Depletion Width

We can now go back to the **charge density as a function of position graph** and easily find the electric field in the depletion region as a function of position. If we integrate **Gauss' Law**, we get for the electric field:

$$\boldsymbol{E}(x)=\frac{1}{\varepsilon}\int\rho(x)\mathrm{d}x$$

(1)

We **could** write down an expression for $\rho(x)$ and then formally integrate it to get $\boldsymbol{E}(x)$ but we can also just do it graphically, which is a lot easier, and gives us a much more intuitive feeling for what is going on. Let's start doing our integral at [x equals -infinity] Whenever we perform an integral such as **Equation 1**, we've got to remember to add a constant to our answer. Since we can not have an electric field which extends to infinity (either plus or minus) however, we can safely assume $\boldsymbol{E}(\text{-infinity})=0$ and remains at that value until we get to the edge of the depletion region at (essentially) x equals zero. Since the charge density is zero all the way up to the edge of depletion region, Gauss tells us that the electric field can not change here either. When we get to x=0 we encounter the large negative delta-function of negative charge at the edge of the depletion region. If you can remember back to your calculus, when you integrate a delta function, you get a step. Since the charge in the p-side delta function is negative, when we integrate it, we get a negative step. Since we don't know (yet) how big the step will be, let's just call it -|Emax|.

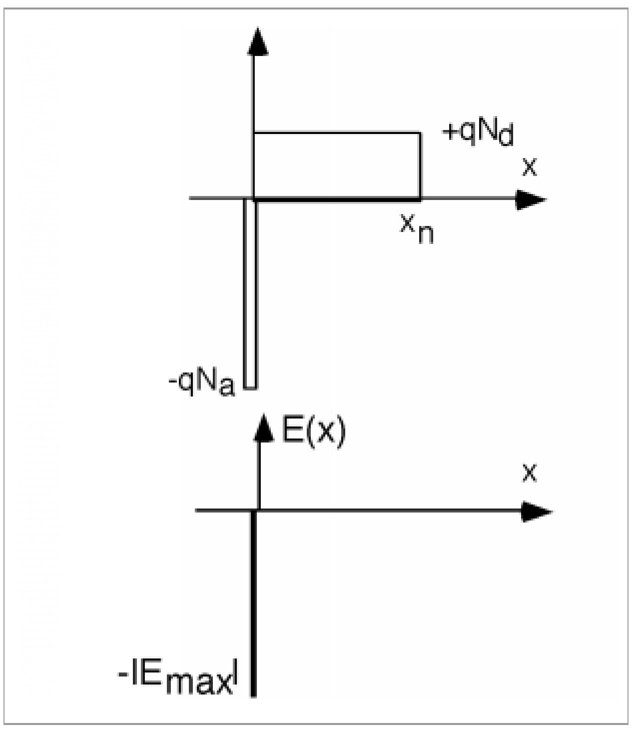

Figure 1: Finding the electric field in the p-type region

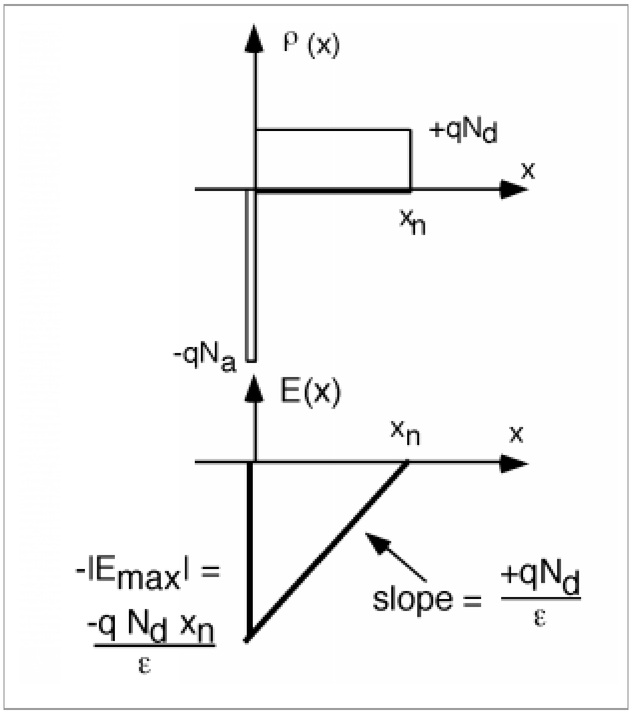

Figure 2: Finishing the integral

In the n-side of the depletion region

$$\rho(x) = (q)N_d = \varepsilon \frac{\partial \mathbf{E}}{\partial x}$$

(2)

and so we plot $E(x)$ with a (positive) slope of $\frac{qN_d}{\varepsilon}$, starting at $E(x) = -E_{max}$ at x = 0. This line continues with this positive slope until it reaches a value of 0 at x = x_n. We know that E(x) must equal 0 at x = x_n because there is no further charge outside of the depletion region and E must be 0 outside this region.

