

Acoustic impedance, intensity and power

Acoustic impedance is the ratio of acoustic pressure to flow. It allows us to calculate acoustic power, intensity and the reflection and transmission at boundaries. This is a background page to the multimedia chapter [Quantifying Sound](#). It uses some results from [The wave equation for sound](#).

Acoustic impedance, which has the symbol Z , is the ratio of acoustic pressure p to acoustic volume flow U . (There is a close analogy with [electrical impedance](#), the ratio of AC voltage V to current I .) The **specific acoustic impedance** is a ratio of acoustic pressure to specific flow, which is the same as flow per unit area, or acoustic flow velocity, u . So we define acoustic impedance Z and specific acoustic impedance z thus:

$$Z = p/U \quad \text{and} \quad z = p/u$$

In all cases, 'acoustic' refers to the oscillating component. With this proviso, we can say that

$$\text{acoustic impedance} = \text{pressure/flow} \quad \text{and} \quad \text{specific acoustic impedance} = \text{pressure/velocity}$$

The specific acoustic impedance, z , is an intensive property of a medium. We can specify the z of air or of water. The acoustic impedance Z is the property of a particular geometry and medium: we can discuss for example the Z of a particular duct filled with air. Usually, Z varies strongly when the frequency changes. The acoustic impedance at a particular frequency indicates how much sound pressure is generated by a given acoustic flow at that frequency.

Now let's get quantitative. We'll need the equation for a travelling sound wave that we found when we set up and solved the [sound wave equation](#). For a one dimensional (plane) wave travelling to the right, these are shown in the animation below.

On the page on the [sound wave equation](#), we wrote a solution for the particle displacement y as a function of position x and time t :

$$y = A \sin(kx - \omega t)$$

where A is the displacement amplitude, k is the wave number ($k = 2\pi/\lambda$) and ω is the angular frequency ($\omega = 2\pi f$). We also showed that the particle velocity u and the acoustic pressure p are given by

$$u = \frac{\partial y}{\partial t} = -y_m \omega \cos(kx - \omega t) \quad \text{and} \quad p = -\kappa \frac{\partial y}{\partial x} = -\kappa y_m k \cos(kx - \omega t)$$

where κ is the adiabatic bulk modulus. We defined z as p/u , so

$$z = \kappa k / \omega.$$

We also showed on that page that $\kappa/\rho = v^2$, above so $\kappa = \rho v^2$ where ρ is the density. We also had $\omega/k = v$. Substituting these gives

$$z = \rho v.$$

Values for air and water. For air, the density is 1.2 kg.m^{-3} and v is 343 m.s^{-1} , so the specific acoustic impedance for air is $420 \text{ kg.s}^{-1}.\text{m}^{-2} = 420 \text{ Pa.s.m}^{-1}$. The values for condensed phases are usually much higher than those of gases. For (fresh) water, the density is 1000 kg.m^{-3} and v is 1480 m.s^{-1} , so the specific acoustic impedance for water is $1.48 \text{ MPa.s.m}^{-1}$. For human soft tissues, the values are comparable with that of water.

Volume flow and acoustic impedance

Let's now consider a one dimensional wave passing through an aperture with area A . The volume flow U is the volume passing per second through the aperture. If the flow moves a distance $dy = udt$, then the volume passing through is $A dy$, so

$$U = \frac{dV}{dt} = A \frac{dy}{dt} = Au$$

The acoustic impedance Z is the ratio of sound pressure to volume flow, so *provided that the wave is only one dimensional*, we should have

$$Z = \frac{p}{U} = \frac{p}{Au} = \frac{z}{A}$$

In practice, it's usually more complicated: if you send a wave into a pipe with uniform cross section A , there are usually reflections from the other end of the pipe, whether open or closed. So there is a sum of waves travelling to the right and left*. In the absence of reflections, we call this the **characteristic acoustic impedance** Z_0 . So

$$Z_0 = \frac{\rho v}{A}.$$

It is possible to have no reflections, however. If the pipe is very long, then it takes a

long time for the reflection to return and, when it does, it is much attenuated by losses at the wall. I mention this because, in our research laboratory, we introduced acoustically infinite acoustic waveguides as calibrations for acoustic measurements. The technique allows precise measurements of impedance spectra over 9 octaves using a single impedance head. It is described in [this scientific paper](#).

* The reflections and resultant standing waves are very important in **musical instruments**. On the page [What is acoustic impedance?](#), we give examples of acoustic impedance spectra for pipes and for musical instruments, and explain the importance of the quantity in music acoustics.

Intensity and power as functions of time

The equations and animations above give the pressure and velocity. These are depicted in the animation below. Consider flow through an area A whose normal is in the direction that the wave is travelling. Consider the force F that is exerted on that plane by the oscillating air. From the definition of pressure, $F = pA$. The velocity of that moving air is u . The [power exerted by a force](#) applied at a point moving with speed v in the same direction is Fu . So the instantaneous power P_{Inst} transmitted is

$$P_{\text{Inst}} = Fu = pAu$$

where the subscript reminds us that P_{Inst} , F , u and p are all quantities that are varying with time, as shown in this animation. Note that, for a one dimensional wave, p and u are in phase so, when they are both positive, P_{Inst} is positive and, when they are both negative, P_{Inst} is still positive.

