Energy - Maximum Available Work Potential Part II

Adiabatic Control Volumes

Examples of Determining the Reversible Work, Irreversibility, and Second Law Efficiency for Adiabatic Control Volumes

This section is mainly concerned with an attempt to develop an intuitive understanding of the energy equations developed in the previous section, by considering reversible equivalent circuits of some common adiabatic components. We repeat equations (3), (4) and (5) developed in Chapter 7a on the energy analysis of a control volume.

$$\boxed{w_{actual} = \underbrace{-\Delta h + T_0 \cdot \Delta s}_{w_{rev}} - \underbrace{T_0 \cdot s_{gen}}_{irrev}} \qquad (3)$$

$$\boxed{irrev = T_0 \cdot s_{gen}} \qquad (4)$$

$$\boxed{w_{rev} = -\Delta h + T_0 \cdot \Delta s} \qquad (5)$$

Note that the reversible work w_{rev} will either be the maximum available output work for work producing devices, or the minimum possible input work (negative value) for work absorbing devices.

We now consider the Energy and Entropy Generation equations for adiabatic components (q = 0):

$$\cancel{q} - w_{actual} = \Delta h = (h_e - h_i) \quad \Rightarrow \quad \boxed{w_{actual} = -\Delta h}$$

$$s_{gen} = \Delta s + \frac{q_{surr}}{T_0} = \Delta s - \frac{q}{T_0}$$

$$\cancel{q} = T_0 \cdot \Delta s - T_0 \cdot s_{gen} \quad \Rightarrow \quad \boxed{irrev = T_0 \cdot s_{gen} = T_0 \cdot \Delta s}$$

Applying all the above analysis to evaluating the Second Law Efficiencies (η_{II}) of adiabatic work absorbing and work producing components we obtain:

$$\eta_{II} = \frac{w_{rev}}{w_{actual}} = \frac{w_{actual} + irrev}{w_{actual}} \quad \Rightarrow \quad \boxed{\eta_{II} = \frac{-\Delta h + T_0 \cdot \Delta s}{-\Delta h}}$$

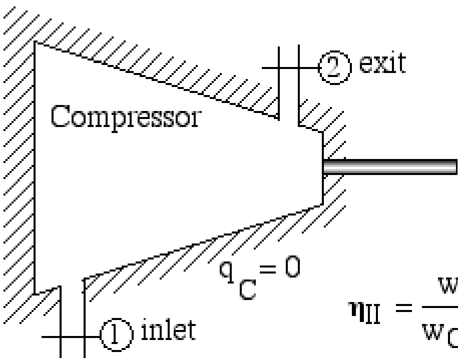
adiabatic work absorbing

$$\eta_{II} = \frac{w_{actual}}{w_{rev}} = \frac{w_{actual}}{w_{actual} + irrev} \quad \Rightarrow \quad \boxed{\eta_{II} = \frac{-\Delta h}{-\Delta h + T_0 . \Delta s}}$$

adiabatic work producing

Second Law Analysis of an Adiabatic Refrigeration Compressor

We now apply the above Second Law analysis to an adiabatic refrigeration compressor. We wish to determine the minimum work $w_{C\ rev}$ required to drive the compressor between the inlet state (1) and the exit state (2). Note that the isentropic compression that we evaluated in **Chapter 6** will not provide the answer, since state (2s) is not the same as the actual state (2).



