Module
3
Quantization and Coding

Lesson
14
Delta Modulation (DM)

After reading this lesson, you will learn about

- ***Principles and features of Delta Modulation;***
- ***Advantages and limitations of Delta Modulation;***
- ***Slope overload distortion;***
- ***Granular Noise;***
- ***Condition for avoiding slope overloading;***

If the sampling interval 'Ts' in DPCM is reduced considerably, i.e. if we sample a band limited signal at a rate much faster than the Nyquist sampling rate, the adjacent samples should have higher correlation (**Fig. 3.14.1**). The sample-to-sample amplitude difference will usually be very small. So, one may even think of only 1-bit quantization of the difference signal. The principle of Delta Modulation (DM) is based on this premise. Delta modulation is also viewed as a 1-bit DPCM scheme. The 1-bit quantizer is equivalent to a two-level comparator (also called as a hard limiter). **Fig. 3.14.2** shows the schematic arrangement for generating a delta-modulated signal. Note that,

$$e(kT_s) = x(kT_s) - \hat{x}(kT_s) \qquad 3.14.1$$

$$= x(kT_s) - u([k-1]T_s) \qquad 3.14.2$$

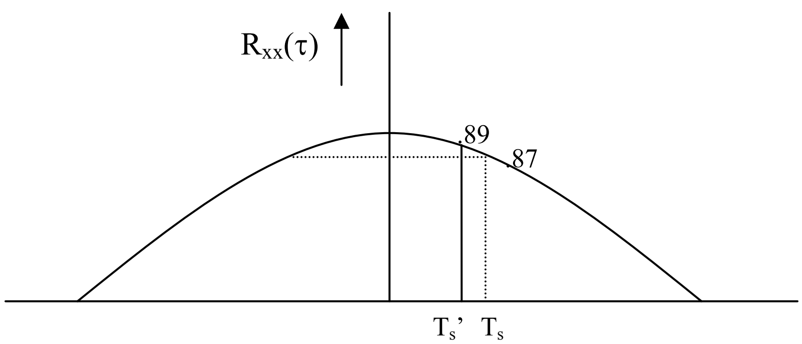

Fig. 3.14.1 *The correlation increases when the sampling interval is reduced*

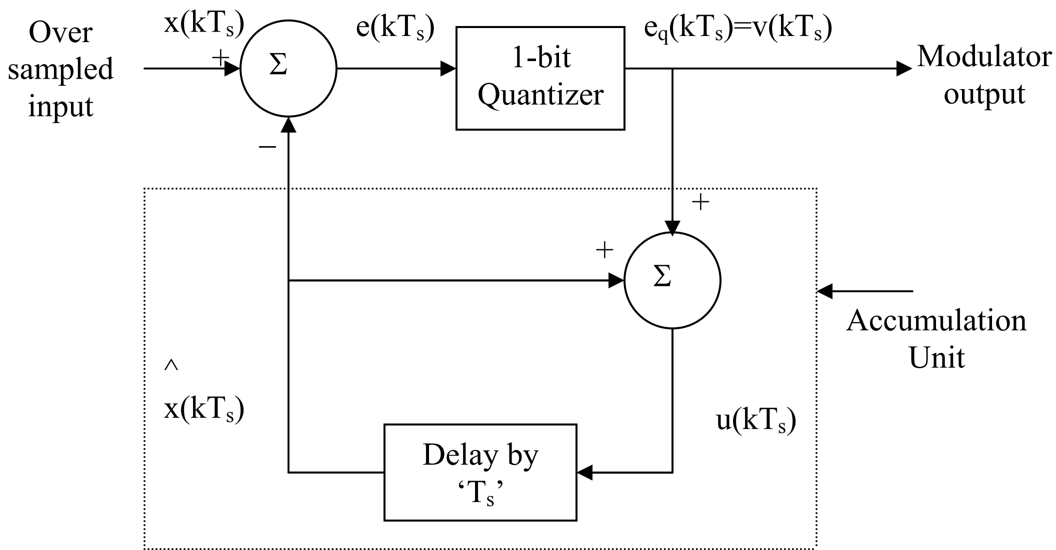

Fig. 3.14.2 *Block diagram of a delta modulator*

Some interesting features of Delta Modulation

- No effective prediction unit – the prediction unit of a DPCM coder (**Fig. 3.13.2**) is eliminated and replaced by a single-unit delay element.
- A 1-bit quantizer with two levels is used. The quantizer output simply indicates whether the present input sample $x(kT_s)$ is more or less compared to its accumulated approximation $\hat{x}(kT_s)$.
- Output $\hat{x}(kT_s)$ of the delay unit changes in small steps.
- The accumulator unit goes on adding the quantizer output with the previous accumulated version $\hat{x}(kT_s)$.
- $u(kT_s)$, is an approximate version of $x(kT_s)$.
- Performance of the Delta Modulation scheme is dependent on the sampling rate. Most of the above comments are acceptable only when two consecutive input samples are very close to each other.

