Energy - Maximum Available Work Potential Part I

Reversible Work, Irreversibility and Second Law Efficiency

In this Section we combine the First and Second Laws of thermodynamics in an attempt to determine theoretical limits of performance of various thermodynamic components and systems. Thus we introduce the concept of Energy (aka Availability) - defined as the maximum work potential of a system or component at a given state in a specified environment. The environment is crucial in this definition since once the system or component has reached total thermodynamic equilibrium with its environment, and has used up all of its potential and kinetic energy relative to that environment, it is said to be in the Dead State. The environment is usually specified in terms of pressure and temperature as P_0 = 1 atmosphere. T_0 = 25°C (77°F). In the following we attempt to introduce the concepts in terms of various examples.

Maximum Available Power Generation by a Wind Turbine

This very intuitive first example defines the theoretical maximum available power from a wind generator as that which occurs when the kinetic energy of the air passing through the turbine rotor is reduced to zero. Clearly this is impractical, and in an interesting discussion of wind power on Wikipedia we find that Betz's Law imposes a theoretical limit of 59.3% of this maximum available power when the wind velocity is reduced by 1/3 while passing through the turbine rotor, and in fact the actual energy usage is much less.

An interesting application of wind power generation for home usage is the project of Dr Greg Kremer of the ME department at Ohio University. He has combined wind and solar power with battery backup connected to the electrical grid in his home. Using the conditions defining Dr. Kremer's wind turbine system (rotor diameter 3.53m) we determine the availability of his system as follows:

$$\text{Power [W]} = \dot{m}\, V^2/2 = \boldsymbol{\rho}\, A\, V^3/2$$

where:

$\dot{m}$ [kg/s] is the mass flow rate of the air through the turbine rotor

$\boldsymbol{\rho}$ [kg/m^3] is the air density (= 1.18 kg/m^3 at 1 atmosphere & 25°C)

A [m^2] is the effective area swept by the turbine rotor

V [m/s] is the wind velocity

Notice the dependence on the cube of the wind velocity. The average annual wind velocity in Athens, Ohio is 7mph (3.11m/s) giving a maximum available power of only 174W. However during the winter months (when the solar energy is lower) the velocity reaches 22.5mph (10m/s) giving a maximum available power of 5.79kW! Thus the wind/solar combination system seems like a compatible match, and so far Dr. Kremer has found that his net electrical power usage from the grid is negative! (His system feeds energy into the grid).

Hydroelectric Power Generation

Our second example is that of hydroelectric power generation due to potential energy. Unlike wind power as described above, all of the available potential energy can be converted directly into work. Our favorite example is that of the Shoshone Hydro power plant in Glenwood Canyon, Colorado. A delightful description of this power plant is presented in Glenwood Canyon: An I-70 Odyssey by Matthew E Salek. The unique aspect of this plant is that unlike traditional plants which have the dam located at the same location, the Shoshone dam is located two miles upstream, and the water flows through a tunnel in the wall of the canyon to the power plant. At the power plant the water exits the Canyon wall and drops to the hydroelectric turbines to generate power.

$$\text{Power [W]} = \dot{m}\, g\, z$$

where:

$\dot{m}$ [kg/s] is the mass flow rate of the water through the turbine rotor

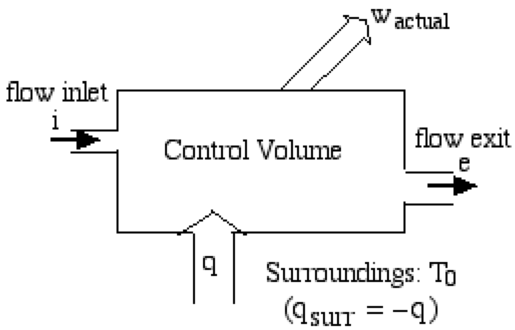
g [m/s^2] is the acceleration due to gravity (= 9.81 m/s^2)

z [m] is the height of the water source above the turbine inlet

The Shoshone plant can provide up to 15MW power, which is enough power for about 15,000 households.

Energy Analysis of a Control Volume

In our third example we do an energy analysis of a single-inlet single-outlet steady-flow control volume and define and evaluate the various concepts used. We have ignored kinetic and potential energy terms which simply directly contribute to the energy as needed. We find it convenient to do the development in terms of specific quantities (by dividing throughout by the mass flow).



Energy (First Law):

$$q - w_{actual} = \Delta h = (h_e - h_i) \qquad (1)$$

Entropy Generation (Second Law):

$$s_{gen} = \Delta s + \frac{q_{surr}}{T_0} = \Delta s - \frac{q}{T_0}$$

$$q = T_0.\Delta s - T_0.s_{gen} \qquad (2)$$

Energy Analysis: we first eliminate q from equations (1) and (2) as follows:

$$T_0.\Delta s - T_0.s_{gen} - w_{actual} = \Delta h$$

$$\boxed{w_{actual} = \underbrace{-\Delta h + T_0.\Delta s}_{w_{rev}} - \underbrace{T_0.s_{gen}}_{irrev}} \qquad (3)$$