$$w_{C\ actual} = -\Delta h = (h_1 - h_2)$$

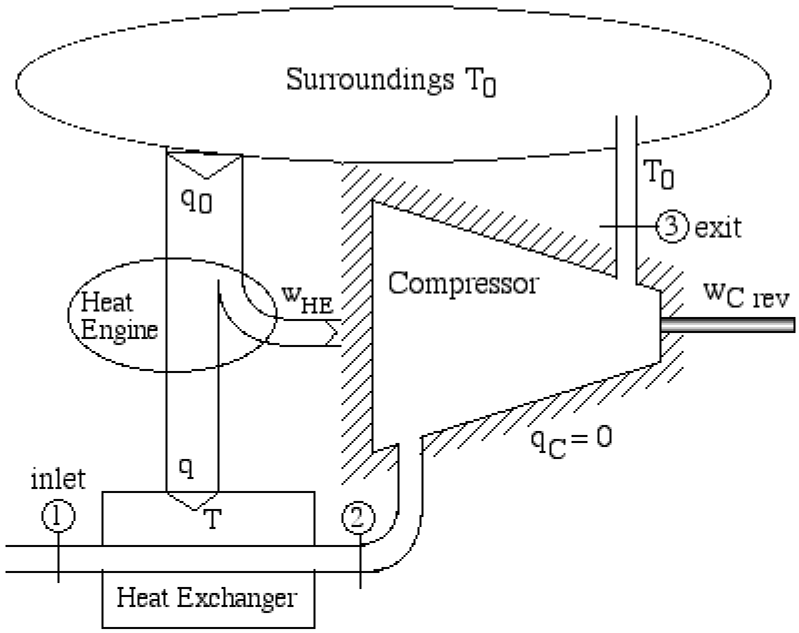
$$irrev = T_0 \Delta s = T_0 (s_2 - s_1)$$

$$w_{C\ rev} = w_{C\ actual} + irrev$$

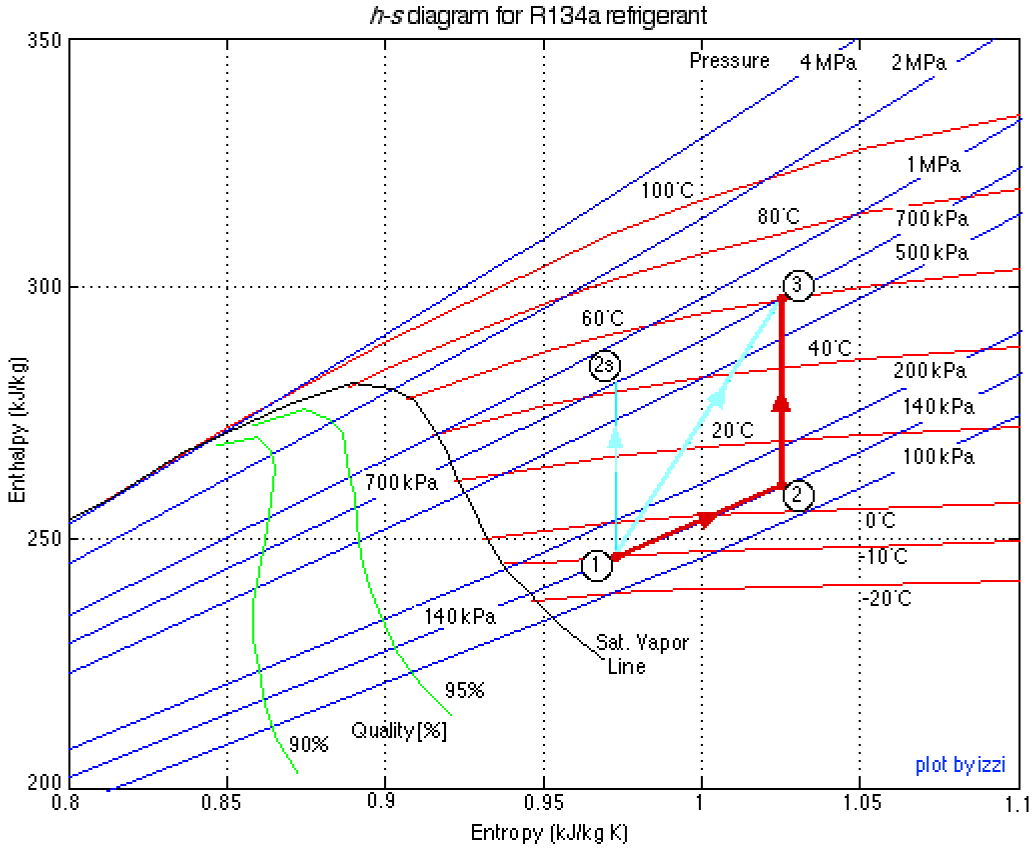
$$\eta_{II} = \frac{w_{C\ rev}}{w_{C\ actual}} = \left(\frac{(h_1 - h_2) + T_0 (s_2 - s_1)}{(h_1 - h_2)} \right)$$

