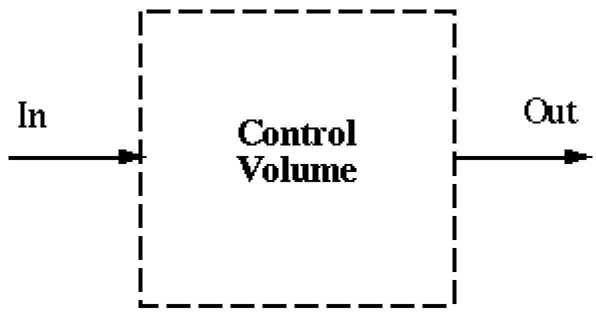
Control Volume (Integral) Technique

<u>Introduction</u>: Techniques for solving flow problems

There are three different techniques for solving these types of problems:

1. **Control Volume Analysis**
    - This is just like in thermodynamics class



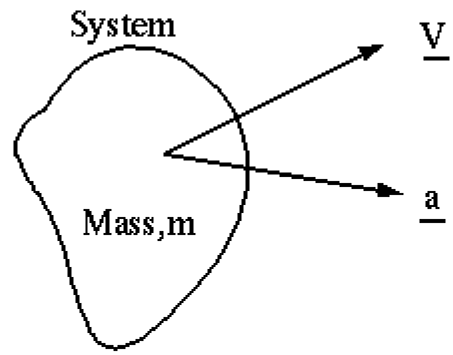
    - One can calculate the **<u>gross properties</u>** (total power output, total heat transfer, etc.)
    - With this however, we do not care about the details inside the control volume (In other words we can treat the control volume as a "black box.")
2. **Differential Analysis**
    - In this technique, one solves differential equations of motion everywhere (i.e. The Navier-Stokes equation).
    - Here we solve for <u>all</u> of the details in the flow
3. **Dimensional Analysis and Experiment**
    - This method is used when methods one or two are not possible.
    - One uses wind tunnels, models, etc. to employ this method.

<u>Introduction to systems and control volumes</u>

<u>System</u>: Consider a <u>system</u> of a fluid:

- Recall from thermo class, that a **<u>system</u>** is defined as a volume of mass of fixed identity.

- Let m = the mass of the system
  Let **V** = the velocity of the system
  Let **a** = the acceleration of the system



- Now we can write three basic conservation laws which apply to this system. Note: These conservation laws apply directly to a system.

2. **Conservation of mass** states that the mass of a system is constant. This can be written as the following equation:

$$\frac{dm_{sys}}{dt} = 0$$

In this equation m = the mass of the system.

3. **Conservation of linear momentum** which is a restatement of

Newton's Second Law.

- In equation form this is written as:

$$\Sigma \underline{F}_{sys} = \frac{d}{dt}(m\underline{V})_{sys}$$

Where m$\underline{V}$ = the linear momentum of the system.

- Note that this is the same as Newton's second law, it is just written a little differently.

- Using the above equation, we can obtain a form of the equation in terms of acceleration:

$$\Sigma \underline{F}_{sys} = \frac{d}{dt}(m\underline{V})_{sys} = m\frac{d\underline{V}}{dt} + \underline{V}\frac{dm}{dt}$$

- This equation is obtained through expansion using the product rule.
- We know that ,

$$\frac{d\underline{V}}{dt} = \underline{a} \quad \text{and} \quad \frac{dm}{dt} = 0$$

  by the conservation of mass found above.

- Using these ideas, we can then see that

$$\Sigma \underline{F}_{sys} = m_{sys}\underline{a}_{sys}$$

4. **Conservation of Energy**
   - For this, use the First Law of Thermodynamics in rate form to obtain the following equation:

