Analysis of Axi-symmetric forging of a disk

R. Chandramouli

Associate Dean-Research

SASTRA University, Thanjavur-613 401

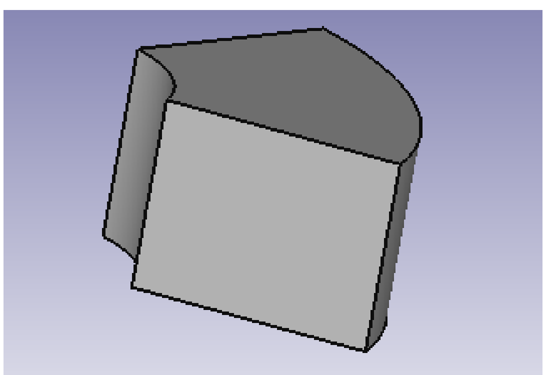
Table of Contents

1. Analysis of Axi-symmetric forging of a disk

1.1 Axi-symmetric forging of a disc-analysis:

Consider a solid circular disk of diameter R and height ho. This disc is subjected to axial upsetting between two dies. The objective is to determine the forging pressure required at any height h of the disk. We have to consider sliding friction at interface, with coefficient of friction taken to be μ.

We also assume that the axial compressive pressure p is constant over the thickness of the disk – because the disk has lower height.



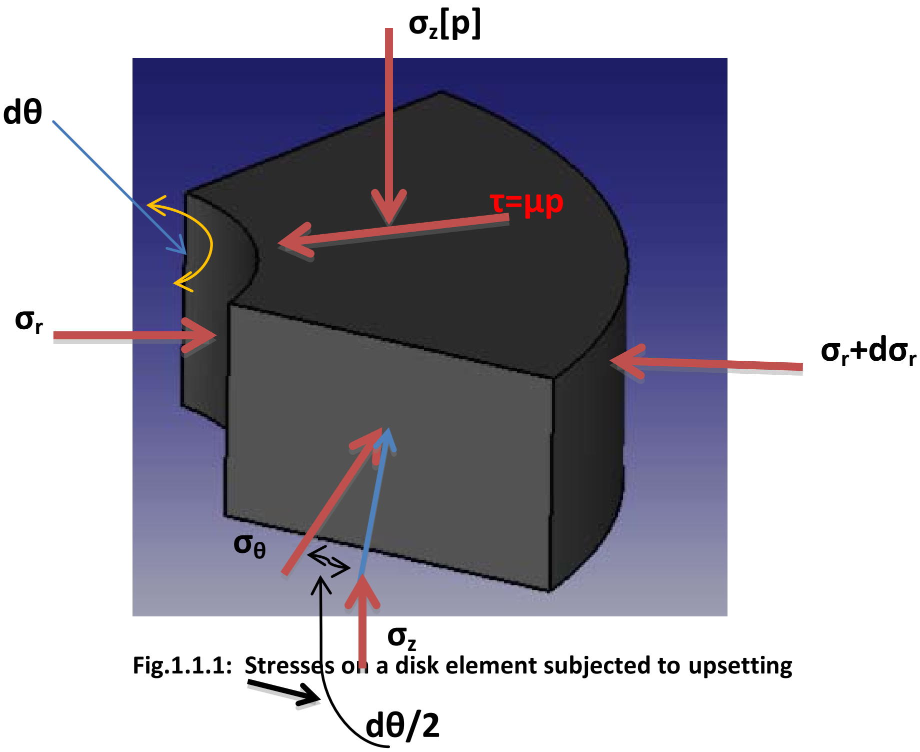


Fig.1.1.1: Stresses on a disk element subjected to upsetting

Consider the disk element as shown above, with height h, radius r and radial thickness dr, angle dθ.

The various stresses acting on the element are shown in figure.

Note that for axial symmetry, we have radial strain = circumferential strain→$d\varepsilon_r = d\varepsilon_x$.

Therefore, we have

$\sigma_r = \sigma_\theta$---------------------------20

Surface shear on top and bottom faces is opposing the radial flow of material. This is shown in figure above.

Due to frictional shear stress, lateral pressure is induced on the material.

We assume that $\sin d\theta/2 = d\theta/2$, because angle $d\theta$ is small.

Equilibrium of forces on the element after applying the approximation said above and the equality of radial and circumferential stresses (equation 20), gives:

$\frac{d\sigma_r}{dr} = -\frac{2\mu p}{h}$----------------------21

Now σ_r has to be eliminated.

We can apply the von Mises yield criterion for the compression. Assuming that all the three stresses are principal stresses, we find that:

$Y = p - \sigma_r$----------------------------22

Hence, $d\sigma_r = dp$--------------------23

Applying eqn 23 in eqn 21, we have

$\frac{dp}{p} = -\frac{2\mu dr}{h}$---------------------------24

Equation 24 can now be integrated, Integrating and applying the boundary condition:

at $r=R$, $\sigma_r = 0$

we obtain the final solution of equation 24 as:

$p = Ye^{\frac{2\mu(R-r)}{h}}$ ----------------- 25

For frictionless compression ($\mu=0$) we get $p = Y$.