The above equations are in fact correct however we have difficulty in understanding their significance. In examining the adiabatic compressor above we cannot understand why the environment (dead space) temperature T_0 features so prominently in the equations, when in fact there seems to be no obvious interaction between the adiabatic compressor and the environment. Note that as far as the adiabatic compressor is concerned we will assume that the surroundings temperature T_0 is equal to the exit temperature.

In an attempt to find some intuitive meaning to these equations we consider a reversible system having the same inlet and exit states as our actual compressor. This comprises a three component system consisting of an inlet heat exchanger, a reversible heat engine and an isentropic compressor as shown below:



A typical *h-s* diagram for this system is shown below, in which we have used typical inlet conditions of 140kPa, -10°C and exit conditions of 700kPa, 60°C. The reversible heat engine will provide extra work to drive the compressor, absorbing its heat from the environment temperature T_0 while rejecting heat to the heat exchanger. The exit state (2) from the heat exchanger has been chosen such that the compression process (2) - (3) will be isentropic.



We now derive the energy equations for the three component system above, and consider first the heat engine. Since the temperature T of the heat exchanger varies from the inlet temperature T_1 to the outlet temperature T_2, we use the differential energy equation form for the reversible heat engine.

$$\delta w_{HE} = \delta q_0 - \delta q = \delta q\left(\frac{\delta q_0}{\delta q} - 1\right) = T.ds\left(\frac{T_0}{T}\right) - \delta q$$

$$\Rightarrow \quad \delta w_{HE} = T_0.ds - \delta q$$

Since T_0 is constant, this equation can be integrated from the inlet state (1) to the outlet state (2), leading to:

$$w_{HE} = T_0(s_2 - s_1) - q, \qquad q = (h_2 - h_1)$$

$$\Rightarrow \quad w_{HE} = T_0(s_2 - s_1) - (h_2 - h_1)$$

This familiar final form was to be expected. The net minimum work required to drive the compressor is thus: determined as follows:

$$w_{HE} = T_0(s_2 - s_1) - (h_2 - h_1)$$

$$w_{C\ isentropic} = -(h_3 - h_2)$$

$$\begin{aligned} w_{C\ rev} &= w_{C\ isentropic} + w_{HE} \\ &= T_0(s_2 - s_1) - (h_2 - h_1) - (h_3 - h_2) \end{aligned}$$

$$\Rightarrow \quad w_{C\ rev} = (h_1 - h_3) + T_0(s_3 - s_1)$$

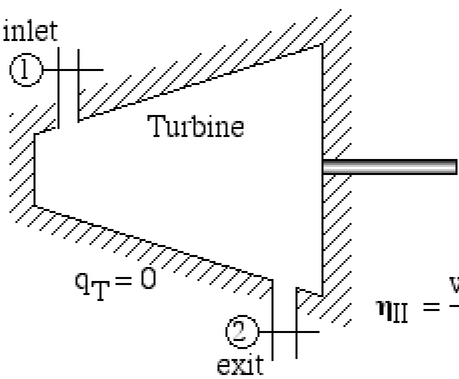
Notice that this result is identical to that shown above for the actual adiabatic compressor, since we added the heat exchanger, thus state (3) is in fact equivalent to the original state (2).

