Analysis of plane strain upset forging of rectangular billet

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1.Analysis of plane strain upset forging of rectangular billet

There are different methods of analysis of bulk deformation processing, like slab analysis, slip line field line, upper bound analysis, FEM analysis. The outcome of all these analyses is the forming load.

In this section we focus on slab method, which is the simplest type of analysis for forming load.

1.1 Upsetting of rectangular plate-analysis

Consider a rectangular billet of height h_o, width (x axis) 2a and unit depth (z axis). Let this billet be subjected to plane strain upsetting. Plane strain condition here means there is no normal and shear strain along the z direction – depth direction. The slab undergoes strain only along the y axis-height direction and along the x direction – width direction.

We can make a force balance on a small elemental strip of width dx, height h and unit depth, as shown.

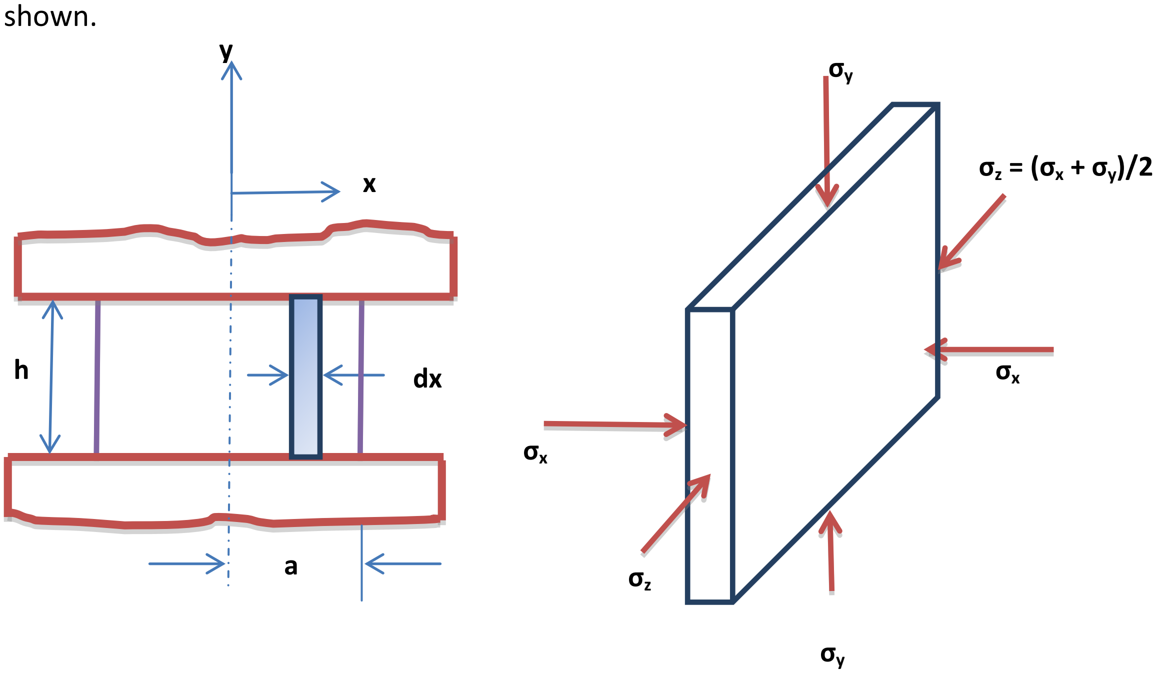

Fig. 3.1.1: Plane strain upsetting of rectangular billet and the stresses acting on the element of thickness dx

Assume that the lateral stress $\boldsymbol{\sigma_x}$is uniform along the height of the element.