$$\frac{dE_{sys}}{dt} = \dot{Q}_{sys} - \dot{W}_{sys}$$

   - Where E = the total energy of the system. In the above equation

$$\frac{dE_{sys}}{dt}$$

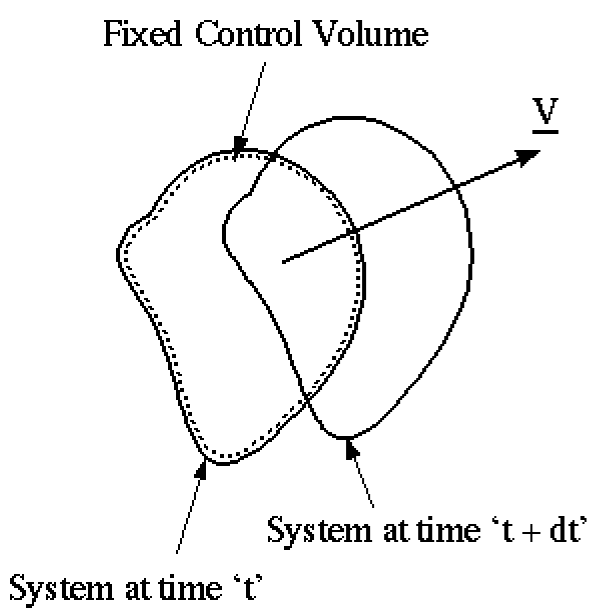
     is the rate of change of system energy.

- $\dot{Q}_{sys}$ is the rate of heat added ***to*** the system $\left(\frac{\delta Q}{dt}\right)$
- $\dot{W}_{sys}$ is the rate of work done ***by*** the system $\left(\frac{\delta W}{dt}\right)$. Because work is done by the system, the negative sign is in the equation for the first law of thermodynamics.
- Now, these conservation laws must always hold ***<u>for a system</u>***.

5. **<u>Conservation of Angular Momentum</u>**
   We will not have time to study this, but see the text for details.

<u>Control Volume</u>:

- One can think of the system approach as the **<u>Lagrangian description</u>**, which if recalled is the description where we follow the individual chunks of the fluid.
- However, the trouble with this is that we prefer to use the **<u>Eulerian Description</u>**, where we define a control volume with fluid moving through it.



- In other words, if we define some control volume, ***<u>fixed in space</u>***, the system will flow ***<u>through</u>*** it.

- Our goal here is to try to write the conservation laws above in a form applicable to a ***control volume*** (Eulerian description) rather than to a ***system***(Lagrangian description).
- There is a way to do this, and it is called the **Reynolds Transport Theorem (R.T.T.)**
- Now our goal is to write all three of these conservation laws in terms of a control volume. But, to save us some work, let's not derive it three times, but just once for a general property B, and then use our result for any of the three conservation laws:
- In other words, let's write R.T.T. for some arbitrary property B.

  Let B = some arbitrary property (vector or scalar) and $\beta \equiv B$ per unit mass.
  This way, we can later substitute B for mass, momentum, energy, etc.
- We are then able to make the following substitutions for β:
- For ***conservation of mass***, let B = m, and $\beta = m/m = 1$.
- For ***conservation of momentum***, $B = m\underline{V}$, and $\beta = \frac{m\underline{V}}{m} = \underline{V}$.
- For ***conservation of energy***, B = E, and $\beta = \frac{E}{m} = e$.
- SEE the text for the derivation.

REYNOLDS TRANSPORT THEOREM (R.T.T.)

- This is for some property B, and is also for a ***fixed*** control volume. Deforming the control volume is more complicated; see the text for details.
- The equation for Reynolds Transport Theorem is written as the following:

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho\beta d\forall + \oint_{CS} \rho\beta(\underline{V}\cdot\underline{n})dA$$

(This is equation 3.12 in the text.)

OR

For a fixed control volume, the ***order*** of differentiation and integration doesn't matter. So alternatively,

$$\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\rho\beta)\,d\forall + \oint_{CS} \rho\beta(\underline{V}\cdot\underline{n})\,dA$$

(This is equation 3.17 in the text.)

- **Comments** on the above equation:

1. $\frac{dB_{sys}}{dt} \equiv$ **The total rate of change of B following the system.** This is the term to which the conservation laws directly apply.
2. $\int_{CV} \frac{\partial}{\partial t}(\rho\beta)\,dV \equiv$ The time rate of change of B within the control volume. This is due to unsteadiness.
3. $\oint_{CS} \rho\beta(\underline{V}\cdot\underline{n})\,dA \equiv$ The net flux of B out of the control surface. Due to **<u>convection</u>**, B changes because system moves to a new part of the flow field, where conditions are different. A circle was put around the integral $\oint_{CS}$ to emphasize that this is an integral over the ***entire*** control surface. (This is not done in the text however).
4. Sometimes $\iiint_{CV}$ and $\oiint_{CS}$ are used to ***emphasize*** that these are ***volume*** and ***area*** integrals. (Again this is not done in the text).

Source:

http://www.mne.psu.edu/cimbala/Learning/Fluid/Control_Volume/index.htm