■

Now, referring back to **Fig. 3.14.2**, we see that,

$$e(kT_S) = x(kT_S) - \{\hat{x}([k-1]T_S) + v([k-1]T_S)\} \qquad 3.14.3$$

Further,

$$v(kT_S) = e_q(kT_S) = s.sign[e(kT_S)] \qquad 3.14.4$$

Here, 's' is half of the step-size δ as indicated in **Fig. 3.14.3.**

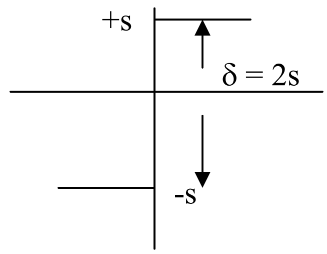

Fig. 3.14.3 *This diagram indicates the output levels of 1-bit quantizer. Note that if δ is the step size, the two output levels are ± s*

Now, assuming zero initial condition of the accumulator, it is easy to see that

$$u(kT_s) = s.\sum_{j=1}^{k} sign[e(jT_s)]$$

$$u(kT_s) = \sum_{j=1}^{k} v(jT_s) \qquad 3.14.5$$

Further,

$$\hat{x}(kT_s) = u([k-1]T_s) = \sum_{j=1}^{k-1} v(jT_s) \qquad 3.14.6$$

Eq. 3.14.6 shows that $\hat{x}(kT_s)$ is essentially an accumulated version of the quantizer output for the error signal $e(kT_s)$. $\hat{x}(kT_s)$ also gives a clue to the demodulator structure for DM. **Fig. 3.14.4** shows a scheme for demodulation. The input to the demodulator is a binary sequence and the demodulator normally starts with no prior information about the incoming sequence.

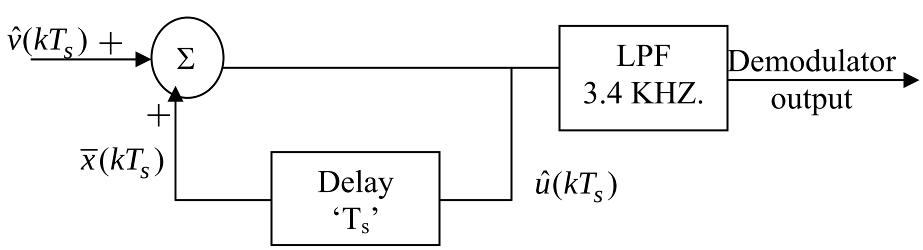

Fig. 3.14.4 *Demodulator structure for DM*

Now, let us recollect from our discussion on DPCM in the previous lesson (Eq. 3.13.3) that, $u(kT_s)$ closely represents the input signal with small quantization error $q(kT_s)$, i.e.

$$u(kT_s) = x(kT_s) + q(kT_s) \qquad 3.14.7$$

Next, from the close loop including the delay-element in the accumulation unit in the Delta modulator structure, we can write

$$u([k-1]T_s) = \hat{x}(kTs) = x(kT_s) - e(kT_s) = x([k-1]T_s) + q([k-1]T_s) \qquad 3.14.8$$

Hence, we may express the error signal as,

$$e(kT_s) = \{x(kT_s) - x([k-1]T_s)\} - q([k-1]T_s) \qquad 3.14.9$$

That is, the error signal is the difference of two consecutive samples at the input except the quantization error (when quantization error is small).

Advantages of a Delta Modulator over DPCM

a) As one sample of $x(kT_s)$ is represented by only one bit after delta modulation, no elaborate word-level synchronization is necessary at the input of the demodulator. This reduces hardware complexity compared to a PCM or DPCM demodulator. Bit-timing synchronization is, however, necessary if the demodulator in implemented digitally.

b) Overall complexity of a delta modulator-demodulator is less compared to DPCM as the predictor unit is absent in DM.

However DM also suffers from a few **limitations** such as the following:

a) **Slope over load distortion***:* If the input signal amplitude changes fast, the step-by-step accumulation process may not catch up with the rate of change (see the sketch in **Fig. 3.14.5**). This happens initially when the demodulator starts operation from cold-start but is usually of negligible effect for speech. However, if this phenomenon occurs frequently (which indirectly implies smaller value of auto-correlation co-efficient $R_{xx}(\tau)$ over a short time interval) the quality of the received signal suffers. The received signal is said to suffer from slope-overload distortion. An intuitive remedy for this problem is to increase the step-size δ but that approach has another serious lacuna as noted in b).

