

Analysis of extrusion

1.1 Material flow in extrusion:

Extrusion pressure depends on the nature of material flow and redundant deformation during extrusion. Metal flow can be studied by etching square grid on the cross-section of one half of the billet. The billet is cut across along the length and after making the grid, the two halves are joined by brazing or simply placed together inside the die. After extrusion the two halves of the billet are separated and the grid lines are inspected for shape change. The nature of material flow pattern depends on the friction at the interface, temperature variations in the billet etc.

A frictionless condition at die-billet interface will result in homogeneous deformation, without formation of dead metal or shear zone. In case of high friction, a region of no-flow or dead metal zone forms in the corner zones. Near the die exit, shear zones are formed, especially on the periphery of the billet. Shear deformation-redundant deformation leads to enhanced extrusion load. Material flow through shear zone may result in defective extrudes. During hot extrusion, the metal near die wall gets chilled out resulting in enhancement of resistance to flow. The material at the centre flows freely. This kind of variation across the billet results in dead metal zone near wall of the die. Under high friction conditions, the shear zone may extend back into the central part of the billet. Larger dead metal zone may form due to high friction, leading to defects in extruded parts. Redundant work will be higher.

Material flow in extrusion

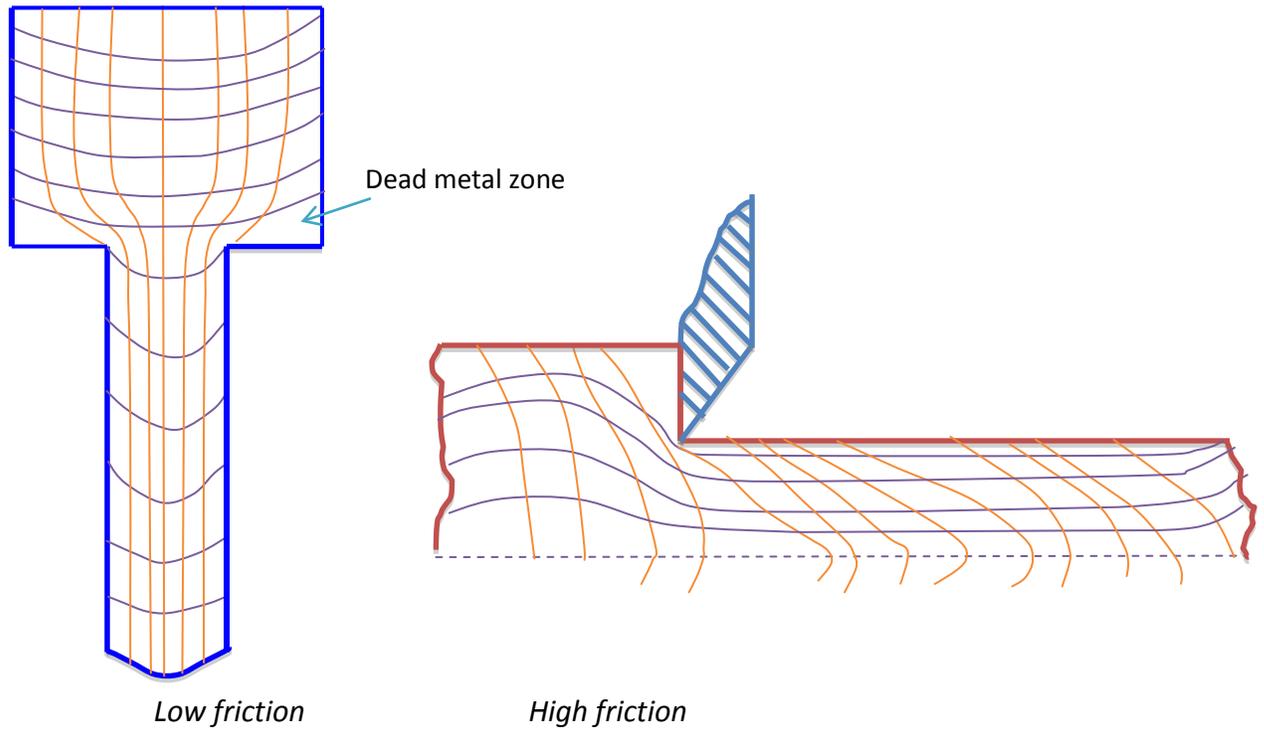


Fig. 2.1.1: Metal flow in extrusion

Redundant work refers to the work done in the redundant shear deformation due to friction. As friction increases, metal flow becomes highly non-homogeneous, enhancing the shear deformation.

