Air - Water Vapor Mixtures Part I

Humidity and the Adiabatic Saturation Process

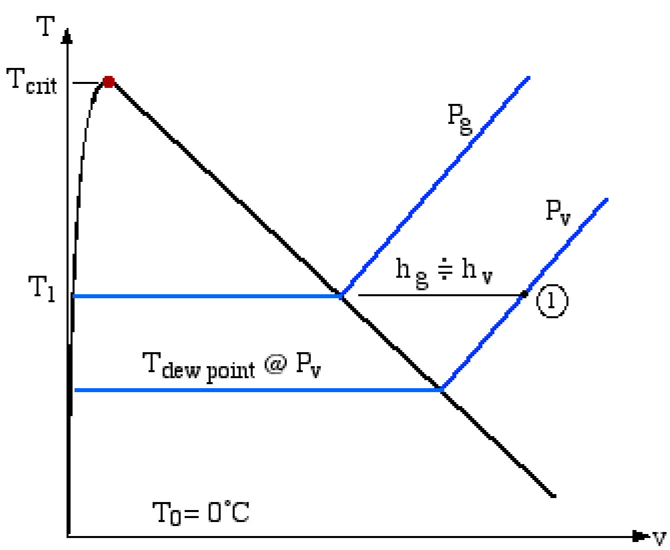
Atmospheric air includes *dry air* and *water vapor*. Recall that for an ideal gas, enthalpy (h) is a function of temperature only ($\Delta h = C_P.\Delta T$). Notice also from the ***h-s* diagram** for steam that at relatively low temperatures (<60°C) the water vapor in the air has a constant enthalpy at constant temperature from saturated vapor through the superheated region, thus can be treated as an ideal gas.

From Dalton's Law of Partial Pressures we have for a dry-air/water-vapor mixture that the total pressure P is given by:

$P = P_a + P_v$

where subscript a refers to the dry air, and v to the water vapor.

We find it convenient to sketch our processes for the water-vapor component on a *T-v* diagram (which we prefer to the ubiquitous *T-s* diagram in common use, since entropy is not considered in this Section)



Consider the water vapor shown at state (1) on the diagram. We will find it convenient throughout this section to evaluate enthalpy with respect to $T_0 = 0°C$, since ultimately we only consider differences in enthalpy. From the above diagram:

$h_{v@T} = h_{g@T}$

where g refers to the saturated vapor state.

Note that $h_{g@T}$ can be obtained from the saturated vapor tables, or one can use Izzi's method, which has a maximum error of 0.5% at 60°C:

$$h_{vapor} = h_{g@T} \approx (2500 + 2T[°C])\ \ [kJ/kg]$$

We also evaluate the enthalpy of the dry air component with respect to T_0 = 0°C, thus:

$$h_{dry\text{-}air} = C_P T \approx T[°C]\ \ [kJ/kg]$$

since at the temperatures under consideration C_P is approximately 1.00 [kJ/kg°C].

In order to evaluate the enthalpy of the atmospheric air we need to first find the mass flow rates of both the dry air and the vapor. We always evaluate these with respect to the mass flow rate of the dry air, and this in turn leads us to the definition of **Specific Humidity** ω, as follows:

$$\dot{m}_a h_{air+vap} = \dot{m}_a h_a + \dot{m}_v h_v$$

$$h_{air+vap} = h_a + \omega . h_v$$

$$\text{Specific humidity} \Rightarrow \omega = \left(\frac{\dot{m}_{vapor}}{\dot{m}_{dry-air}}\right) = \left(\frac{\dot{m}_v}{\dot{m}_a}\right)$$

Note that other terms in common usage are *humidity ratio* or *absolute humidity* to denote specific humidity. The specific humidity can be conveniently determined in terms of the partial pressures P_a and P_v as follows:

$$\omega = \left(\frac{\dot{m}_v}{\dot{m}_a}\right) = \left(\frac{m_v}{m_a}\right) = \left(\frac{P_v V/(R_v T)}{P_a V/(R_a T)}\right) = \left(\frac{P_v/(0.4615[kJ/kg.K])}{P_a/(0.287[kJ/kg.K])}\right)$$

$$\omega = 0.622\left(\frac{P_v}{P_a}\right) = 0.622\left(\frac{P_v}{P - P_v}\right)\ \left[\frac{\text{kg vapor}}{\text{kg dry air}}\right]$$