With various coefficients of friction, the variation of forging pressure along the radial direction is shown in figure below:

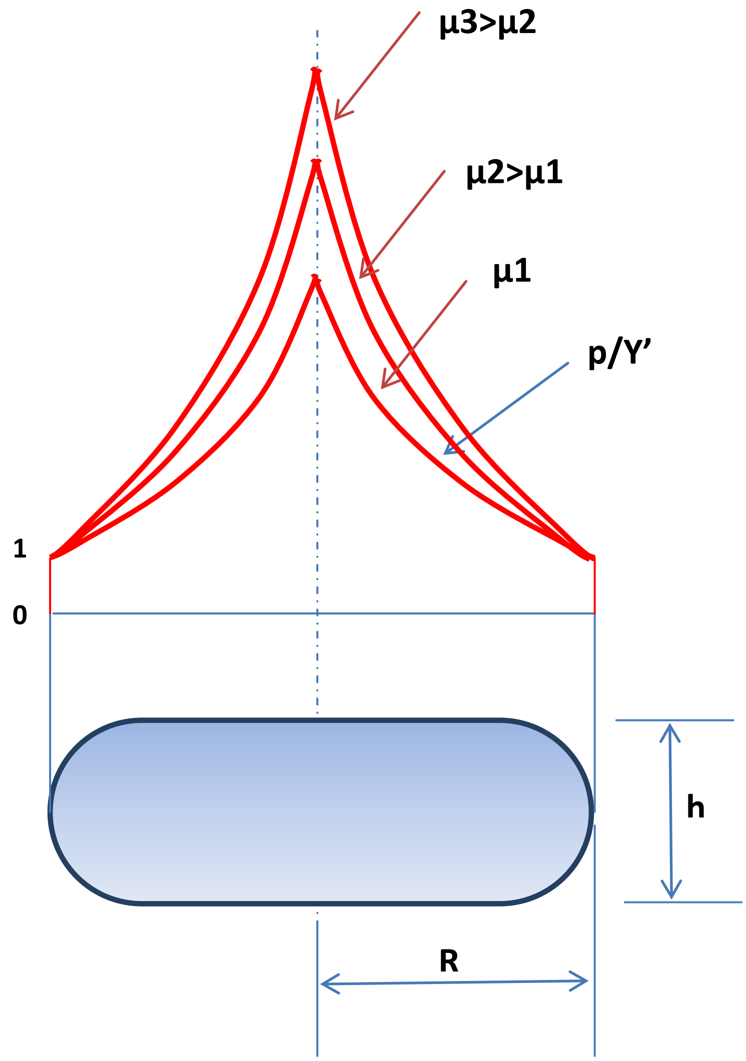


Fig. 1.1.2: Friction hill for sliding friction

The average forging pressure can be determined from the following integration:

$$P_{av.} = \frac{1}{\pi R^2}\int_0^R p2\pi r dr \text{ ----------------26}$$

$$= \frac{1}{2}\left(\frac{h}{\mu R}\right)^2 Y\left[e^{\frac{2\mu R}{h}} - \frac{2\mu R}{h} - 1\right]$$

Approximately,

The average pressure can be obtained as:

$\bar{p}$= Y(1+$\frac{2\mu R}{3h}$)--------------------27

For materials which undergo strain hardening, Y is replaced by the corresponding flow stress .

Coefficient of friction values for various forming operations are given in table:

Table 4.1.1: Coefficient of friction values for various forming operations

Process	μ - Cold forming	μ - Hot forming
Forging	0.05 to 0.1	0.2 to 0.7
Rolling	0.05 to 0.1	0.1 to 0.2
Drawing	0.03 to 0.1	0.1 to 0.2
Sheet metal working	0.05 to 0.1	

As the coefficient of friction increases, the forming pressure also increases.

The aspect ratio of the billet also has notable effect on the forging pressure.