We are now done doing the integral. We would know everything about this problem, if we just knew what x_n was. Note that since we know the slope of the triangle now, we can find $-E_{max}$ in terms of the slope and x_n. We can derive an expression for x_n, if we remember that the integral of the electric field over a distance is the potential drop across that distance. What is the potential drop in going from the p-side to the n-side of the diode?

As a reminder, **Figure 3** shows the junction band diagram again. The potential drop must just be V_{bi} the "built-in" potential of the junction. Obviously V_{bi} can not be greater than 1.1 V, the band-gap potential. On the other hand, by looking at **Figure 3**, and remembering that the bandgap in silicon is 1.1 eV, it will not be some value like 0.2 or 0.4 volts either. Let's make life easy for ourselves, and say $V_{bi} = 1\text{Volt}$. This will not be too far off, and as you will see shortly, the answer is not very sensitive to the **exact** value of V_{bi} anyway.

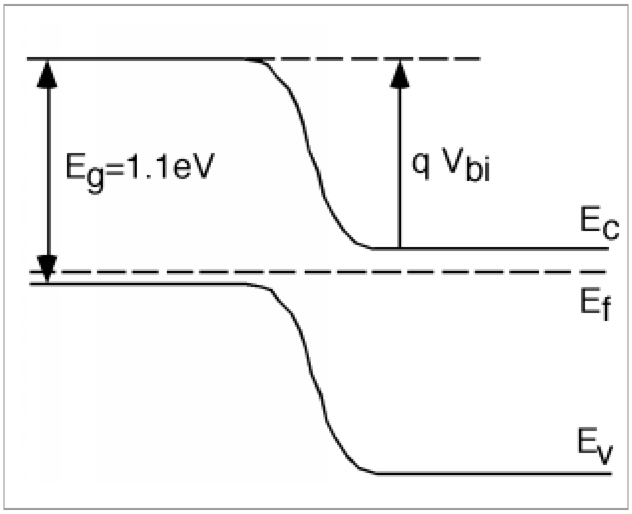

Figure 3: Band diagram for a p-n junction

The integral of $E(x)$ is now just the area of the triangle in **Figure 2**. Getting the area is easy:

$$\begin{aligned} \text{area} &= \frac{1}{2}\text{base}\times\text{height} \\ &= 1/2\, x_n \frac{q N_d x_n}{\varepsilon} \\ &= \frac{q N_d x_n^2}{2\varepsilon} \\ &= V_{\text{bi}} \end{aligned}$$

(3)

We can simply turn **Equation 3** around and solve for x_n.

$$x_n = \sqrt{\frac{2\varepsilon V_{\text{bi}}}{q N_d}}$$

(4)

As we said, for silicon, $\varepsilon_{\text{Si}} = 1.1\times10^{-12}$. Let's let $N_d = 10^{16}\text{cm}^{-3}$ donors. As we already know from before, $q = 1.6\times10^{-19}$ Coulombs. This makes $x_n = 3.7\times10^{-5}\text{cm}$ or 0.37 μm long. Not a very wide depletion region! How big is $|E_{\max}|$? Plugging in

$$E_{\max} = \frac{q N_d x_n}{\varepsilon}$$

(5)

We find $|E_{\max}| = 53{,}000\frac{V}{\text{cm}}$! Why such a big electric field? Well, we've got to shift the potential by about a volt, and we do not have much distance to do it in (less than a micron), and so there must be, by default, a fairly large field in the depletion region. Remember, potential is electric field **times** distance.

Enough p-n junction electrostatics. The point of this exercise was two-fold; • **a):** so you would know something about the details of what is really going on in a p-n junction ; • **b):** to show you that with just some very simple electrostatics and a little thinking, it is not so hard to figure these things out!

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