EQUATORIAL BULGE

The Earth has an equatorial bulge of 42.72 km (26.5 miles) due to its rotation. That is, its diameter measured across the equatorial plane (12756.28 km, 7,927 miles) is 42.72 km more than that measured between the poles (12713.56 km, 7,900 miles).

Kinetic energy and potential energy

Picture 1. Image
The circular band is elastic. When the model is not in motion the tension of the elastic band keeps it in a circular shape.

Picture 2. Animation
Spinning the ring-shaped band makes it bulge out.

Image 1 shows a device that illustrates the physics of equatorial bulge. The ring is a band, made out of a springy metal. The elasticity of the metal makes the band resist deformation.

An elastic deformation of the metal band has a corresponding potential energy. That is, elastic deformation of the metal bands acts as a repository of potential energy.

So what happens when you spin up the device? By driving the wheel you are doing work, which means the kinetic energy of the band increases.

As the band bulges out some of the energy goes to elastic potential energy. In fact, once the device has settled onto a constant rotation rate the energy that was put into the system is divided equally: half the energy is kinetic energy, half the energy is elastic potential energy.

Why does the energy divide evenly like that?
The key is to think in terms of two repositories of energy: kinetic energy and elastic potential energy.
When the band bulges out kinetic energy is transformed to elastic potential energy. Conversely, when the spin rate goes down elastic potential energy is transformed to kinetic energy.
When the spin rate goes down the elastic force of the band is doing work, contracting the rotating system to a smaller diameter, and thus increasing the kinetic energy.
The overall dynamics is somewhat similar to the case of communicating vessels; if there is more of one kind of the energy than the other then some energy will flow over until the two are in balance.

Actual planet

For a planet the lowest possible state of gravitational potential energy is to be perfectly spherical. To be deformed away from perfectly spherical is a state of higher gravitational potential energy.

The equatorial bulge of a spinning celestial body is a balance of two repositories of energy: kinetic energy and gravitational potential energy.

Discussion

Most people sense that the Earth's equatorial bulge must be due to some kind of equilibrium, but recognizing what kind of equilibrium it is is not straightforward. Very often people try to understand the bulge in terms of a force equilibrium: often there are statements about a balance between a centripetal force and a centrifugal force. However, it's not a good idea to try and understand equatorial bulge in terms of an equilibrium between forces. It's when you understand it in terms of an energy equilibrium that you can see hoe it ties in with other phenomena in physics.

Differences in gravitational acceleration

Because of a planet's rotation around its own axis, the gravitational acceleration is less at the equator than at the poles. In the 17th century, following the invention of the pendulum clock, French scientists found that clocks sent to French Guiana, on the northern coast of South America, ran slower than their exact counterparts in Paris.

Any object that is stationary with respect to the surface of the Earth is in actual fact following a circular trajectory, circumnavigating the Earth's axis. Pulling an object into such a circular trajectory requires a force. The acceleration that is required to circumnavigate the Earth's axis along the equator at one revolution per sidereal day is 0.0339 m/s². Providing this acceleration decreases the effective gravitational acceleration. At the equator, the effective gravitational acceleration is 9.7805 m/s². This means that the true gravitational acceleration at the equator must be 9.8144 m/s² (9.7805 + 0.0339 = 9.8144). At the poles, the gravitational acceleration is 9.8322 m/s². The difference of 0.0178 m/s² between the gravitational acceleration at the poles and the true gravitational acceleration at the equator is predominantly due to the fact that objects located on the equator are about 21 kilometers further away from the center of mass of the Earth than at the poles, which corresponds to a smaller gravitational acceleration. (I have glossed over differences in density now. The Earth's core is much denser, and there are other, smaller density variations.)

In summary, there are two contributions to the fact that the effective gravitational acceleration is less strong at the equator than at the poles. About 70 percent of the difference is contributed by the fact that objects circumnavigate the Earth's axis, and about 30 percent is due to the non-spherical shape of the Earth.

Centre of gravitation

Let a satellite be in an equatorial orbit. If all of the Earth's mass would be concentrated in a single point, where would that point mass have to be to exert exactly the same gravitational force on the satellite as the Earth does?

In the case of a celestial body that is a perfect sphere the answer is that all of the mass can be treated as concentrated at the geometrical center. In the case of an oblate spheroid, flattened due to rotation, this does not apply. In the case of a satellite orbiting the Earth in an equatorial orbit, the center of gravitational attraction is about 10 kilometers away from the Earth's geometrical center. For an orbiting satellite this means that at every point in time a different point inside the Earth is the effective center of gravitational attraction. For a satellite orbiting the Earth, the center of gravitation is a point that circumnavigates the geometrical center of the Earth at a distance of about ten kilometers away from the geometrical center of the Earth.

It is not a coincidence that Saturn's rings orbit Saturn in Saturn's equatorial plane. The fact that the effective center of gravitational attraction moves around in a circle tends to align satellites with the planet's equatorial plane.

Satellite orbits

The fact that the Earth's gravitational field slightly deviates from being spherically symmetrical also affects the orbits of satellites and changes their orbits away from pure ellipses. This is especially important in the case of the trajectories of GPS-satellites.