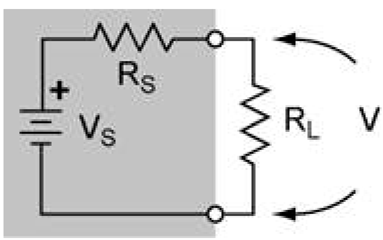
MAXIMUM POWER TRANSFER

Real power sources include devices such as your audio amplifier. When a load is connected to any power source it is often desired to transfer the maximum power to the load (speakers for example). Because real sources have an internal source resistance, some power is ALWAYS lost within the source. In the example below we have attached a load R_L to a voltage source V_S with internal resistance R_S. This is the*load matching problem*. The second example is for a *ideal power supply* where we are simply trying transfer the maximum power to the load.

Maximum power transfer for fixed non zero R_S, the load matching problem.

Given a source with an internal resistance R_S what value of R_L will give the maximum power transferred to R_L? Reducing R_L increases the current in the circuit, but decreases the voltage across R_L. IncreasingR_L increases the voltage, but decreases the current. Because the power delivered to the load P_L is the *product* of the current and the voltage, there must be a value of R_L that maximizes P_L.



$$0 = \frac{dP_L}{dR_L} \qquad P_L = \frac{V^2}{R_L} \qquad V = V_S\left(\frac{R_L}{R_s + R_L}\right)$$

$$P_L = \frac{V^2}{R_L} = \frac{1}{R_L}\left[V_S\left(\frac{R_L}{R_s + R_L}\right)\right]^2 = \frac{V_S^2 R_L}{(R_s + R_L)^2}$$

$$0 = \frac{dP_L}{dR_L} = \frac{d}{dR_L}\left(\frac{V_S^2 R_L}{(R_s + R_L)^2}\right) = \frac{V_S^2}{(R_s + R_L)^2} + (-2)\frac{V_S^2 R_L}{(R_s + R_L)^3}$$

$$\frac{V_S^2}{(R_s + R_L)^2} = 2\frac{V_S^2 R_L}{(R_s + R_L)^3}$$

$$R_s + R_L = 2R_L$$

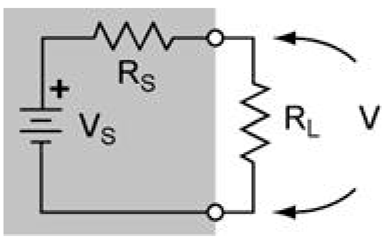
$$R_L = R_s$$

The maximum power transfer occurs when the load resistance equals the source resistance. Audio amplifiers specify the impedance of the speakers; this is precisely the same concept. For example, an amplifier designed to drive an 8 ohm speaker can also drive two 4 ohm speakers placed series or two 16 ohm speaks placed in parallel.

Any combination that changes the equivalent load resistance will result in less volume.

Maximum power transfer for fixed non zero R_L, the ideal power source problem.

Now let's consider the case where R_L is fixed and R_S can be varied. That is, given R_L, what R_S will maximize the P_L? Another way to state the problem is: What value of R_S will minimize the P_S, power dissipated in the source? Clearly the smaller we make R_S, the smaller P_S will become. Hence if $R_S = 0$, $P_S = 0$, so all the power from the source is transferred to the load. Therefore the answer is $R_S = 0$. The mathematical proof is below.



$$0 = \frac{dP_L}{dR_S} \qquad P_L = \frac{V^2}{R_L} \qquad V = V_S\left(\frac{R_L}{R_s + R_L}\right)$$

$$P_L = \frac{V^2}{R_L} = \frac{1}{R_L}\left[V_S\left(\frac{R_L}{R_s + R_L}\right)\right]^2 = \frac{V_S^2 R_L}{(R_s + R_L)^2}$$

$$0 = \frac{dP_L}{dR_S} = \frac{d}{dR_S}\left(\frac{V_S^2 R_L}{(R_s + R_L)^2}\right) = (-2)\frac{V_S^2 R_L}{(R_s + R_L)^3}$$

$$0 = \frac{1}{(R_s + R_L)} = \frac{1}{(R_s + R_L)}\frac{\frac{1}{R_L}}{\frac{1}{R_L}} = \frac{\frac{1}{R_L}}{\left(\frac{R_s}{R_L} + 1\right)}$$

$$0 = \frac{1}{\left(\frac{R_s}{R_L} + 1\right)}$$

$$\frac{R_s}{R_L} \to 0$$

$$R_s = 0 \quad \text{for} \quad R_L \neq 0$$

Source : http://www.nhn.ou.edu/~bumm/ELAB/Lect_Notes/Max_Power_Transfer_v2_0_1.html