1.2 Extrusion pressure for ideal extrusion:

Consider a cylindrical billet of initial diameter d_o , length l_o , being subjected to axisymmetric extrusion without friction and redundant deformation. Let the final diameter be d_f .

The ideal plastic work done per unit volume is given by:

$$w = \int \sigma d\varepsilon \text{ -----2.1}$$

Assuming that the stress is equal to average flow stress of the material in compression, we can write,

$$w = \bar{Y}' \int d\varepsilon = \bar{Y}' \varepsilon \quad \text{-----2.2}$$

We consider the average flow stress here because the material undergoes strain hardening during extrusion. Therefore its flow stress increases from entrance to exit.

We know that $\bar{Y}' = k \frac{\varepsilon^n}{1+n}$

And the strain during extrusion is given as:

$$\varepsilon = \ln(A_o/A_f), \quad \text{because,} \quad A_o L_o = A_f L_f \quad \text{for constancy of volume}$$

Extrusion ratio R is defined as $R = A_o/A_f$

Therefore, $w = \bar{Y}' \ln R$

The total work done during extrusion is given by:

$$w \times \text{Volume} = AL\bar{Y}' \ln R \quad \text{-----2.3}$$

Also we can write work done = Pressure X Area X Displacement = pAL

From the above expressions for work, we get the extrusion pressure as:

$$p = \bar{Y}' \ln R \quad \text{-----2.4}$$

If redundant work due to friction is assumed, the extrusion pressure is expected to be higher than that predicted by equation 4.

$$\text{Extrusion force} = F = pA_o = p \frac{\pi}{4} d_o^2$$

We define the extrusion efficiency as the ratio of ideal work of deformation to actual work of deformation.

$$\eta = \frac{w_{ideal}}{w_{actual}} \quad \text{and} \quad W_{actual} = W_{ideal} + W_{friction} + W_{redundant}$$

Or, $p_{actual} = \bar{Y}' \ln R / \eta \quad \text{-----2.5}$

Where η is the extrusion efficiency.

If one has to consider the friction between the container and the billet alone, then the total extrusion pressure can be taken to be the sum of the die pressure and the pressure required to overcome friction in the container.

$p = \text{die pressure} + \text{friction pressure}$

The friction pressure is given by:

$$p_f = \frac{4\tau L}{D}, \quad (\text{assuming sliding friction})$$

where L is length of billet in container and D is diameter of billet and τ is interface shear stress.

Note: As seen from equation 4, as the extrusion ratio increases, the extrusion force also increases.

Example: A certain material has a strength coefficient of 400 MPa and a strain hardening exponent of 0.16. A billet of this material has a diameter of 30 mm and a length of 80 mm. This billet is extruded to a ratio of 4. Assuming square die, estimate the extrusion force required, ignoring friction. Use the following formula for extrusion pressure:

$$p = \bar{Y} \left[(a + b \ln R) + \frac{2L}{D_0} \right], \quad \text{where } a = 0.8, b = 1.5$$

Solution:

$$R = 4 \text{ (given)}, \quad \bar{Y} = k\varepsilon^n = 421.64 \text{ MPa.}$$

$$\varepsilon = \ln R = 1.39$$

$$p = 3461 \text{ MPa}$$

$$\text{Extrusion force} = 2.45 \text{ MN}$$

1.3 Direct extrusion - More accurate analysis - Slab analysis:

Slab analysis can be used for determining the extrusion pressure during direct extrusion. In order to simplify the analysis, we may assume that the die angle is small. Consider axis-symmetric extrusion. Consider an elemental cylinder of thickness dx within the deformation zone of extrusion. The radius of the cylinder reduces from $R+dR$ to R as a result of extrusion. We can make a force balance on the cylinder, assuming that

constant sliding friction exists between the billet and die wall. The semi-cone angle of the die is considered as α . Let R_0 and R_f be the initial and final radii of the billet.