Referring to the *T-v* diagram above, we now define **Relative Humidity** φ as follows:

$$\text{Relative humidity} \Rightarrow \phi = \left(\frac{m_v}{m_g}\right) = \left(\frac{P_v.V/(R_v.T)}{P_g.V/(R_v.T)}\right) = \frac{P_v}{P_g} \quad \text{at temperature T}$$

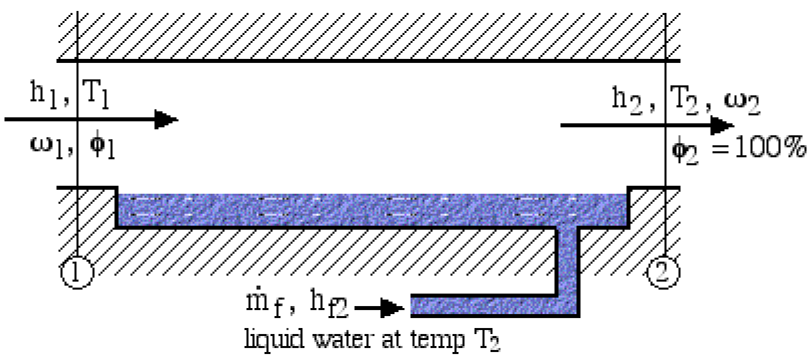
Furthermore, we can determine the specific humidity in terms of the relative humidity, and vice versa, as follows:

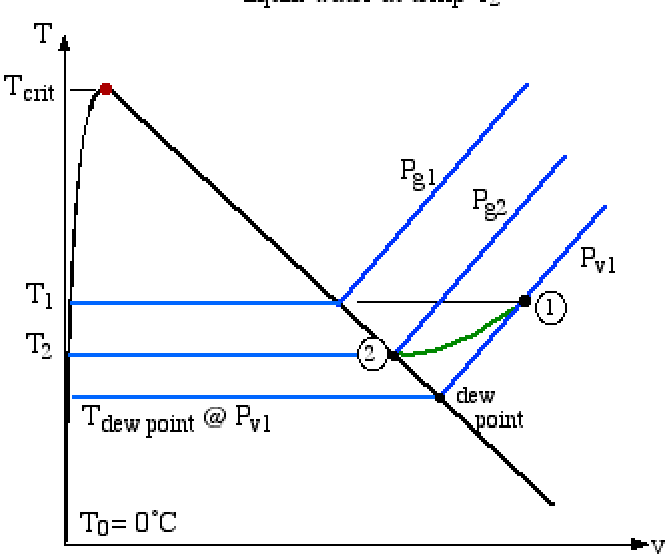
$$\omega = 0.622\left(\frac{P_v}{P_a}\right) = 0.622\left(\frac{\phi . P_g}{P_a}\right) = 0.622\left(\frac{\phi . P_g}{P - \phi . P_g}\right)$$

$$\phi = \left(\frac{\omega . P_a}{0.622\, P_g}\right) = \left(\frac{\omega . P}{(0.622 + \omega) P_g}\right)$$

The Adiabatic Saturation Process

There is *no* direct method of measuring specific humidity ω or relative humidity φ thus in this section we develop the **Adiabatic Saturation Process** leading to the practical **Wet & Dry Bulb Thermometer**, or **Sling Psychrometer**. Consider the channel below in which air of unknown humidity enters at station (1) and after absorbing moisture from the liquid pool, exits at 100% relative humidity at station (2). This process is shown on the *T-v* diagram below.