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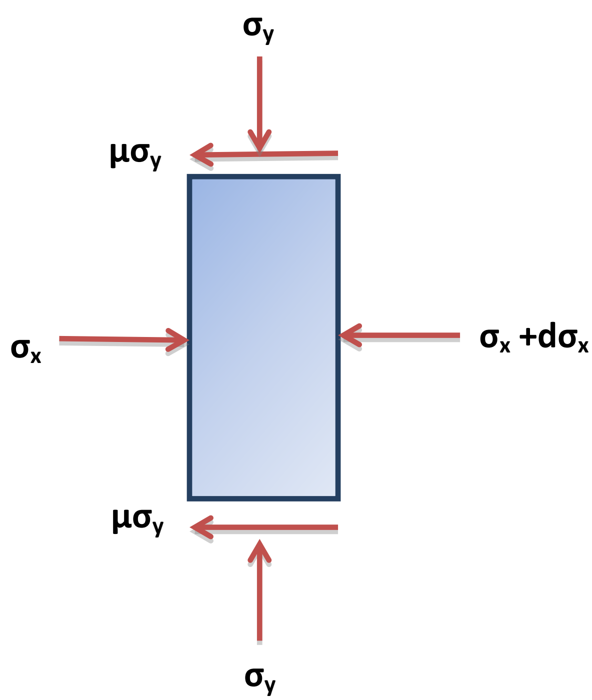

Fig. 3.1.2: Stresses acting on a small elemental billet of thickness dx and unit depth

Assumptions:

compressive stresses are positive.

Sliding Columbic friction

Coefficient of friction is low

The height of the billet is small so that the forging pressure is constant over the height of the billet.

Assume that σ_x and σ_y are principal stresses [Though σ_y can not be assumed as principal stress as a shear stress is also acting on the plane on which the normal stress is acting]

Here σ_y is the forging stress necessary at any height h of the billet.

Force balance on the element gives:

Assuming the dimension of the billet perpendicular to the plane of the paper,

$(\sigma_x + d\sigma_x)h + 2\mu\sigma_y dx - \sigma_x h = 0$ -----------------9

We have to eliminate σ_x because there are two unknowns in the above equation.

For eliminating σ_x we can apply the von Mises yield criterion for plane strain.

According to this criterion, we have:

$\sigma_y - \sigma_x = \frac{2}{\sqrt{3}}Y = Y'$ --------------------10

From this we have $d\sigma_y = d\sigma_x$

The force balance equation now becomes:

$\frac{d\sigma_y}{\sigma_y} = -\frac{2\mu}{h}dx$ ------------------------------11

Upon integration, we get:

σ_y = A$e^{-\frac{2\mu x}{h}}$ ----------------------------------12

To solve the constant A, we need a boundary condition.

At x = a, σ_x = 0 [free surface]

From the yield criterion we have: At x=a, $\sigma_y = Y'$

Substituting this in equation 12 and simplifying we get,

$p = \sigma_y = Y'[e^{\frac{2\mu(a-x)}{h}}]$ -----------------------13

P is the forging pressure

Equation 13 can also be written as:

P = Y'[$e^{\frac{L\mu(1-\frac{2x}{L})}{h}}$] -------------------14

Where L = 2a → width of the billet

From the above equation we find that as L/h increases, the forging pressure increases – resistance to compressive deformation increases. This fact is utilized in closed die forging where the deformation resistance of flash, being high [due to high L/h] the die filling is effective.

Note: Y' is plane strain yield strength of the material

If the material is work hardening type of material, we have to replace Y' with Y'_f which is the flow stress of the material

The variation of forging pressure normalized with plane strain yield strength Y' is shown with respect to the billet thickness:

