Active Feedback Control

Introduction:

The turbulence is an unstructured process, because there´s no pattern in it. The Fourier transform gives poor results when is used so that´s the reason why the turbulence is associated to a noise, and sometimes to a white noise. The phenomenon of bistable flow is turbulence induced transition process, because the reason of the jumping between two flow values is the turbulence. Because turbulence and noise are physically the same, mathematically the phenomenon can be simulated with a noise-induced transition model. Active vibration control is the active application of force in an equal and opposite fashion to the forces imposed by external vibration. With this application, a precision industrial process can be maintained on a platform essentially vibration-free. Active Feedback Vibration Control system provides fast response and effective control of vibrations magnitude.

FEEDBACK CONTROL SYSTEMS

Feedback, or closed loop systems feedback information from the process to control the operation of the machine. One of the earliest closed loop systems was that used by the Romans to maintain water levels in their aqueducts by means of floating valves. The concept of the feedback loop to control the dynamic behavior of the system: this is negative feedback, because the sensed value is subtracted from the desired value to create the error signal, which is amplified by the controller. Any system in which the output quantity is monitored and compared with the input, any difference being used to actuate the system until the output equals the input is called a closed-loop or feedback control system.

For a general robotic system, the models can be as

- For given motor commands, what is the outcome? ------------- forward model
- For a desired outcome, what are the motor commands? -------- inverse model
- From observing the outcome, how should we adjust the motor commands to achieve a goal? -------------- Feedback control

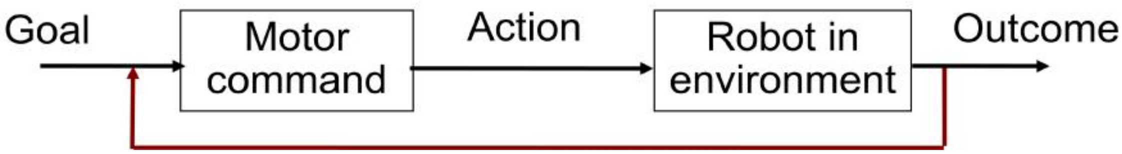

Fig. 3.1 (a) Scheme of feedback control

The essential feature of an automatic control system is the existence of a feedback loop to give good performance. This is a closed loop system; if the measured output is not compared with the input the loop is open. Usually it is required to apply a specific input to a system and for some other part of the system to respond in the desired way. The error between the actual response and the ideal response is detected and feed back to the input to modify it so that the error is reduced, as shown in Fig. 3.1 (a). The output of a device represented by a block in a block diagram cannot affect the input to that device unless a specific feedback loop is provided. The output of a device represented by a block in a block diagram cannot affect the input to that device unless a specific feedback loop is provided.

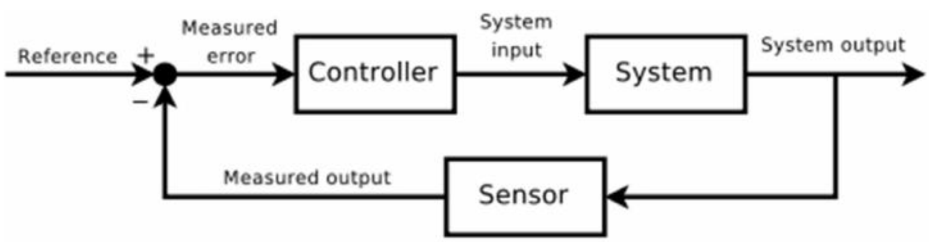

Fig. 3.1 (b) Scheme of feedback control

In general both input and output vary with time, and the control system can be mechanical, pneumatic, hydraulic and electrical in operation, or any combination of these or other power sources. The system should be absolutely stable so that if excited it will settle to some steady value, and it should be accurate in the steady state. The vibrations of many structures and devices are controlled by sophisticated control methods. Examples of the use of feedback control to remove vibrations range from machine tools to tall buildings and large spacecraft. One popular way to control the vibrations of a structure is to measure the position and velocity vectors of the structure and to use that information to drive the system in direct proportion to its positions and velocities.

