Vibration Generations Mechanisms: Damping- Models and Measures

Introduction:

A rotating machine has one or more machine elements that turn with a shaft, such as rolling-element bearings, impellers, and other rotors. In a perfectly balanced machine, all rotors turn true on their centerline and all forces are equal. However, in industrial machinery, it is common for an imbalance of these forces to occur. In addition to imbalance generated by a rotating element, vibration may be caused by instability in the media flowing through the rotating machine.

Damping is the dissipation of energy from a vibrating structure. In this context, the term dissipate is used to mean the transformation of energy into the other form of energy and, therefore, a removal of energy from the vibrating system. The type of energy into which the mechanical energy is transformed is dependent on the system and the physical mechanism that cause the dissipation. For most vibrating system, a significant part of the energy is converted into heat. The specific ways in which energy is dissipated in vibration are dependent upon the physical mechanisms active in the structure. These physical mechanisms are complicated physical processes that are not totally understood. The types of damping that are present in the structure will depend on which mechanisms predominate in the given situation. Thus, any mathematical representation of the physical damping mechanisms in the equations of motion of a vibrating system will have to be a generalization and approximation of the true physical situation. As Scanlan (1970) has observed, any mathematical damping model is really only a crutch which does not give a detailed explanation of the underlying physics.

For our mathematical convenience, we divide the elements that dissipate energy into three classes: (a) damping in single degree-of-freedom (SDOF) systems, (b) damping in continuous systems, and (c) damping in multiple degree-of-freedoms (MDOF) systems. Elements such as dampers of a vehicle-suspension fall in the first class. Dissipation within a solid body, on the other hand, falls in the second class, demands a representation which accounts for both its intrinsic properties and its spatial distribution. Damping models for MDOF systems can be obtained by discretization of the equations of motion. There have been attempt to mathematically describe the damping in SDOF, continuous and MDOF systems.

Single Degree-of-freedom Systems

Free oscillation of an undamped SDOF system never dies out and the simplest approach to introduce dissipation is to incorporate an ideal viscous dashpot in the model. The damping force (F_d) is assumed to be proportional to the instantaneous velocity, that is

$$F_d = C\,\dot{x} \tag{5.1}$$

and the coefficient of proportionality, c is known as the dashpot-constant or viscous damping constant. The loss factor, which is the energy dissipation per radian to the peak potential energy in the cycle, is widely accepted as a basic measure of the damping.

For a SDOF system, this loss factor can be given by;

$$\eta = \frac{c\,\omega}{k} \tag{5.2}$$

where k is the stiffness. The expression similar to this equation has been discussed by Ungar and Kerwin (1962) in the context of viscoelastic systems. Equation (5.2) shows a linear dependence of the loss factor on the driving frequency. This dependence has been discussed by Crandall (1970) where it has been pointed out that the frequency dependence, observed in practice, is usually not of this form. In such cases one often resorts to an equivalent ideal dashpot. Theoretical objections to the approximately constant value of damping over a range of frequency, as observed in aeroelasticity problems, have been raised by Naylor (1970). On the lines of equation (5.2) one is tempted to define the frequency-dependent dashpot as

$$c\,(\omega) = \frac{k\,\eta\,(\omega)}{|\omega|} \tag{5.3}$$

This representation however has some serious physical limitations. Crandall (1970, 1991), Newland (1989) and Scanlan (1970) have pointed out that such a representation violates causality, a principle which asserts that the states of a system at a given point of time can be affected only by the events in the past and not by those of the future.

Now for the SDOF system, the frequency domain description of the equation of motion can be given by

$$[-m\,\omega^2 + \omega\, c\,(\omega) + k]X\,(\omega) = F\,(\omega) \qquad (5.4)$$

Where X(ω) and F(ω)are the response and excitation respectively, represented in the frequency domain. Note that the dashpot is now allowed to have frequency dependence Inserting equation (5.3) into (5.4) we obtain

$$[-m\,\omega^2 + k\{1 + \text{¦}\,(\omega)\sin(\omega t)\}]X\,(\omega) = F\,(\omega) \qquad (5.5)$$

The ‘time-domain’ representations of equations (5.5) and (5.6) are often taken as

$$m\ddot{x} + c(\omega)\dot{x} + kx = f \qquad (5.6)$$

and

$$[-m\,\ddot{x} + kx\{1 + \eta\,(\omega)\sin(\omega t)\}] = f \qquad (5.7)$$

It has been pointed out by Crandall (1970) that these are not the correct Fourier inverses of equations (5.5) and (5.6). The reason is that the inertia, the stiffness and the forcing function are inverted properly, while the damping terms in equations (5.7) and (5.8) are obtained by mixing the frequency-domain and time-domain operations. Crandall (1970) calls 5.7) and (5.8) the ‘non-equations’ in time domain. It has been pointed out by Newland (1989) that only certain forms of frequency dependence for $\eta(\omega)$are allowed in order to satisfy causality. Crandall (1970) has shown that the impulse response function for the ideal hysteretic dashpot (η independent of frequency), is given by

$$h(t) = \frac{1}{\pi k \eta_0}.\frac{1}{t}, \quad -\infty < t < \infty. \qquad (5.8)$$