mass flow:

$$\dot{m}_{v1} + \dot{m}_f = \dot{m}_{v2}, \quad \dot{m}_a \text{ (dry air)}$$

$$\dot{m}_a\omega_1 + \dot{m}_f = \dot{m}_a\omega_2 \quad \Rightarrow \boxed{\dot{m}_f = \dot{m}_a(\omega_2 - \omega_1)}$$

energy:

$$\dot{m}_a h_1 + \dot{m}_f h_{f2} = \dot{m}_a h_2$$

$$\dot{m}_a h_1 + \dot{m}_a(\omega_2 - \omega_1)h_{f2} = \dot{m}_a h_2$$

$$\Rightarrow h_1 + (\omega_2 - \omega_1)h_{f2} = h_2$$

$$h_1 = C_p T_1 + \omega_1 h_{g1}, \; h_2 = C_p T_2 + \omega_2 h_{g2}$$

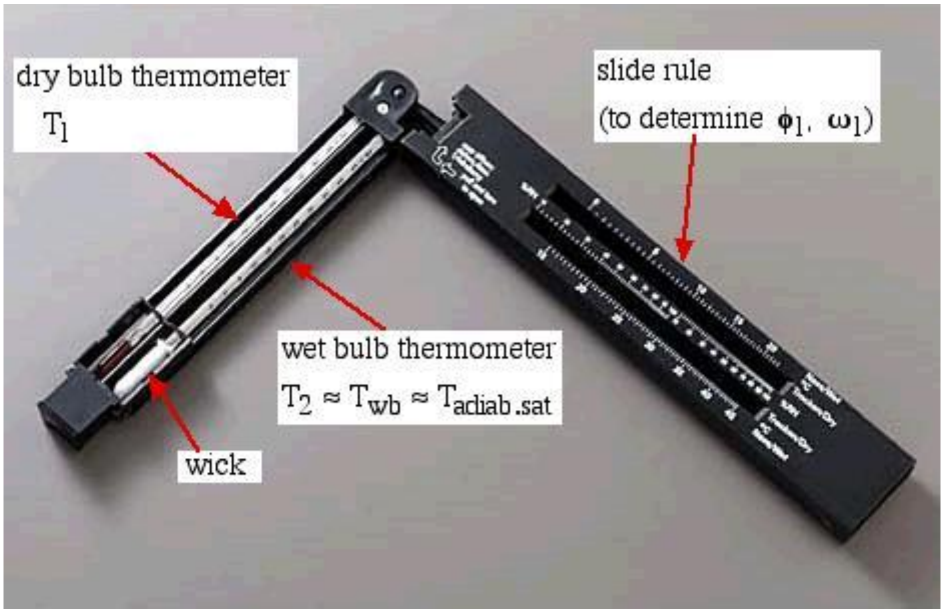
$$\omega_1(h_{g1} - h_{f2}) = C_p(T_2 - T_1) + \omega_2(h_{g2} - h_{f2})$$

$$\boxed{\omega_1 = \frac{C_p(T_2 - T_1) + \omega_2 h_{fg2}}{(h_{g1} - h_{f2})}} \Leftarrow \text{ how to determine: } T_2?\ \omega_2?$$

Referring to the *T-v* diagram above, since φ_2=100%, P_{v2}=P_{g2}, thus:

$$\boxed{\begin{array}{l}\omega_2 = 0.622\left(\dfrac{P_{g2@T2}}{P_2 - P_{g2@T2}}\right) \\ P_2 \approx P_1 \approx P\end{array}}$$

In order to determine T_2 and T_1 we use a **Sling Psychrometer** (or **Wet & Dry Bulb Thermometer**), typically as in the following figure. The wet bulb is wrapped in a cotton wick saturated with water, and one swings the thermometer in the air until a steady temperature is attained. The wet bulb temperature T_{wb} is then very closely equal to the adiabatic saturation temperature T_2.



Note that the relative humidity is then determined by means of a slide-rule on the handle of the sling-psychrometer, as shown in the above diagram.

Source: http://www.ohio.edu/mechanical/thermo/Applied/Chapt.7_11/Chapter10a.html