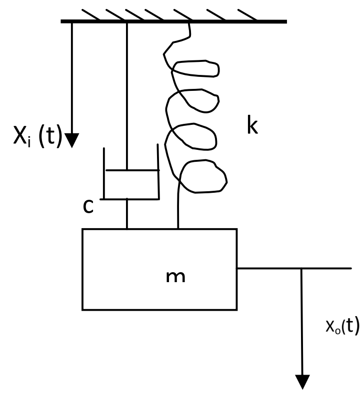

Fig. 3.2 Spring mass suspension system

The spring mass damper system comprises a body of mass m connected to an input with controlled platform by a spring and viscous damper. A input $x_i(t)$ is applied to this platform, as a control system, the response of the body or output $x_o(t)$ should be identical to the input. Considering the Newton's law of forces acting on the body, the equation of motion can be written as;

$$m\ddot{x}_o = k(x_i - x_o) + c(\dot{x}_i - \dot{x}_o) \qquad (3.1)$$

Equations of motion of this type have been solved for a harmonic input. For a general solution irrespective of input it is convenient to use the D-operator.

$$mD^2x_o = k(x_i - x_o) + cD(x_i - x_o) = (k + cD)(x_i - x_o) \qquad (3.2)$$

It should be noted that although using the D-operator is a neat and compact form of writing the equation it does not help with the solution of the response problem. Now the force ***F*** on the body is mD^2x_0 acted as,

$$F\left(\frac{1}{mD^2}\right) = x_o \qquad (3.3)$$

The transfer function of a system is the function by which the input is multiplied to give the output, so that since F is the input to the body and x_0 the output, (l/mD^2) is the transfer function (TF) of the body.

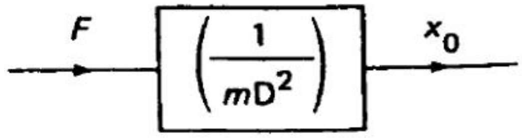

Fig. 3.3 Rigid body block diagram.

For the spring/damper unit F = (k + cD) (x_i – x_0)

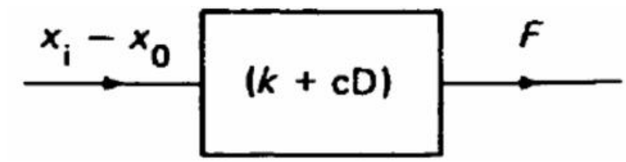

Fig. 3.4 Spring/damper block diagram.

Because the input to the spring/damper unit is (x_i – x_0) and the output is F, the TF is (k + cD). These systems can be combined as

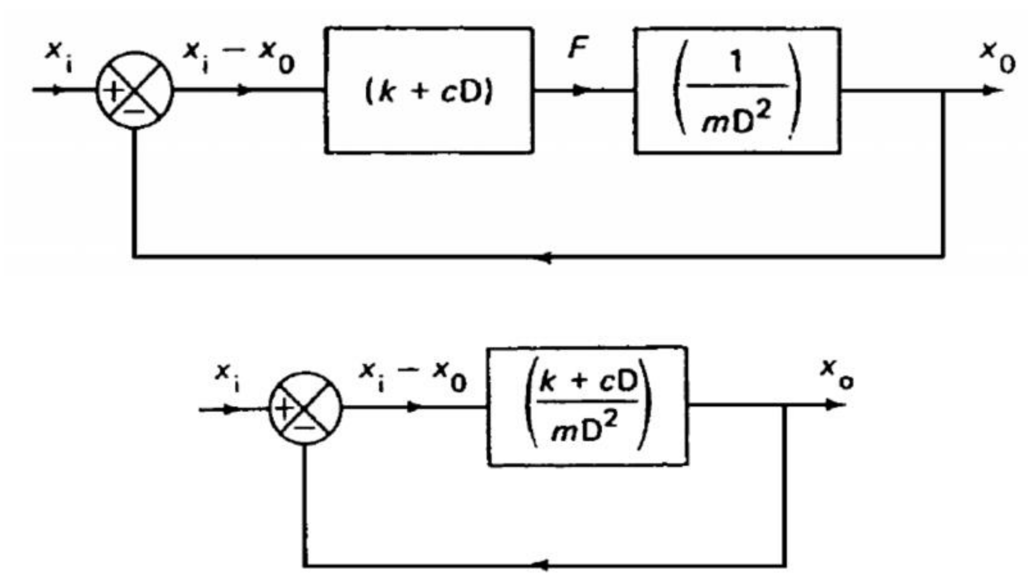

Fig. 3.5 System block diagram with transfer function

Above Fig. 3.5 shows the conventional unity feedback loop form. Essentially, the spring/damper acts as an error-sensing device and generates a restoring force related to that error. Since

$$(x_i - x_o)\left(\frac{k + cD}{mD^2}\right) = x_o$$

(3.4)

$$x_o = \left(\frac{cD + k}{mD^2 + cD + k}\right)x_i$$

$$\frac{x_o}{x_i} = \left(\frac{cD + k}{mD^2 + cD + k}\right).$$

(3.5)