This response function is clearly non-causal since it states that the system responds before the excitation (or the cause) takes place. This non-physical behaviour of the hysteretic damping model is a flaw, and further attempts have been made to cure this problem. Bishop and Price (1986) introduced the band limited hysteretic damper and suggested that it might satisfy the causality requirement. However, Crandall (1991) has further shown that the band-limited hysteretic dashpot is also non-causal. In view of this discussion it can be said that the most of the hysteretic damping model fails to satisfy the casualty condition. Recently, based on the analyticity of the transfer function, Makris (1999) has shown that for causal hysteretic damping the real and imaginary parts of the dynamic stiffness matrix must form a Hilbert transform pair1. He has shown that the causal hysteretic damping model is the limiting case of a linear viscoelastic model with nearly frequency-independent dissipation that was proposed by Biot (1958). It was also shown that there is a continuous transition from the linear viscoelastic model to the ideally hysteretic damping model.

The physical mechanisms of damping, including various types of external friction, fluid viscosity, and internal material friction, have been studied rather extensively in some detail and are complicated physical phenomena. However, a certain simplified mathematical formulation of damping forces and energy dissipation can be associated with a class of physical phenomenon. Coulomb damping, for example is used to represent dry friction present in sliding surfaces, such as structural joints. For this kind of damping, the force resisting the motion is assumed to be proportional to the normal force between the sliding surfaces and independent of the velocity except for the sign. The damping force is thus

$$F_d = \frac{\dot{x}}{|\dot{x}|} F_r = \sin(\dot{x})\, F_r \qquad (5.9)$$

Where F_r is the frictional force, in the context of finding equivalent viscous damping, Bandstra (1983) has reported several mathematical models of physical damping mechanisms in SDOF systems.

For example, velocity squared damping, which is present when a mass vibrates in a fluid or when fluid is forced rapidly through an orifice, the damping force in this case is;

$$F_d = \sin(\dot{x})\ a\dot{x}^2 \qquad (5.10)$$

It can be expressed more generally as;

$$F_d = C\,\dot{x}\,|\dot{x}|^{n-1} \tag{5.11}$$

Where C is the damping proportionality constant, viscous damping is a special case of this type of damping. If the fluid flow is relatively slow *i.e.* laminar, then by letting $n = 1$ the above equation reduces to the case of viscous damping (5.6).

Continuous Systems

Construction of damping models becomes more difficult for continuous systems. Banks and Inman (1991) have considered four different damping models for a composite beam. These models of damping are:

1. *Viscous air damping:* For this model the damping operator in the Euler-Bernoulli equation for beam vibration becomes

$$L_1 = \gamma\,\frac{\partial}{\partial t} \tag{5.12}$$

Where γ is the viscous damping constant.

2. *Kelvin-Voigt damping:* For this model the damping operator becomes

$$L_1 = C_d\, I\,\frac{\partial^5}{\partial x^4\,\partial t} \tag{5.13}$$

Where I is the moment of inertia and C_d is the strain-rate dependent damping coefficient. A similar damping model was also used by Manohar and Adhikari (1998) and Adhikari and Manohar (1999) in the context of randomly parametered Euler-Bernoulli beams.

3. *Time hysteresis damping:* For this model the damping operator is assumed as

$$L_1 = \int_{-\infty}^{t} g(\tau)u_{xx}(x, t+\tau)\mathrm{d}\tau \quad \text{where } g(\tau) = \frac{\alpha}{\sqrt{-\tau}}\exp(\beta\tau) \tag{5.14}$$

4. Spatial hysteresis damping:

$$L_1 = \frac{\partial}{\partial x}\left[\int_0^L h(x,\xi)\{u_{xx}(x,t) - u_{xt}(\xi,t)\}\mathrm{d}\xi\right] \tag{5.15}$$

The kernel function $h(x, \xi)$ is defined as

$$h(x,\xi) = \frac{a}{b\sqrt{\pi}}\exp[-(x-\xi)^2/2b^2] \tag{5.16}$$

It was observed by them that the spatial hysteresis model combined with a viscous air damping model results in the best quantitative agreement with the experimental time histories. Again, in the context of Euler-Bernoulli beams, Bandstra (1983) has considered two damping models where the damping term is assumed to be of the forms

$\{\operatorname{sgn} u_t(x,t)\}\, b_1 u^2(x,t)$ and $\{\operatorname{sgn} u_t(x,t)\}\, b_2 |u(x,t)|$.

Multiple Degrees-of-freedom Systems

The most popular approach to model damping in the context of multiple degrees-of-freedom (MDOF) systems is to assume viscous damping. This approach was first introduced by Rayleigh (1877). By analogy with the potential energy and the kinetic energy, Rayleigh assumed the *dissipation function*, given by

$$\mathcal{F}(\mathbf{q}) = \frac{1}{2}\sum_{j=1}^{N}\sum_{k=1}^{N} C_{jk}\dot{q}_j\dot{q}_k = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{C}\dot{\mathbf{q}}. \tag{5.17}$$

In the above expression $C \epsilon R^{NXN}$ is a non-negative definite symmetric matrix, known as the viscous damping matrix. It should be noted that not all forms of the viscous damping matrix can be handled within the scope of classical modal analysis. Based on the solution method, viscous damping matrices can be further divided into classical and non-classical damping.

