GEOMETRIC OBJECTS AND TRANSFORMATIONS – II

5.1 Transformations in homogeneous coordinates

More generally, we consider the <*x*, *y*, *z*> position vector to be merely a special case of the four-component <*x*, *y*, *z*, *w*> form. This type of four-component position vector is called a *homogeneous position*. When we express a vector position as an <*x*, *y*, *z*> quantity, we assume that there is an implicit 1 for its *w* component.

Mathematically, the *w* value is the value by which you would divide the *x*, *y*, and *z* components to obtain the conventional 3D (nonhomogeneous) position, as shown in Equation 4-1.

Equation 4-1 Converting Between Nonhomogeneous and Homogeneous Positions

$$\left\langle \frac{x}{w}, \frac{y}{w}, \frac{z}{w}, 1 \right\rangle = \langle x, y, z, w \rangle$$

Expressing positions in this homogeneous form has many advantages.

- For one, multiple transformations, including projective transformations required for perspective 3D views, can be combined efficiently into a single 4x4 matrix.
- Also, using homogeneous positions makes it unnecessary to perform expensive intermediate divisions and to create special cases involving perspective views.
- Homogeneous positions are also handy for representing directions and curved surfaces described by rational polynomials.

1	0	dx		x
0	1	dy	·y	
0	0	1		1

1 * x	+	0 * y	+	dx * 1
0 * x	+	1 * y	+	dy * 1
0 * x	+	0 * y	+	1 * 1

Concatenation of transformations

- Rotate a house about the origin
- Rotate the house about one of its corners
  - translate so that a corner of the house is at the origin
  - rotate the house about the origin

– translate so that the corner returns to its original position

All these operations could be carried out at once by multiplying the corresponding matrices and obtaining one single matrix which would then be multiplied with the projection matrix of the object to obtain the final result.

World Space

Object space for a particular object gives it no spatial relationship with respect to other objects. The purpose of *world space* is to provide some absolute reference for all the objects in your scene. How a world-space coordinate system is established is arbitrary. For example, you may decide that the origin of world space is the center of your room. Objects in the room are then positioned relative to the center of the room and some notion of scale (Is a unit of distance a foot or a meter?) and some notion of orientation (Does the positive *y*-axis point "up"? Is north in the direction of the positive *x*-axis?).

The Modeling Transform

The way an object, specified in object space, is positioned within world space is by means of a modeling transform. For example, you may need to rotate, translate, and scale the 3D model of a chair so that the chair is placed properly within your room's world-space coordinate system. Two chairs in the same room may use the same 3D chair model but have different modeling transforms, so that each chair exists at a distinct location in the room.

You can mathematically represent all the transforms in this chapter as a 4x4 matrix. Using the properties of matrices, you can combine several translations, rotations, scales, and projections into a single 4x4 matrix by multiplying them together. When you concatenate matrices in this way, the combined matrix also represents the combination of the respective transforms. This turns out to be very powerful, as you will see.

If you multiply the 4x4 matrix representing the modeling transform by the object-space position in homogeneous form (assuming a 1 for the *w* component if there is no explicit *w* component), the result is the same position transformed into world space. This same matrix math principle applies to all subsequent transforms discussed in this chapter.

Figure 4-2 illustrates the effect of several different modeling transformations. The left side of the figure shows a robot modeled in a basic pose with no modeling transformations applied. The right side shows what happens to the robot after you apply a series of modeling transformations to its various body parts. For example, you must rotate and translate the right arm to position it as shown. Further transformations may be required to translate and rotate the newly posed robot into the proper position and orientation in world space.

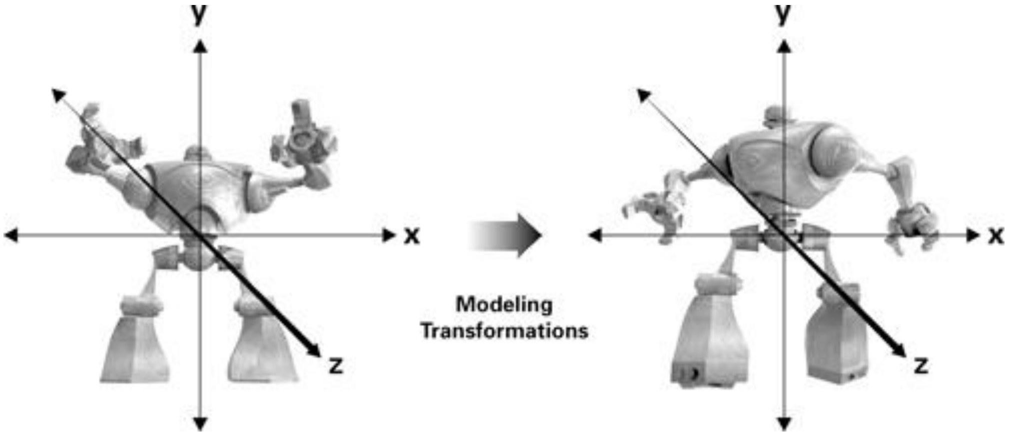

Figure 4-2 The Effect of Modeling Transformations

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