation cost Z becomes fuzzy as well. The conventional transportation problem defined then turns into the fuzzy transportation problem.

III. FUZZY TRANSPORTATION MODEL ILLUSTRATION

Consider transportation with *m* fuzzy origins (rows) and *n* fuzzy destinations (columns). Let $C_{ij} = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]$ be the cost of transporting one unit of the product from i^{th} fuzzy origin to j^{th} fuzzy destination. $S_i = [s_i^{(1)}, s_i^{(2)}, s_i^{(3)}, s_i^{(4)}]$ be the quantity of commodity available at fuzzy origin i , $D_j = [d_j^{(1)}, d_j^{(2)}, d_j^{(3)}, d_j^{(4)}]$ the quantity of commodity needed at fuzzy destination j. $X_{ij} = [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$ is quantity transported from i^{th} fuzzy origin to j^{th} fuzzy destination. The above fuzzy transportation problem can be stated in the below tabular form.

1	**1**	**2…**	**n**	**Fuzzy Supply**
1	C_{11} X_{11}	C_{12} X_{12}	C_{1n} X_{1n}	S_1
2 . .	C_{21} . . X_{21}	C_{22} . . X_{22}	C_{2n} . . X_{2n}	S_2 . .
m	C_{m1} X_{m1}	C_{m2} X_{m2}	C_{mn} X_{mn}	S_m
Fuzzy Dem	d_1	$d_{1..}$	d_n	$\sum_{j=1}^{n} d_j = \sum_{i=1}^{m} S_i$

Where

$C_{ij} = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]$,

$X_{ij} = [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$,

$a_i = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}]$ and

$b_j = [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}]$

3.1 Methodology

The approach used to solve the transportation problem has following steps:

i. Identify the lowest cost in each column and row of the transportation matrix and subtract the lowest cost identified for each row or column from the respective row and column.

ii. Check if each column fuzzy demand is less than to the sum of fuzzy supply whose reduced cost in that column are fuzzy zero. Also check if each row fuzzy supply is less than to sum of the column fuzzy demands whose reduced costs in that row are fuzzy zero. If so go to step 4 of go to step 3.

iii. Draw minimum number of horizontal & vertical lines to cover all fuzzy zeros. Find the smallest fuzzy cost not covered by any line & subtract it from all uncovered fuzzy costs and add the same to all fuzzy costs lying at the intersection of any two lines. Apply this step till fuzzy supply satisfies fuzzy demand for all rows & columns.

iv. Allocate the maximum quantity to be transported where the costs have been zero depending on the fuzzy demand and fuzzy supply.

v. Repeat the procedure till all supply and demand quantities are exhausted.

vi. Use the MODI method for optimal solution.

Solution

To obtain the initial basic solution the Hungarian method is applied to the following Fuzzy transportation problem.

	D1	D2	D3	Supp
S1	[6 8 9]	[9 10 14]	[11 12 13]	[12 14 14]
S2	[14 16 17]	[8 10 11]	[10 14 15]	[15 16 17]
S3	[8 9 10]	[16 17 20]	[4 6 7]	[9 10 12]
Dem	[9 10 11]	[13 14 15]	[14 16 17]	[36 40 43]

This is balanced fuzzy transportation problem. Applying the step 1and 2 of solution we obtain the allocated matrix as below. The lowest cost cell of the row S1 is S1D4 hence subtracting the lowest cost from all the costs of the cell and so on for all remaining rows and columns. We obtain the final fuzzy cost matrix.

	D1	**D2**	**D3**	**Supp**
S1	[0 0 0]	[3 2 5]	[5 4 4]	**[12 14 14]**
S2	[6 6 6]	[0 0 0]	[2 4 4]	**[15 16 17]**
S3	[4 3 3]	[12 11 13]	[0 0 0]	**[9 10 12]**
Dem	**[9 10 11]**	**[13 14 15]**	**[14 16 17]**	**[36 40 43]**

As fuzzy Supply does not satisfy fuzzy demand go to step 3

	D1	**D2**	**D3**	**Supp**
S1	[0 0 0]	[3 2 5]	[5 4 4]	**[12 14 14]**
S2	[6 6 6]	[0 0 0]	[2 4 4]	**[15 16 17]**
S3	[4 3 3]	[12 11 13]	[0 0 0]	**[9 10 12]**
Dem	**[9 10 11]**	**[13 14 15]**	**[14 16 17]**	**[36 40 43]**