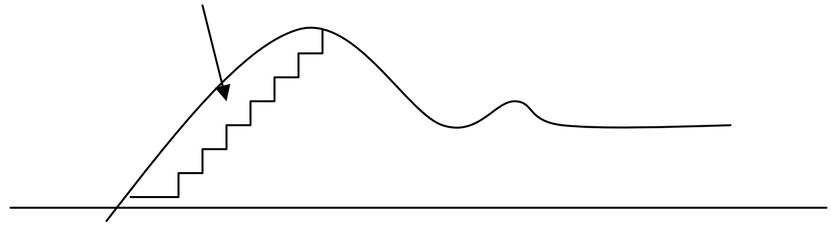

Fig. 3.14.5 *A sketch indicating slope-overload problem. The horizontal axis represents time. The continuous line represents the analog input signal, before sampling and the stair-case represents the output* $\hat{x}(kT_s)$ *of the delay element.*

b) **Granular noise:** If the step-size is made arbitrarily large to avoid slope-overload distortion, it may lead to 'granular noise'. Imagine that the input speech signal is fluctuating but very close to zero over limited time duration. This may happen due to pauses between sentences or else. During such moments, our delta modulator is likely to produce a fairly long sequence of 101010…., reflecting that the accumulator output is close but alternating around the input signal. This phenomenon is manifested at the output of the delta demodulator as a small but perceptible noisy background. This is known as 'granular noise'. An expert listener can recognize the crackling sound. This noise should be kept well within a tolerable limit while deciding the step-size. Larger step-size increases the granular noise while smaller step size increases the degree of slope-overload distortion. In the first level of design, more care is given to avoid the slope-overload distortion. We will briefly discuss about this approach while keeping the step-size fixed. A more efficient approach of adapting the step-size, leading to Adaptive Delta Modulation (ADM) , is excluded.

Condition for avoiding slope overload: From **Fig. 3.14.3** we may observe that if an input signal changes more than half of the step size (i.e. by 's') within a sampling interval, there will be slope-overload distortion. So, the desired limiting condition on the input signal x(t) for avoiding slope-overloading is,

$$\frac{dx(t)}{dt}\Big|_{\max} \le \frac{s}{T_s} \qquad 3.14.10$$

Quantization Noise Power

Let us consider a sinusoid representing a narrow band signal $x(t) = a_m \cos(2\pi ft)$ where 'f' represents the maximum frequency of the signal and 'a_m' its peak amplitude. There will be no slope-overload error if

$$\frac{s}{T_s} \geq 2\pi a_m f \quad \text{or} \quad a_m \leq \frac{s}{2\pi f T_s}$$

The above condition effectively limits the power of x(t). The maximum allowable power of x(t) = $P_{max} = \frac{a_m^2}{2} = \frac{s^2}{8\pi^2 f^2 T_s^2}$.

Once the slope overload distortion has been taken care of, one can find an estimate of $SQNR_{max}$. Assuming uniform quantization noise between +s and –s, the quantization noise power is

$$N_Q = \frac{4s^2}{12} = \frac{s^2}{3}$$

Let us now recollect that the sampling frequency $f_s = 1/T_s$ is much greater than 'f' . The granular noise due to the quantizer can be approximated to be of uniform power spectral density over a frequency band upto f_s (**Fig. 3.14.6)**. The low pass filter at the output end of the delta demodulator is designed as per the bandwidth of x(t) and much of the quantization noise power is filtered off. Hence, we may write,

the in-band quantization noise power $\approx \frac{f}{f_s}.N_Q$

Therefore, $SQNR_{max}$= (Maximum signal power) / (In-band quantization noise power)

$$= \left(\frac{3}{8\pi^2}\right).\left(\frac{f_s}{f}\right)^3$$

The above expression indicates that we can expect an improvement of about 9dB by doubling the sampling rate and it is not a very impressive feature when compared with a PCM scheme. Typically, when the permissible data rate after quantization and coding of speech signal is more than 48 Kbps, PCM offers better SQNR compared to linear DM.

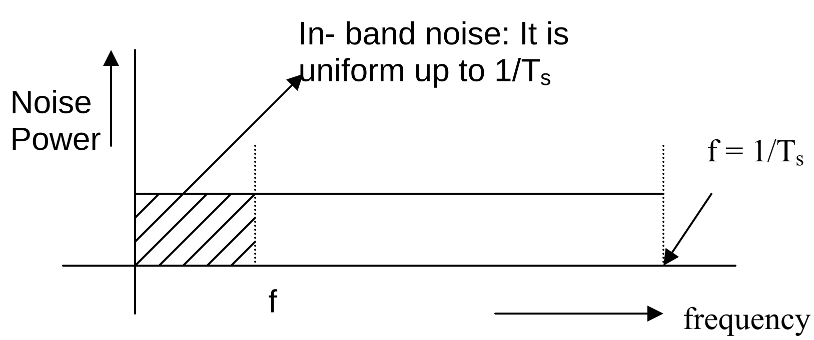

Fig. 3.14.6 *In-band noise power at the output of the low pass filter in a delta demodulator is shown by the shaded region*

Problems

Q3.14.1) Comment if a delta modulator can also be called as a 1-bit DPCM scheme.

Q3.14.2) Mention two differences between DPCM and Delta Modulator

Q3.14.3) Suggest a solution for controlling the granular noise at the output of a delta modulator.

Q3.14.4) Let $x(t) = 2 \cos (2\pi \times 100t$. If this signal is sampled at 1 KHz for delta modulator, what is the maximum achievable SQNR in dB?