It is important to avoid the widespread misconception that viscous damping is the *only* linear model of vibration damping in the context of MDOF systems. Any causal model which makes the energy dissipation functional non-negative is a possible candidate for a damping model. There have been several efforts to incorporate non-viscous damping models in MDOF systems. Bagley and Torvik (1983), Torvik and Bagley (1987), Gaul *et al.* (1991), Maia *et al.* (1998) have considered damping modeling in terms of fractional derivatives of the displacements.

Following Maia *et al.* (1998), the damping force using such models can be expressed by

$$\mathbf{F}_d = \sum_{j=1}^{l} \mathbf{g}_j D^{\nu_j}[\mathbf{q}(t)]. \tag{5.18}$$

Here g_j are complex constant matrices and the fractional derivative operator

$$D^{\nu_j}[\mathbf{q}(t)] = \frac{\mathrm{d}^{\nu_j}\mathbf{q}(t)}{\mathrm{d}t^{\nu_j}} = \frac{1}{\Gamma(1-\nu_j)}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t \frac{\mathbf{q}(t)}{(t-\tau)^{\nu_j}}\mathrm{d}\tau \tag{5.19}$$

Where υ_j is a fraction and $\Gamma(\bullet)$ is the Gamma function. The familiar viscous damping appears as a special case when $\upsilon_j = 1$. We refer the readers to the review papers by Slater *et al.* (1993), Rossikhin and Shitikova (1997) and Gaul (1999) for further discussions on this topic. The physical justification for such models, however, is far from clear at the present time.

Possibly the most general way to model damping within the linear range is to consider nonviscous damping models which depend on the past history of motion via convolution integrals over some kernel functions. A *modified dissipation function* for such damping model can be defined as

$$\mathcal{F}(\mathbf{q}) = \frac{1}{2}\sum_{j=1}^{N}\sum_{k=1}^{N} \dot{q}_k \int_0^t \mathcal{G}_{jk}(t-\tau)\dot{q}_j(\tau)\mathrm{d}\tau = \frac{1}{2}\dot{\mathbf{q}}^T \int_0^t \boldsymbol{\mathcal{G}}(t-\tau)\dot{\mathbf{q}}(\tau)\mathrm{d}\tau. \tag{5.20}$$

Here $G(t) \epsilon R^{NXN}$ is a symmetric matrix of the damping kernel functions, $G_{jk}(t)$. The kernel functions, or others closely related to them, are described under many different names in the literature of different subjects: for example, retardation functions, heredity functions, after-effect functions, relaxation functions *etc*. In the special case when $G(t-\tau) = C\delta(t-\tau)$ where $\delta(t)$ the Dirac-delta function, equation (5.20) reduces to the case of viscous damping as in equation (5.17). The damping model of this kind is a further generalization of the familiar viscous damping. By choosing suitable kernel functions, it can also be shown that the fractional derivative model discussed before is also a special case of this damping model. Thus, as pointed byWoodhouse (1998), this damping model is the most general damping model within the scope of a linear analysis.

Golla and Hughes (1985), McTavis and Hughes (1993) have used damping model of the form (1.33) in the context of viscoelastic structures. The damping kernel functions are commonly defined in the frequency/Laplace domain. Conditions which **G**(s), the Laplace transform of $G(t)$ must satisfy in order to produce dissipative motion were given by Golla and Hughes (1985). Several authors have proposed several damping models and they are summarized in Table 1.1.

Table 1.1: Summary of damping functions in the Laplace domain

Damping functions	Author, Year
$G(s) = \sum_{k=1}^{n} \frac{a_k s}{s + b_k}$	Biot (1955, 1958)
$G(s) = as \int_0^\infty \frac{\gamma(\rho)}{s+\rho} \mathrm{d}\rho$ $\gamma(\rho) = \begin{cases} \frac{1}{\beta - \alpha} & \alpha \le \gamma \le \beta \\ 0 & \text{otherwise} \end{cases}$	Buhariwala (1982)
$G(s) = \frac{E_1 s^\alpha - E_0 b s^\beta}{1 + b s^\beta}$ $0 < \alpha < 1, \quad 0 < \beta < 1$	Bagley and Torvik (1983)
$sG(s) = G^\infty \left[1 + \sum_k \alpha_k \frac{s^2 + 2\xi_k \omega_k s}{s^2 + 2\xi_k \omega_k s + \omega_k^2} \right]$	Golla and Hughes (1985) and McTavis and Hughes (1993)
$G(s) = 1 + \sum_{k=1}^{n} \frac{\Delta_k s}{s + \beta_k}$	Lesieutre and Mingori (1990)
$G(s) = c \frac{1 - e^{-st_0}}{st_0}$	Adhikari (1998)
$G(s) = c \frac{1 + 2(st_0/\pi)^2 - e^{-st_0}}{1 + 2(st_0/\pi)^2}$	Adhikari (1998)

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