This is the Transfer Function (TF) of the dynamic system with feedback, that is, it is the closed loop TF. Equation for control system becomes;

$$M\ddot{\mathbf{q}}(t) + (G + D)\dot{\mathbf{q}}(t) + (K + H)\mathbf{q}(t) = -K_p\mathbf{q}(t) - K_v\dot{\mathbf{q}}(t) + \mathbf{f}(t) \quad (3.6)$$

This is the vector version of Equation. Here, Kp and Kv are called feedback gain matrices. Thus, analysis performed on Equation will also be useful for studying the vibrations of structures controlled by position and velocity feedback (called state feedback). Most of the work carried out in linear systems has been developed for systems in state-space form. The state-space form is as;

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$

(3.7)

where **x** is called the state vector, A is the state matrix, and B is the input matrix. Here, **u** is the applied force, or control, vector. Much software and many theoretical developments exist for systems in the form of Equation. Equation (3.6) can be written in this form by several very simple transformations. To this end, let **x**1 =**q** and **x**2 = ˙**q**; then, Equation (3.7) can be written as the two coupled equations

$$\dot{\mathbf{x}}_1(t) = \mathbf{x}_2(t)$$
$$M\dot{\mathbf{x}}_2(t) = -(D + G)\mathbf{x}_2(t) - (K + H)\mathbf{x}_1(t) + \mathbf{f}(t)$$

(3.8)

This form allows the theory of control and systems analysis to be directly applied to vibration problems. Now suppose there exists a matrix, M^{-1}, called the inverse of M, such that $M^{-1}M = I$, the n×n identity matrix. Then, Equation (3.7) can be written as

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & I \\ -M^{-1}(K+H) & -M^{-1}(D+G) \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \mathbf{f}(t) \qquad (3.8)$$

where the state matrix A is

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}(K+H) & -M^{-1}(D+G) \end{bmatrix}$$

and the input matrix B is

$$B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$$

and where $\mathbf{x} = [\mathbf{x}_1 \;\; \mathbf{x}_2]^T = [\mathbf{q} \;\; \dot{\mathbf{q}}]^T$

The state-space approach has made a big impact on the development of control theory and, to a lesser but still significant extent, on vibration theory. This state-space representation also forms the approach used for numerical simulation and calculation for vibration analysis.

The matrix inverse M^{-1} can be calculated by a number of different numerical methods readily available in most mathematical software packages along with other factorizations. A simple calculation will show that for second order matrices of the form

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the inverse is given by

$$M^{-1} = \frac{1}{\det(M)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad (3.9)$$

Where $det(M) = ad - cb$

This indicates that, if ad=cb, then M is called singular and M^{-1} does not exist. In general, it should be noted that, if a matrix inverse exists, then it is unique. Furthermore, the inverse of a product of square matrices is given by

$$(AB)^{-1} = B^{-1}A^{-1} \qquad (3.10)$$

Example 3.1:

The first example (Meirovitch, 1980) consists of a rotating ring of negligible mass containing an object of mass m that is free to move in the plane of rotation, as indicated in Figure 3.6. In the figure, k_1 and k_2 are both positive spring stiffness values, c is a damping rate (also positive), and Ω is the constant angular velocity of the disc. The linearized equations of motion are

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \ddot{\mathbf{q}} + \left\{ \begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix} + 2\, m\Omega \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\} \ddot{\mathbf{q}} + \begin{bmatrix} k_1 + k_2 - m\Omega^2 & 0 \\ 0 & 2k_2 - m\Omega^2 \end{bmatrix} \mathbf{q} = \mathbf{0} \qquad (3.11)$$

where $\mathbf{q} = [x(t) \quad y(t)]^T$ is the vector of displacements. Here, M, D, and K are symmetric, while G is skew-symmetric, so the system is a damped gyroscopic system.

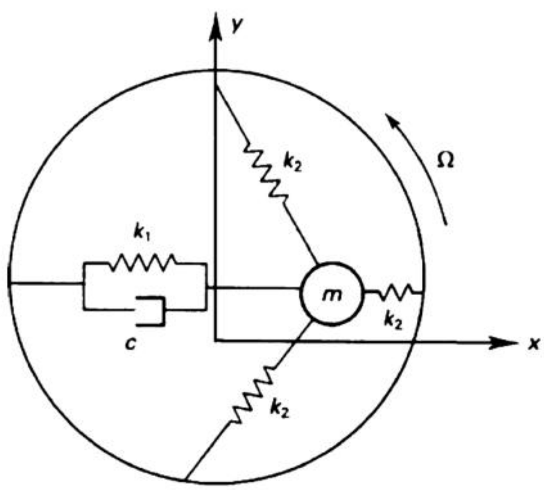

Figure 3.6 Schematic of a Simplified model of spinning satellite

$$\mathbf{x}^T M \mathbf{x} = [x_1 \; x_2] \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = m(x_1^2 + x_2^2) > 0 \qquad (3.12)$$