Repeating the step 3 we get the reduced fuzzy cost matrix as below

	D1	**D2**	**D3**	**Supp**
S1	[0 0 0] **[9 10 11]**	[3 2 5]	[5 4 4] **[3 4 3]**	**[12 14 14]**
S2	[6 6 6]	[0 0 0] **[13 14 15]**	[2 4 4] **[2 2 2]**	**[15 16 17]**
S3	[4 3 3]	[12 11 13]	[0 0 0] **[9 10 12]**	**[9 10 12]**
Dem	**[9 10 11]**	**[13 14 15]**	**[14 16 17]**	**[36 40 43]**

Optimal solution for fuzzy transportation problem. Therefore, the initial fuzzy transportation minimum cost is,

	D1	**D2**	**D3**	**Supp**
S1	[6 8 9] **[9 10 11]**	[9 10 14]	[11 12 13] **[3 4 3]**	**[12 14 14]**
S2	[14 16 17]	[8 10 11] **[13 14 15]**	[10 14 15] **[2 2 2]**	**[15 16 17]**
S3	[8 9 10]	[16 17 20]	[4 6 7] **[9 10 12]**	**[9 10 12]**
Dem	**[9 10 11]**	**[13 14 15]**	**[14 16 17]**	**[36 40 43]**

Applying the fuzzy modified distribution method, we determine a set of numbers $[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}]$ and $[v_j^{(1)}, v_j^{(2)}, v_j^{(3)}]$ each row and column such that

$[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}] = [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}] + [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}]$ for each occupied cell.

Since 3rd column has maximum numbers of allocations, we give fuzzy number $[v_3^{(1)}, v_3^{(2)}, v_3^{(3)}] = [-1,0,1]$. The remaining numbers can be obtained as given below.

$[c_{13}^{(1)}, c_{13}^{(2)}, c_{13}^{(3)}] = [u_1^{(1)}, u_1^{(2)}, u_1^{(3)}] + [v_3^{(1)}, v_3^{(2)}, v_3^{(3)}]$

□ $[u_1^{(1)}, u_1^{(2)}, u_1^{(3)}] = [4\ \ 4\ \ 5]$

$[c_{23}^{(1)}, c_{23}^{(2)}, c_{23}^{(3)}] = [u_2^{(1)}, u_2^{(2)}, u_2^{(3)}] + [v_3^{(1)}, v_3^{(2)}, v_3^{(3)}]$

□ $[u_2^{(1)}, u_2^{(2)}, u_2^{(3)}] = [1\ \ 4\ \ 5]$

$[c_{33}^{(1)}, c_{33}^{(2)}, c_{33}^{(3)}] = [u_3^{(1)}, u_3^{(2)}, u_3^{(3)}] + [v_3^{(1)}, v_3^{(2)}, v_3^{(3)}]$

□ $[u_3^{(1)}, u_3^{(2)}, u_3^{(3)}] = [3\ \ 6\ \ 8]$

$[c_{22}^{(1)}, c_{22}^{(2)}, c_{22}^{(3)}] = [u_2^{(1)}, u_2^{(2)}, u_2^{(3)}] + [v_2^{(1)}, v_2^{(2)}, v_2^{(3)}]$

□ $[v_2^{(1)}, v_2^{(2)}, v_2^{(3)}] = [7\ \ 6\ \ 6]$

$[c_{11}^{(1)}, c_{11}^{(2)}, c_{11}^{(3)}] = [u_1^{(1)}, u_1^{(2)}, u_1^{(3)}] + [v_1^{(1)}, v_1^{(2)}, v_1^{(3)}]$

□ $[v_1^{(1)}, v_1^{(2)}, v_1^{(3)}] = [2\ \ 4\ \ 4]$

We calculate penalties by using following formula,

$[\Delta c_{ij}^{(1)}, \Delta c_{ij}^{(2)}, \Delta c_{ij}^{(3)}] = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}] - \{[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}] + [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}]\}$

We find, for each empty cell of the sum $[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}]$ and $[v_j^{(1)}, v_j^{(2)}, v_j^{(3)}]$.