Notice in equation (3) that we have defined reversible work (w_{rev}) as that in which no entropy is generated. We thus define a new term Irreversibility (irrev) as follows:

$$\boxed{irrev = T_0.s_{gen}} \qquad (4)$$

Thus from equation (3), when the irreversibility irrev = 0, the resulting Reversible Work is given by:

$$\boxed{w_{rev} = -\Delta h + T_0.\Delta s} \qquad (5)$$

We now define the Second Law Efficiency (η_{II}) for either a work producing or a work absorbing device as follows:

$$\eta_{II} = \frac{w_{actual}}{w_{rev}} \quad \text{(for a work producing device)}$$
$$\eta_{II} = \frac{w_{rev}}{w_{actual}} \quad \text{(for a work absorbing device)} \qquad (6)$$

The Energy ψ (or Availability) of the working fluid at either the inlet or the outlet port is defined as the maximum available work when the state of that working fluid is reduced to the Dead State 0, thus:

$$\psi = (h - h_0) - T_0(s - s_0) \qquad (7)$$

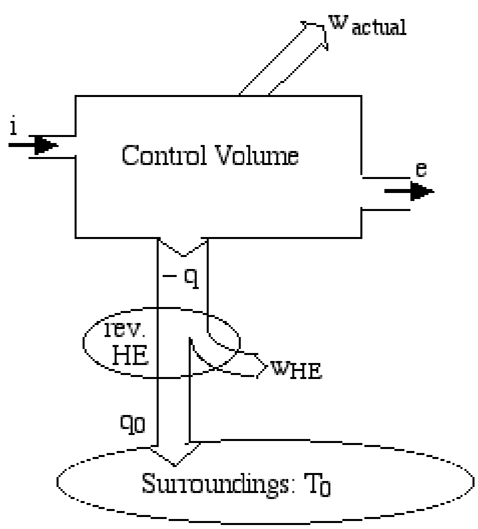
Notice that on referring to the Control Volume diagram above, the reversible work equation (5) can be written in terms of the inlet (i) and outlet (e) states as follows:

$$w_{rev} = (h_i - h_e) - T_0(s_i - s_e) \qquad \text{(5 alternate)}$$

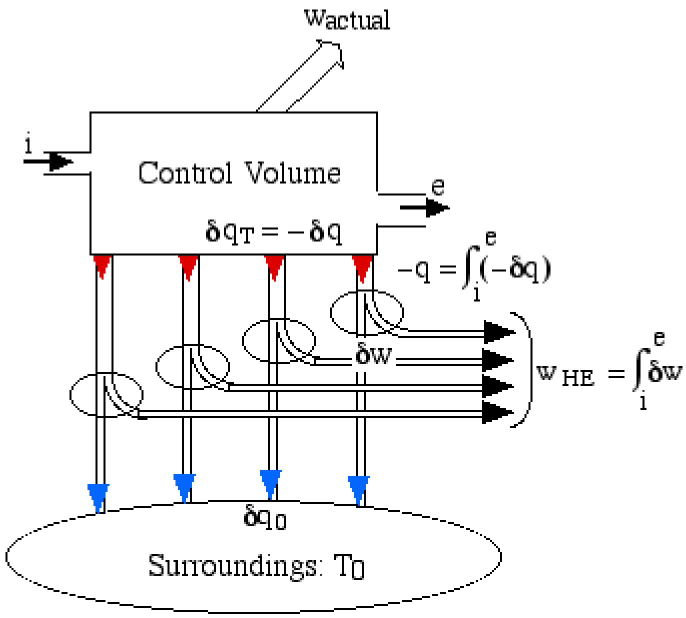
Thus the Reversible Work of the control volume can also be defined in terms of the difference in energy between the inlet and exit ports, thus:

$$w_{rev} = (\psi_i - \psi_e) \qquad (8)$$

In order to get an inuitive understanding of this analysis, consider the following equivalent system in which we use the heat transfer between the system and the surroundings in order to obtain reversible work.



However this reversible work w_{HE} is a function of the temperature T of the control volume, which can vary significantly between the inlet state (i) and the outlet state (e). Thus we will need to sum the work output of an infinite number of elemental reversible heat engines, as shown in the equivalent diagram which follows:



This analysis was first presented to me by the late Gary Graham (of Ohio University) in 1995. Thus:

$$\delta w = \delta q_T - \delta q_0 = \delta q_T\left(1 - \frac{\delta q_0}{\delta q_T}\right) = (-\delta q)\left(1 - \frac{T_0}{T}\right) = -\delta q + T.ds\left(\frac{T_0}{T}\right) = -\delta q + T_0.ds$$

Since T_0 is constant, this can be integrated, thus:

$$w_{HE} = \int_i^e \delta w = \int_i^e (-\delta q + T_0.ds) = -q + T_0\Delta s$$

$$w_{rev} = w_{actual} + w_{HE} = (-\Delta h + q) + (-q + T_0\Delta s)$$

$$w_{rev} = -\Delta h + T_0\Delta s = (h_i - h_e) - T_0(s_i - s_e)$$ as in equation (5) above.

Source: http://www.ohio.edu/mechanical/thermo/Applied/Chapt.7_11/Chapter7a.html