Note that, for any arbitrary nonzero vector **x**, the quadratic form associated with M becomes

$$\mathbf{x}^T \begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix} \mathbf{x} = cx_1^2 > 0 \tag{3.13}$$

Therefore, $\mathbf{x}^T M \mathbf{x}$ is positive for all nonzero choices of **x** and the matrix M is (symmetric) positive definite (and nonsingular, meaning that M has an inverse). Likewise, the quadratic form for the damping matrix becomes Note here that, while this quadratic form will always be nonnegative, the quantity $\mathbf{x}^T D\mathbf{x} = cx_1^2 = 0$ for the nonzero vector $\mathbf{x} = [0 \quad 1]^T$, so that D is only positive semi-definite (and singular). Easier methods for checking the definiteness of a matrix are given in the next chapter. The matrix G for the preceding system is obviously skew-symmetric. It is interesting to calculate its quadratic form and note that for any real value of **x**

$$\mathbf{x}^T G\mathbf{x} = 2\ m\Omega(x_1x_2 - x_2x_1) = 0 \tag{3.14}$$

This is true in general. The quadratic form of any-order real skew-symmetric matrix is zero.

LYAPUNOV STABILITY

A rough idea concerning the concept of stability was introduced for single-degree-of-freedom systems in the first chapter. It was pointed out that the sign of the coefficients of the acceleration, velocity, and displacement terms determined the stability behavior of a given single-degree-of-freedom system. That is, if the coefficients have the proper sign, the motion will always remain within a given bound. This idea is extended in this chapter to the multiple degree- of-freedom systems described in the previous two chapters. As in the case of the oscillatory behavior, the criterion based on the sign of the coefficients, is translated into a criterion based on the definiteness of certain coefficient matrices.

It should be noted that no universal definition of stability exists, but rather variations are adopted depending on the nature of the particular problem under consideration. However, all definitions of stability are concerned with the response of a system to certain disturbances and whether or not the response stays within certain bounds. The majority of the work done on the stability behavior of dynamical systems is based on a formal definition of stability given by Lyapunov (Hahn, 1962). This definition is stated with reference to the equilibrium point, $\mathbf{x}_0$, of a given system. In the case of the linear systems considered in this chapter, the equilibrium point can always be taken to be the zero vector. In addition, the definition of Lyapunov is usually stated in terms of the state vector of a given system rather than in physical coordinates directly, so that the equilibrium point refers to both the position and velocity.

Let $\mathbf{x}(0)$ represent the vector of initial conditions for a given system (both position and velocity). The system is said to have a *stable equilibrium* if, for any arbitrary positive numberε, there exists some positive number $\delta(\varepsilon)$ such that, whenever $\|X(0)\| < \delta$ then $\|X(t)\| < \varepsilon$ for all values of t > 0. A physical interpretation of this mathematical definition is that, if the initial state is within a certain value, i.e., $\|X(0)\| < \delta(\varepsilon)$, then the motion stays within another bound for all time, i.e., $\|X(t)\| < \varepsilon$. Here, $\|X(t)\|$ called the norm of $\mathbf{x}$, is defined by $\|X(t)\| = (X^T X)^{1/2}$.

To apply this definition to the single-degree-of-freedom system of Equation (1.1), note that $X(t) = [x(t)x(t)]^T$

Hence $\|X(t)\| = (X^T X)^{1/2} = \sqrt{x^2(t) + x'^2(t)}$ (3.12)

For the sake of illustration, let the initial conditions be given by $X(0) = 0$ and $X \cdot (0) = \omega = \sqrt{k / m}$. Then the solution is given by $x(t) = \sin \omega_n t$. Intuitively, this system has a stable response as the displacement response is bounded by 1, and the velocity response is bounded by ω_n. The following simple calculation illustrates how this solution satisfies the Lyapunov definition of stability.

First, note that

$$\|X(0)\| = [x^2(0) + x'(0)]^{\frac{1}{2}} = (0 + \omega_n^2)^{\frac{1}{2}} = \omega_n \quad (3.14)$$

and that

$$\|X(t)\| = [\sin^2 \omega_n t + \omega_n^2 \cos^2 \omega_n t]^{\frac{1}{2}} < (1 + \omega_n^2)^{1/2} \quad (3.15)$$

These expressions show exactly how to choose δ as a function of ε for this system.