$[\Delta c_{21}^{(1)}, \Delta c_{21}^{(2)}, \Delta c_{21}^{(3)}] = [c_{21}^{(1)}, c_{21}^{(2)}, c_{21}^{(3)}] - \{[u_2^{(1)}, u_2^{(2)}, u_2^{(3)}] + [v_1^{(1)}, v_1^{(2)}, v_1^{(3)}]\}$

□ $[\Delta c_{21}^{(1)}, \Delta c_{21}^{(2)}, \Delta c_{21}^{(3)}] = [11\ \ 8\ \ 8]$

$[\Delta c_{31}^{(1)}, \Delta c_{31}^{(2)}, \Delta c_{31}^{(3)}] = [c_{31}^{(1)}, c_{31}^{(2)}, c_{31}^{(3)}] - \{[u_3^{(1)}, u_3^{(2)}, u_3^{(3)}] + [v_1^{(1)}, v_1^{(2)}, v_1^{(3)}]\}$

□ $[\Delta c_{31}^{(1)}, \Delta c_{31}^{(2)}, \Delta c_{31}^{(3)}] = [3\ \ -1\ \ -2]$

$[\Delta c_{32}^{(1)}, \Delta c_{32}^{(2)}, \Delta c_{32}^{(3)}] = [c_{32}^{(1)}, c_{32}^{(2)}, c_{32}^{(3)}] - \{[u_3^{(1)}, u_3^{(2)}, u_3^{(3)}] + [v_2^{(1)}, v_2^{(2)}, v_2^{(3)}]\}$

□ $[\Delta c_{32}^{(1)}, \Delta c_{32}^{(2)}, \Delta c_{32}^{(3)}] = [6\ \ 5\ \ 6]$

$[\Delta c_{12}^{(1)}, \Delta c_{12}^{(2)}, \Delta c_{12}^{(3)}] = [c_{12}^{(1)}, c_{12}^{(2)}, c_{12}^{(3)}] - \{[u_1^{(1)}, u_1^{(2)}, u_1^{(3)}] + [v_2^{(1)}, v_2^{(2)}, v_2^{(3)}]\}$

□ $[\Delta c_{12}^{(1)}, \Delta c_{12}^{(2)}, \Delta c_{12}^{(3)}] = [-2\ \ 0\ \ 3]$

Next we find net evaluation $[Z_{ij}^{(1),} Z_{ij}^{(2)}, Z_{ij}^{(3)}]$ is given by

	D1	D2	D3	U_j
S1	[6 8 9] **[9 10 11]**	[9 10 14]	[11 12 13] **[3 4 3]**	**[12 14 14]**
S2	[14 16 17]	[8 10 11] **[13 14 15]**	[10 14 15] **[2 2 2]**	**[15 16 17]**
S3	[8 9 10]	[16 17 20]	[4 6 7] **[9 10 12]**	**[9 10 12]**
V_j	**[9 10 11]**	**[13 14 15]**	**[14 16 17]**	**[36 40 43]**

Where,

$U_i = [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}]$, $V_j = [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}]$ and

Since all $[\Delta c_{12}^{(1)}, \Delta c_{12}^{(2)}, \Delta c_{12}^{(3)}] > 0$ the solution is fuzzy optimal and unique.

Therefore, the fuzzy optimal solution in terms of trapezoidal fuzzy numbers is given by:

$[x_{11}^{(1)}, x_{11}^{(2)}, x_{11}^{(3)}] =$ **[9 10 11]**

$[x_{22}^{(1)}, x_{22}^{(2)}, x_{22}^{(3)}] =$ **[13 14 15]**

$[x_{13}^{(1)}, x_{13}^{(2)}, x_{13}^{(3)}] =$ **[3 4 3]**

$[x_{23}^{(1)}, x_{23}^{(2)}, x_{23}^{(3)}] =$ **[2 2 2]**

$[x_{33}^{(1)}, x_{33}^{(2)}, x_{33}^{(3)}] =$ **[9 10 12]**

Total Fuzzy Transportation cost=

[9 (6+8+9) 10 (6+8+9) 11 (6+8+9)] +

[13(8+10+11) 14 (8+10+11) 15 (8+10+11)] +

[3 (11+12+13) 4 (11+12+13) 3 (11+12+13)] +

[2 (10+14+15) 2 (10+14+15) 2 (10+14+15)] +

[9 (4+6+7) 10 (4+6+7) 12(4+6+7)]

= [207 230 253] + [377 406 435] + [108 120 130] + [60 84 90] + [153 170 204]

= 336.33

IV. CONCLUSION

Transportation models have wide applications in logistics and supply chain for reducing the cost. Some previous studies have devised solution procedures for fuzzy transportation problems. In this paper we have thus obtained an optimal solution for a fuzzy transportation problem using trapezoidal fuzzy number. A new approach called fuzzy modified computational procedure to find the optimal solution is also discussed. The new arithmetic operations of trapezoidal fuzzy numbers are employed to get the fuzzy optimal solutions.

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