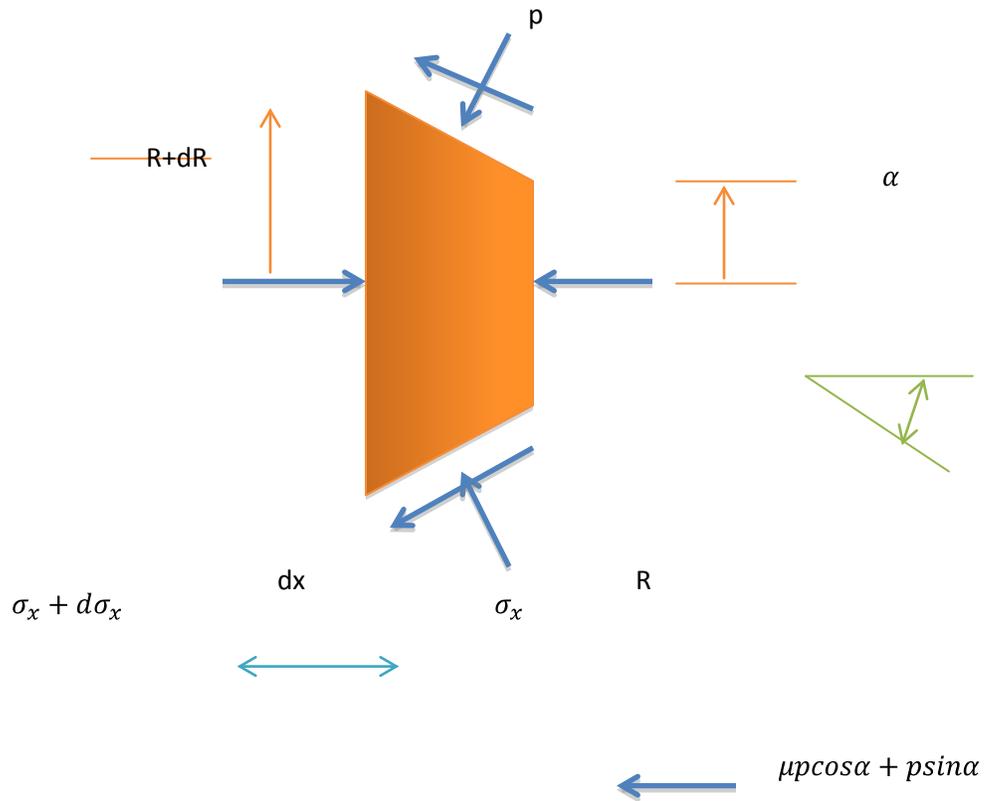


Fig.2.3.1: Stresses acting on elemental billet subjected to axi-symmetric extrusion

The slant surface area is given as:

$$\frac{\pi(R+dR)^2 - \pi R^2}{\sin \alpha} = \frac{2\pi R dR}{\sin \alpha} \quad (\text{neglecting small terms}) \text{ -----2.6}$$

The force balance equation for the elemental slab along the direction of extrusion may be written as:

$$(\sigma_x + d\sigma_x)\pi(R + dR)^2 - \sigma_x\pi R^2 = (\mu p \cos\alpha + p \sin\alpha) \frac{2\pi R dR}{\sin\alpha} \text{ -----2.7}$$

Simplifying, neglecting the terms involving square and product of small quantities, we get

$$d\sigma_x + 2\sigma_x \frac{dR}{R} = 2p(1 + \mu \cot\alpha) \frac{dR}{R} \text{ -----2.8}$$

Applying the Tresca yield criterion:

Note: Treat applied stress as positive and induced stress as negative

That is: p is negative and the applied stress σ_x will have a negative induced stress = $-\sigma_x$

Therefore the yield criterion is written in the form:

$$-\sigma_x + p = Y \rightarrow p = Y + \sigma_x$$

Substituting for p into equation 8 above, integrating we get:

$$\frac{1}{B} \ln[(B)\sigma_x + (1 + B)y] = 2 \ln R + c \text{ -----2.9}$$

applying the boundary condition:

At $R = R_f$, $\sigma_x = 0$, Solving for c and simplifying, we get:

$$\sigma_x = Y \left[\frac{1+B}{B} \right] \left[\left(\frac{R}{R_f} \right)^{2B} - 1 \right] \text{ -----2.10}$$

where, $B = \mu \cot\alpha$, Y is yield strength of material in compression., σ_x is extrusion pressure.

Equation 10 gives the extrusion pressure at any location x along the deformation zone.

The punch pressure to be applied can be obtained by substituting $R = R_i$ in equation 10

And also noting that the punch pressure = $-\sigma_x$

$$i.e. p_{ext} = Y \left[\frac{1+B}{B} \right] \left[1 - \left(\frac{R_o}{R_f} \right)^{2B} \right] \text{ -----2.11}$$

Extrusion force at punch now becomes: $F = A_o p_{ext}$

1.4 Effect of redundant deformations on extrusion force:

Due to non-homogeneous deformation as a result of friction, material undergoes shear at entry and exit sections of the die. Shear deformation consumes some energy. The work to be spent on the shear is known as redundant work.