From Equation (2.2) note that, if $(1 + \omega_n^2)^{\frac{1}{2}} < \varepsilon$ then $\|x(t)\| = \varepsilon$. From Equation (2.1) note that, if $\delta(\varepsilon)$ is chosen to be $\delta(\varepsilon) = \varepsilon\omega_n(1 + \omega_n^2)^{-\frac{1}{2}}$ then the definition can be followed directly to show that, if

$$\|X(0)\| = \omega_n < \delta(\varepsilon) = \frac{\varepsilon\omega_n}{\sqrt{1+\omega_n^2}} \quad (3.16)$$

is true, then $\omega_n < \frac{\varepsilon\omega_n}{\sqrt{1+\omega_n^2}}$. This last expression yields

$$\sqrt{1+\omega_n^2} < \varepsilon$$

That is, if $\|X(0)\| < \delta(\varepsilon)$, then $\sqrt{1+\omega_n^2} < \varepsilon$ must be true, and Equation (2.2) yields that

$$\|X(t)\| \leq \sqrt{1+\omega_n^2} < \varepsilon \quad (3.17)$$

Hence, by a judicious choice of the function $\delta(\varepsilon)$, it has been shown that, if $\|X(0)\| < \delta(\varepsilon)$,then $\|X(t)\| < \varepsilon$ for all t>0. This is true for any arbitrary choice of the positive number ε. The preceding argument demonstrates that the undamped harmonic oscillator has solutions that satisfy the formal definition of Lyapunov stability. If dissipation, such as viscous damping, is included in the formulation, then not only is this definition of stability satisfied, but also

$$\lim_{t\to\infty}\|X(t)\| = 0 \quad (3.18)$$

Such systems are said to be *asymptotically stable*. As in the single-degree-of-freedom case, if a system is asymptotically stable it is also stable. In fact, by definition, a system is asymptotically stable if it is stable and the norm of its response goes to zero as t becomes large. This can be seen by examining the definition of a limit (see Hahn, 1962). The procedure for calculating $\delta(\varepsilon)$ is similar to that of calculating ε and δ for limits and continuity in beginning calculus. As in the case of limits in calculus, this definition of stability does not provide the most efficient means of checking the stability of a given system.

Hence, the remainder of this chapter develops methods to check the stability properties of a given system that require less effort than applying the definition directly. There are many theories that apply to the stability of multiple-degree-of-freedom systems, some of which are discussed here. The most common method of analyzing the stability of such systems is to show the existence of a Lyapunov function for the system. A *Lyapunov function*, denoted by V(**x**), is a real scalar function of the vector **x**(t), which has continuous first partial derivatives and satisfies the following two conditions:

*1. V(**x**)>0 for all values of* $X(t) \neq 0$.

*2. V'(**x**)<0 for all values of* $X(t) \neq 0$

Here, V'(**x**) denotes the time derivative of the function V(**x**). Based on this definition of a Lyapunov function, several extremely useful stability results can be stated. The first result states that, if there exists a Lyapunov function for a given system, then that system is *stable*.

If, in addition, the function V'(**x**) is strictly less than zero, then the system is *asymptotically stable*. This is called the *direct*, or *second, method* of Lyapunov. It should be noted that, if a Lyapunov function cannot be found, nothing can be concluded about the stability of the system, as the Lyapunov theorems are only sufficient conditions for stability. The stability of a system can also be characterized by the eigenvalues of the system. In fact, it can easily be shown that a given linear system is stable if and only if it has no eigenvalue with a positive real part. Furthermore, the system will be asymptotically stable if and only if all of its eigenvalues have negative real parts (no zero real parts allowed). These statements are certainly consistent with the discussion in Section 2.1. The correctness of the statements can be seen by examining the solution using the expansion theorem (modal analysis) of the previous chapter [Equation (2.68)]. The eigenvalue approach to stability has the attraction of being both necessary and sufficient. However, calculating all the eigenvalues of the state matrix of a system is not always desirable.

The preceding statements about stability are not always the easiest criteria to check. In fact, use of the eigenvalue criteria requires almost as much calculation as computing the solution of the system. The interest in developing various different stability criteria is to find conditions that (1) are easier to check than calculating the solution, (2) are stated in terms of the physical parameters of the system, and (2) can be used to help design and/or control systems to be stable. Again, these goals can be exemplified by recalling the single-degree-of-freedom case, where it was shown that the sign of the coefficients *m*, *c* and *k* determine the stability behavior of the system. To this end, more convenient stability criteria are examined on the basis of the classifications of a given physical system.

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