Aspect ratio = diameter / height or = width / height

Effect of friction and aspect ratio on forging pressure is shown in figure below

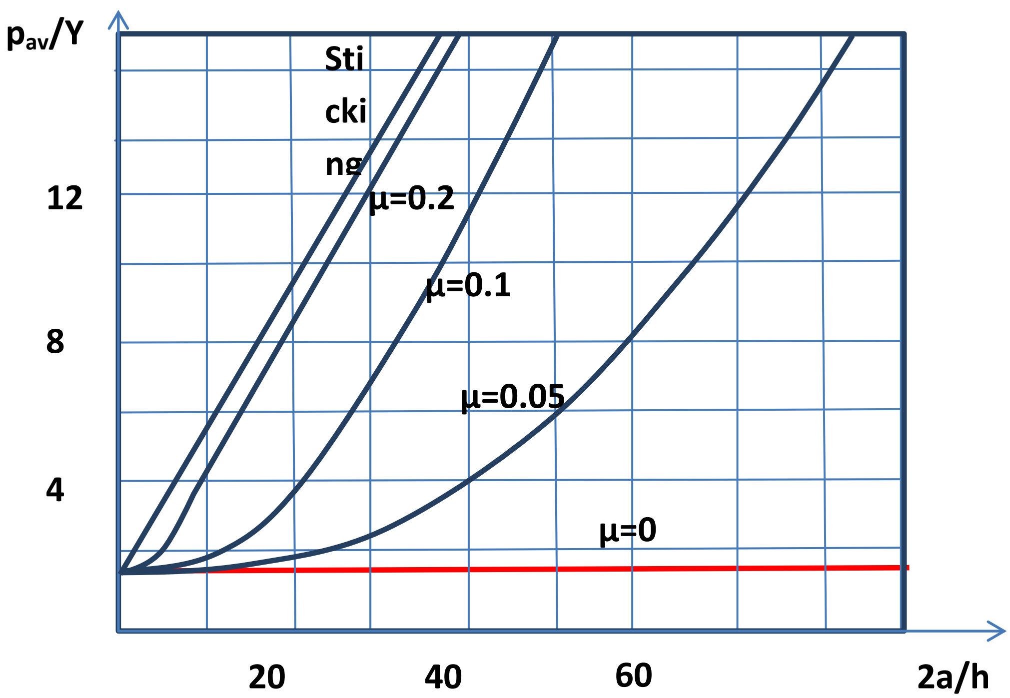


Fig. 1.1.3: Variation of forging pressure with aspect ratio of billet and coefficient of friction

Sticking friction:

Taking τ =k

We can get the forging pressure p as:

$$\frac{p}{k} = 1 + \frac{(R-r)}{h}$$ ------------------------28

If combination of sliding and sticking friction occurs, the distance from the axis where the sticking friction changes to sliding friction can be determined as followed:

At the location where the change occurs, namely, r, we can equate the shear stress due to sliding friction to that due to sticking friction:

$\tau = \mu p = K$ (assuming m=1)

Substituting for p from eqn. 25, we can solve for r:

$$r = R - \frac{h}{2\mu}\ln\left(\frac{\mu k}{Y}\right)$$ ----------------29

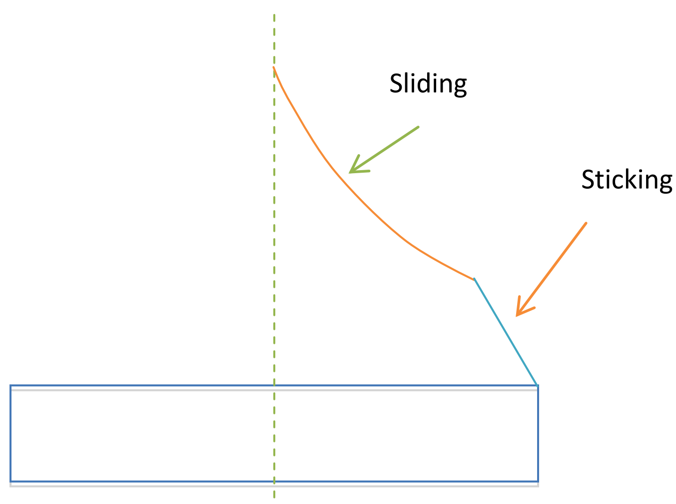


Fig. 1.1.4: Combined sliding and sticking friction

Example: A 40 mm diameter disk of initial height of 40 mm is upset forged between a pair of platens. The coefficient of friction at the interfaces is found to be 0.22. The material of the billet has a strength coefficient of 650 MPa and a strain hardening exponent of 0.16. What is the instantaneous forging force just at the point of yielding (assuming yield point strain = 0.002)? Determine the average force at the height reduction of 30%.

Solution:

Given disk with do = 40 mm, ho = 40 mm, μ=0.22, k=650 MPa, n=0.16.

To determine: a] the forging load at the commencement of yielding and b] average force at height reduction of 30%

We can use the expression for forging pressure for axisymmetric forging for solving this problem.

The average forge pressure is given by:

$$\bar{p} = Y\left(1+\frac{2\mu R}{3h}\right)$$

a] At yielding

$$\varepsilon = 0.002$$

Y = 240.48 MPa

$$\varepsilon = \ln\left(\frac{ho}{hf}\right)$$

Therefore hf = 39.92 mm

Rf = 20.02 mm

Average pressure = **258.17** MPa

b] At 30% height reduction:

hf = (1-0.3)ho = 28 mm

Rf = 23.9 mm

Strain = 0.358

Y = 551.41 MPa

Average pressure = 620.44 MPa

Average forging force = 620.44 X Final Area = 1.11 MN

Source:http://nptel.ac.in/courses/112106153/16