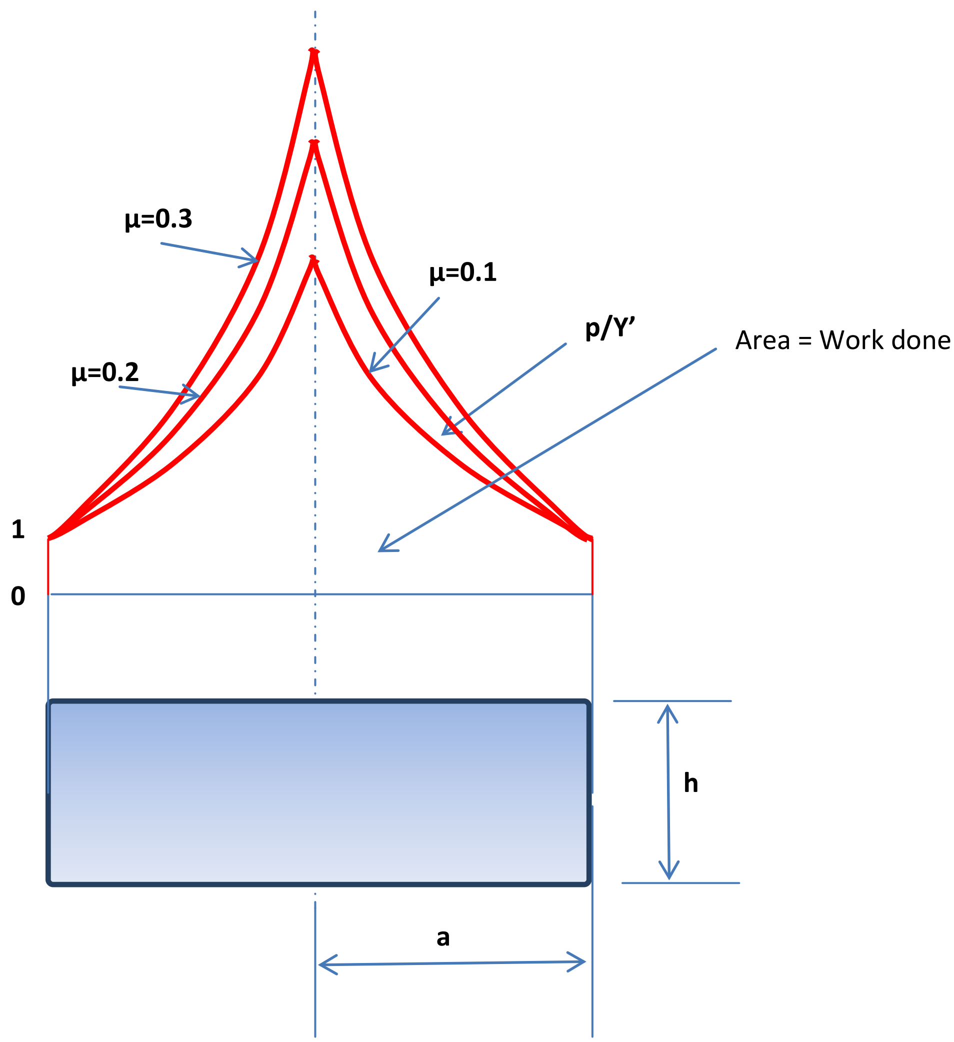

Fig. 3.1.3: Friction hill in plane strain upsetting under sliding friction

Forging pressure variation across the billet due to friction is shown above. The pressure distribution curve is called friction hill. Area under the friction hill represents the forging work done.

As shown in figure, as the coefficient of friction increases, the forging pressure increases and hence the work done.

Average forging pressure:

The average forging pressure is given as:

$\bar{p} = \frac{1}{a}\int_0^a p dx$ ------------15

Substituting for p from equation 13, we get:

$$\bar{p} = \frac{\bar{Y}}{\frac{2\mu a}{h}}(e^{\frac{2\mu a}{h}} - 1)$$

We can get approximate expression for average forging load by expanding exponential function as infinite series. We get:

$\bar{p} = Y'(1 + \frac{\mu a}{h})$ ----------------16

Note that the forge pressure is a function of instantaneous height of billet. As height gets reduced, after successive plastic flow, forging pressure increases.

If the rectangular billet is subjected to plane stress compression – stress acting along the height axis and the length axis, there will be material flow in the width direction. It is found that the extent of flow along width direction is several times greater than the flow along longitudinal direction. Because of lower friction along width, material flows freely along width direction.

If a rectangular block is compressed, due to friction and non-uniform flow, bulging and barreling take place. Bulging refers to the non-uniform flow considered on the plane of the loading, while barreling refers to the non-uniform deformation along the height of the specimen. The reason for bulging and barreling is the material flow along the diagonal direction is rather sluggish, compared to the other directions.

Sticking friction:

The frictional shear stress – μp increases towards the axis as the forging pressure p increases.

However, the maximum frictional shear stress can not exceed the shear yield strength of the material. When the limiting condition of $\tau = k$, we can say sticking exists at the interface.

Generally, we can relate the friction shear stress with shear yield strength by the relation:

$\tau = mk$

m is friction factor, which can not exceed 1.