Second Law Analysis of an Adiabatic Steam Turbine

We now apply the above Second Law Analysis to an adiabatic steam turbine. We wish to determine the maximum available turbine work output $w_{T\ rev}$ between the inlet state (1) and the exit state (2). We will then be able to determine the second law efficiency by comparing the actual work output to the reversible (maximum available) work output as follows:

$$\eta_{II} = \frac{w_{actual}}{w_{rev}} = \frac{w_{actual}}{w_{actual} + irrev} \quad \Rightarrow \quad \boxed{\eta_{II} = \frac{-\Delta h}{-\Delta h + T_0 . \Delta s}}$$

adiabatic work producing



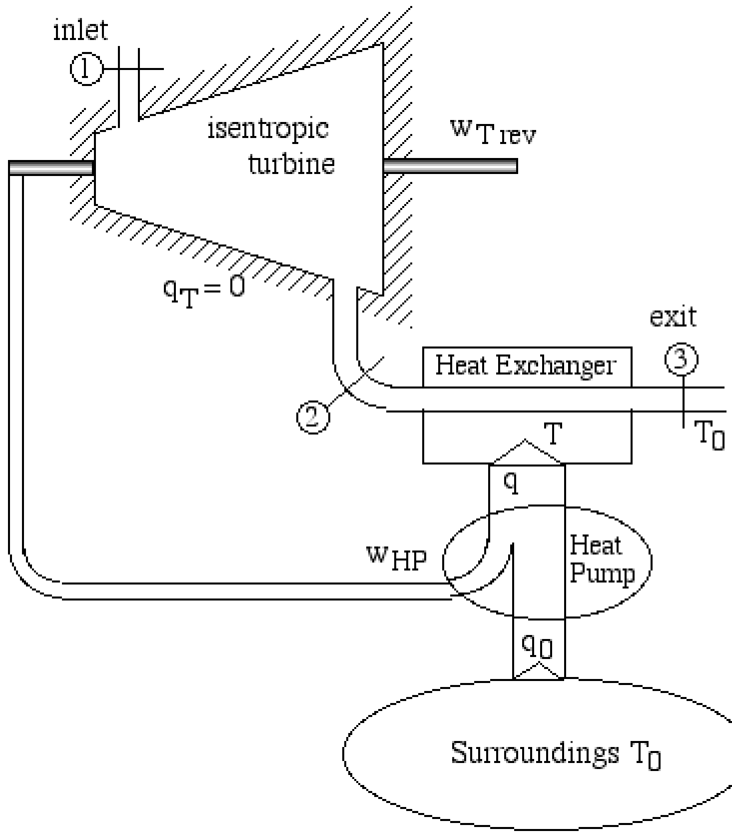
$$w_{T\ actual} = -\Delta h = (h_1 - h_2)$$