It can be shown that the stress due to shear at entry, $\sigma_{si} = \frac{2}{3\sqrt{3}} \alpha Y$ -----2.12

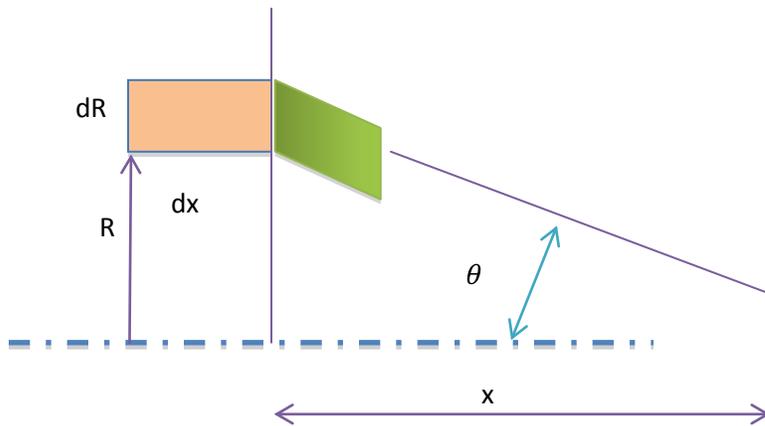


Fig. 2.4.1: Redundant shear of an element of thickness dx during extrusion

Consider a small ring element of length dx at a radius of R , with thickness dR , located at X from the reference point.

This element gets sheared through an angle of θ as shown above.

Let the yield strength of the material in shear be $\frac{Y}{\sqrt{3}} = Y'$ (From Von-Mises criterion)----2.13

The volume of the small element is $= 2\pi R dR dx$ -----2.14

Shear strain undergone by the element $= \tan\theta = R/x$ ----2.15

Shear Work done on the elemental ring $= dW = \text{shear stress} \times \text{shear strain}$

$dW = Y' 2\pi R dR dx R/x$ -----2.16

Total work done on many such elemental rings can be written as:

$W = \int_0^{R_0} 2\pi R dR dx R/x = Y' (dx/x) \frac{2}{3} \pi R^3$ -----2.17

We know the total volume of the ring of thickness $dx = \pi R^2 dx$

Work done per unit volume now becomes:

$$w = \frac{2}{3} Y' \frac{R}{x} \text{ -----18}$$

We can write $\tan \alpha = R/x \approx \alpha$

Therefore, shear stress at entry now becomes:

$$\sigma_{si} = \frac{2}{3\sqrt{3}} \alpha Y \quad \text{This is same as equation 2.12.}$$

Similarly we can obtain shear stress at exit of the conical section of the die as:

$$\sigma_{se} = \frac{2}{3\sqrt{3}} \alpha Y \quad (\text{If no work hardening happens}) \text{ ----2.19}$$

Another approximate expression for extrusion can be written assuming that sticking friction exists at billet-die interface and that the material flows at 45 degrees.

$$P_{\text{ext}} = Y[1.75 \ln R + 2L/D_0] \text{ -----2.20}$$

It can be noted from the above expression that as ram travel increases, L gets reduced and hence extrusion pressure.

The actual extrusion punch pressure is a function of friction, shear etc.

Therefore, a general expression for extrusion pressure can be written of the form:

$$P_{\text{ext}} = Y(a + b \ln R) \text{ -----2.21}$$

Value of $a = 0.5$ and $b = 1.2$ to 1.5

Example: For an alloy of aluminium the flow stress at a temperature of 420°C is given by the expression: $\sigma = C \dot{\epsilon}^m$ where $C = 200 \text{ MPa}$ and $m = 0.11$. This alloy is hot extruded from an initial diameter of 180 mm to a final diameter of 60 mm . Length of billet is 400 mm . The speed of extrusion is 60 mm/s . Assuming square die and poor lubrication determine the extrusion force. Consider the friction in the container also.

Solution:

Assuming the semi-die angle as 45°

Extrusion pressure can be found from the expression:

$$p_{\text{ext}} = Y \left[\frac{1+B}{B} \right] \left[1 - \left(\frac{R_o}{R_f} \right)^{2B} \right]$$

$$B = \mu \cot \alpha = 0.15$$

Assuming $\mu=0.15$

$$R = 9$$

Average Strain rate can be calculated from:

$$6v \ln R / D_o = 4.39$$

$$Y = 235.34 \text{ MPa}$$

$$p = 703.67 \text{ MPa}$$

Friction pressure in container = $4kL/D_o$ (Assuming sticking friction)

$$K = Y/\sqrt{3} = 235.34/1.732 = 135.87$$

$$P_f = 1207 \text{ MPa}$$

$$\text{Extrusion force} = 48.6 \text{ MN}$$

Source:

<http://nptel.ac.in/courses/112106153/24>