We previously defined the intensity as the power transmitted per unit area. (See [Travelling Waves II](#) to revise.) So the instantaneous intensity due to a sound wave is

$$I_{\text{inst}} = \frac{\text{Instantaneous power}}{A} = \frac{Fu}{A} = p u$$

Above we defined $z = p/u$. We may rearrange this and use the result above that $z = \rho v$ to obtain

$$u = \frac{p}{z} = \frac{p}{\rho v}$$

which we substitute in the previous equation to give

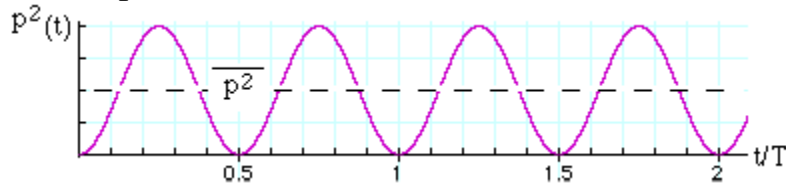
$$I_{\text{inst}} = \frac{p^2}{z} = \frac{p^2}{\rho v}$$

For the sinusoidal oscillation we are considering, let the pressure amplitude be p_m , so

$$I_{\text{inst}} = \frac{p_m^2 \sin^2 \omega t}{z} = \frac{p_m^2 \sin^2 \omega t}{\rho v}$$

Average intensity and RMS values

So the instantaneous power is proportional to the square of the pressure, as shown in this graph. We are usually interested in the average value of I , for which we need the mean value of $p^2(t)$.



We've seen in [Travelling Waves II](#) that the average of a sine squared or cos squared function, over a whole number of cycles, is $\frac{1}{2}$.

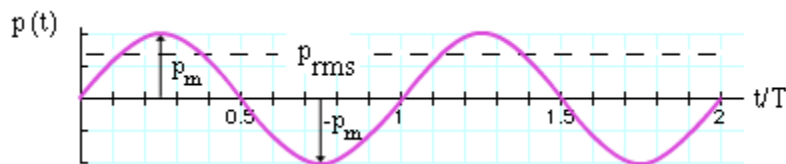
$$\text{Average intensity } I = \frac{1}{2} \frac{P_m^2}{z} = \frac{1}{2} \frac{P_m^2}{\rho v}$$

The magnitude of a sound wave is sometimes described in terms of the pressure amplitude p_m . Much more common, however, is to specify the RMS pressure: this is the square root of the mean of the square of the pressure. (See [Power and RMS](#) for more details.) Because the average value of $p^2(t)$ is $\frac{1}{2}$,

$$P_{\text{rms}} = P_{\text{Root Mean Square}} = P = \frac{P_m}{\sqrt{2}}$$

which we can represent

graphically



If you see the acoustic pressure of a sound wave written as p , with no indication that it is the instantaneous value, it will nearly always mean the RMS value. So that allows us to drop the $\frac{1}{2}$ in the previous expression and write the result for the average intensity

$$I = \frac{p^2}{z} = \frac{p^2}{\rho v}$$

Note that this equation is also true for instantaneous values.

Applications and implications

As we noted above, z varies greatly among media, especially between gas and condensed phases. Water is 800 times denser than air and its speed of sound is 4.3 times greater than that of air. So the specific acoustic impedance of water is 3500 times higher than that of air. This means that a sound in water with a given pressure amplitude is 3500 times less intense than one in air with the same pressure. This is because the air, with its lower z , moves with a much greater velocity and displacement amplitude than does water. Reciprocally, if a sound in water and another in air have the same intensity, then the pressure is much smaller in air.

Remember that the specific acoustic impedance, z , is an intensive property of a medium while acoustic impedance Z is the property of a particular geometry and medium: we can discuss for example the Z of a particular duct.

As mentioned above Z usually depends strongly on frequency. A few simple cases are worth examining in detail. When a small volume is compressed by a sound wave, the volume flow in acts to compress the air and raise its pressure. So it acts as a **compliance**. When a small volume is accelerated by the difference in sound pressure, acoustic flow is determined by pressure difference and we have an acoustic **inertance**. We introduce these in [Compliance, Inertance and Impedance](#).

On the site [What is acoustic impedance?](#), we give examples of acoustic impedance spectra for pipes and for musical instruments, and explain the importance of the quantity in music acoustics. See [this web site](#) for the research lab and [this scientific paper](#) for some technical information. See also [Acoustic impedance measurement techniques](#).

Source: <http://www.animations.physics.unsw.edu.au/jw/sound-impedance-intensity.htm>