$$irrev = T_0 \Delta s = T_0(s_2 - s_1)$$

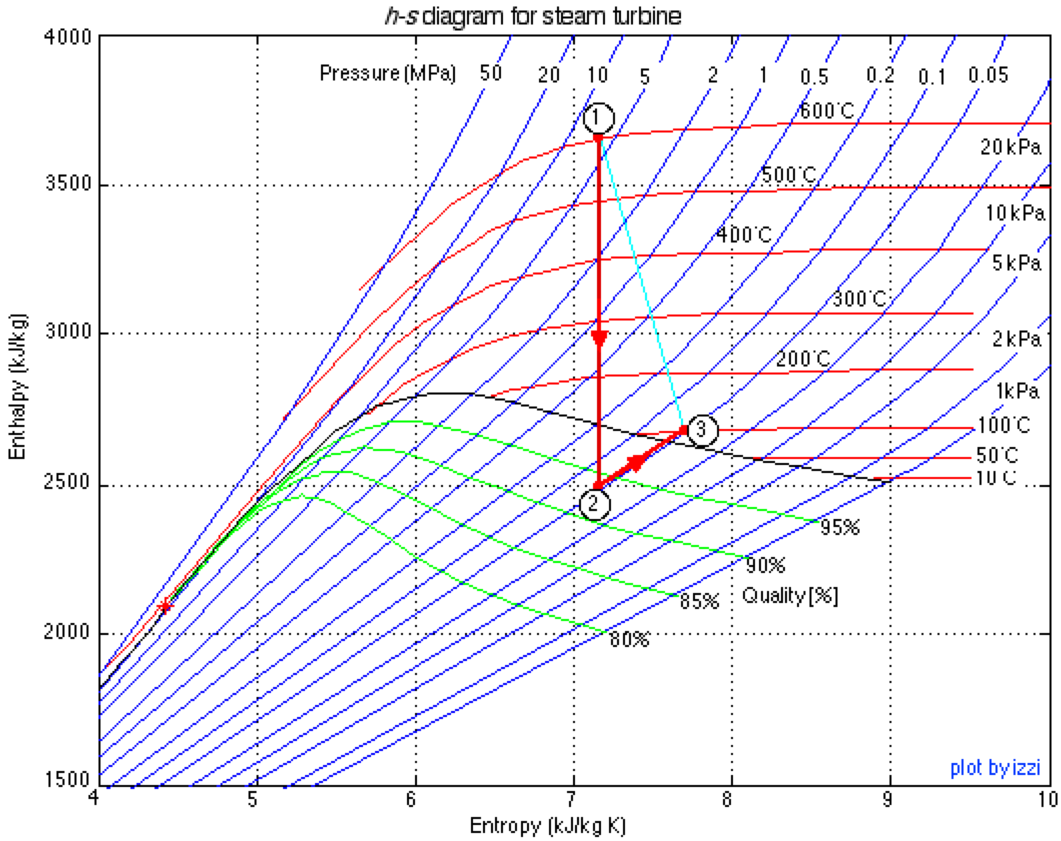
$$w_{T\ rev} = w_{T\ actual} + irrev$$

$$\eta_{II} = \frac{w_{T\ actual}}{w_{T\ rev}} = \left(\frac{(h_1 - h_2)}{(h_1 - h_2) + T_0(s_2 - s_1)} \right)$$

Once again, in an attempt to find some intuitive meaning to these equations we consider a reversible system having the same inlet and exit states as the actual turbine, comprising an isentropic turbine, a heat pump pumping heat from the surroundings to the heat exchanger in the exit stream. Note that as far as the adiabatic turbine is concerned we will assume that the surroundings temperature T_0 is equal to the exit temperature.



The *h-s* diagram for this system is shown below, in which we have chosen as an example a steam turbine having inlet conditions 6MPa, 600°C, and outlet conditions 50kPa, 100°C.



Notice from the h-s diagram that the heat exchanger temperature varies from the saturation temperature at 50 kPa (81°C) to 100°C at state (3). In order to accommodate that change we develop the differential form of the heat pump as follows:

$$\delta w_{HP} = \delta q - \delta q_0 = \delta q\left(1 - \frac{\delta q_0}{\delta q}\right) = \delta q - T.ds\left(\frac{T_0}{T}\right)$$

$$\Rightarrow \quad \delta w_{HP} = \delta q - T_0.ds$$

Since T_0 is constant, this differential equation can be integrated from state (2) to state (3).

$$w_{HP} = q - T_0(s_3 - s_2), \qquad q = (h_3 - h_2)$$

$$\Rightarrow \quad w_{HP} = (h_3 - h_2) - T_0(s_3 - s_2)$$

We can then subtract the work provided to the heat pump from the output work of the turbine leading to the final form of the maximum available work, as follows:

$$
\begin{aligned}
w_{T\,rev} &= w_{T\,isentropic} - w_{HP} \\
&= (h_1 - h_2) - [(h_3 - h_2) - T_0(s_3 - s_2)]\ , \quad s_2 = s_1 \\
\Rightarrow \quad w_{T\,rev} &= (h_1 - h_3) + T_0(s_3 - s_1)
\end{aligned}
$$

Notice that this result is identical to that shown in the box above for the actual adiabatic turbine, since state (3) is in fact equivalent to the original state (2).

Source: http://www.ohio.edu/mechanical/thermo/Applied/Chapt.7_11/Chapter7b.html