Under sticking friction the friction shear stress and shear yield strength are related as:

$\tau = k$ ------------------------17

where k is shear yield strength. For sticking friction, the limit of friction shear stress is the shear yield strength of the material→ m=1.

In general, with $\tau = mk$, the forging pressure is given by:

$P = Y'\frac{m}{h}(a - x) + Y'$ --------------------------------18

As per the above equation, with sticking friction [m = 1], one can write the forging pressure as:

$P = Y'\frac{(a-x)}{h} + Y'$ ----------------------19

This is a linear relation, which is shown in figure below:

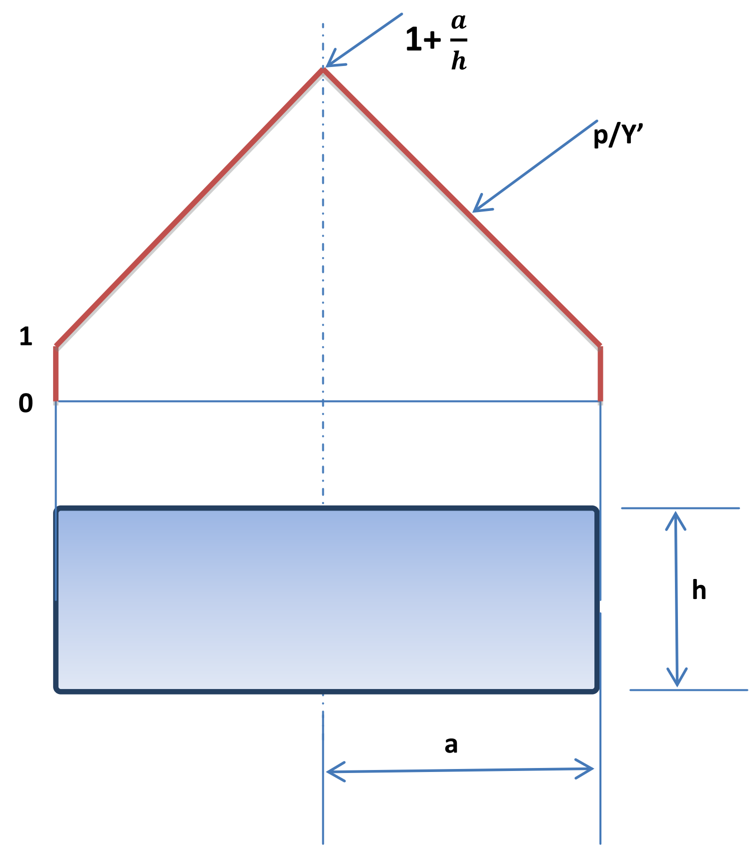

Fig. 3.1.4: Upset forging with sticking friction – variation of forging pressure

Example: A rectangular block of height 40 mm, width 100 mm and depth 30 mm is subjected to upset forging under sliding friction condition, with a friction coefficient of 0.2. The material of the billet has flow stress expressed as: $Y = 300\varepsilon^{0.2}$. Calculate the forging load required at the height reduction of 30%, assuming plane strain compression.

Solution: Due to plane strain assumption, the depth side of the block remains without deformation. We can use the solution obtained for plane strain compression. The average forging pressure is given by:

$$\bar{p} = \frac{\bar{Y}}{\frac{2\mu a}{h}}\left(e^{\frac{2\mu a}{h}} - 1\right)$$

Given: a_o = 50 mm, h_o = 40 mm, $h_f = (1-0.3)h_o$ = 28 mm, depth = w = 30 mm.

To find width after the deformation, we can use volume constancy.

$2a_oh_o = 2ah$ → a = 71.43 mm

True strain = $\ln(h_o/h_f)$ = 0.357

Average flow stress = $\frac{k\varepsilon^n}{n+1}$ = 203.4 MPa

Average forging pressure = 199.41X1.773 = 353.59 MPa.

Average Forging load = 353.59 X 71.43 X 30 = 757. 7 kN(For one half of the bar)

Total forging load = 2X757.7kN.

Source:http://nptel.ac.in/courses/112106153/15