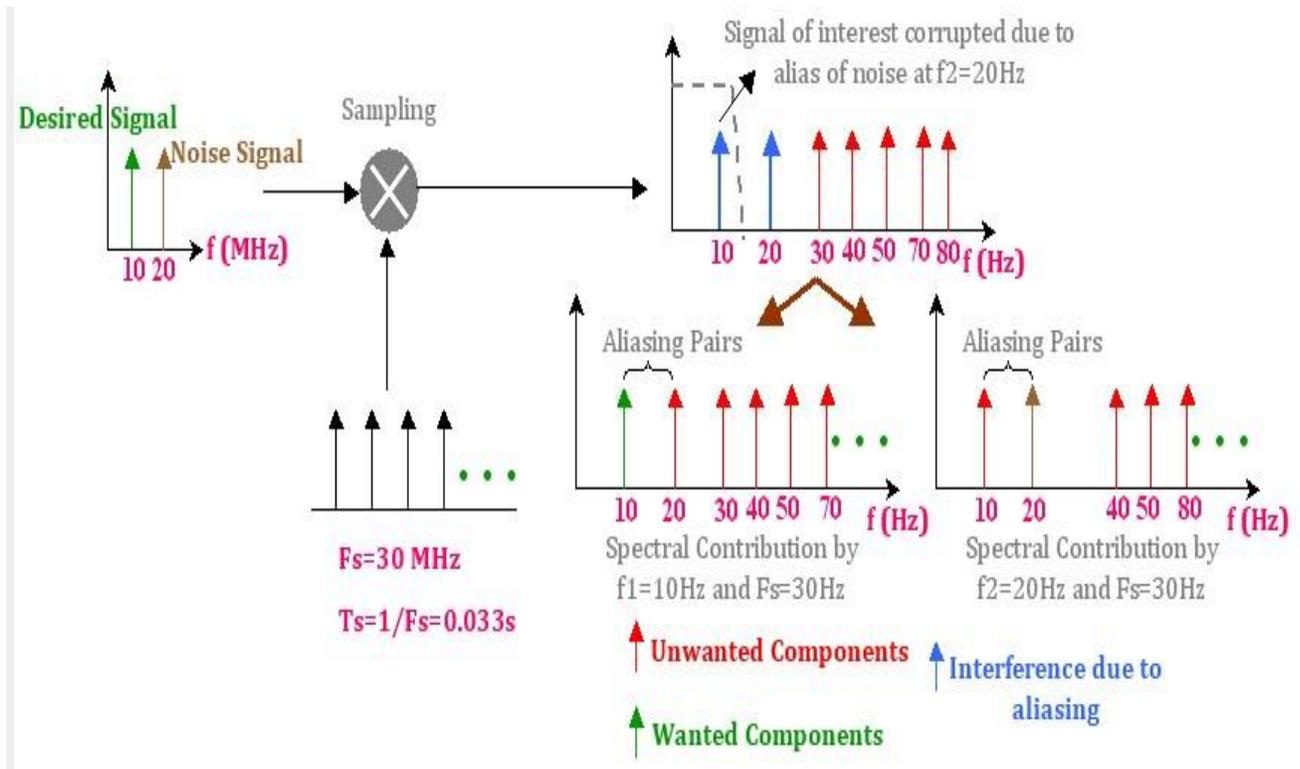


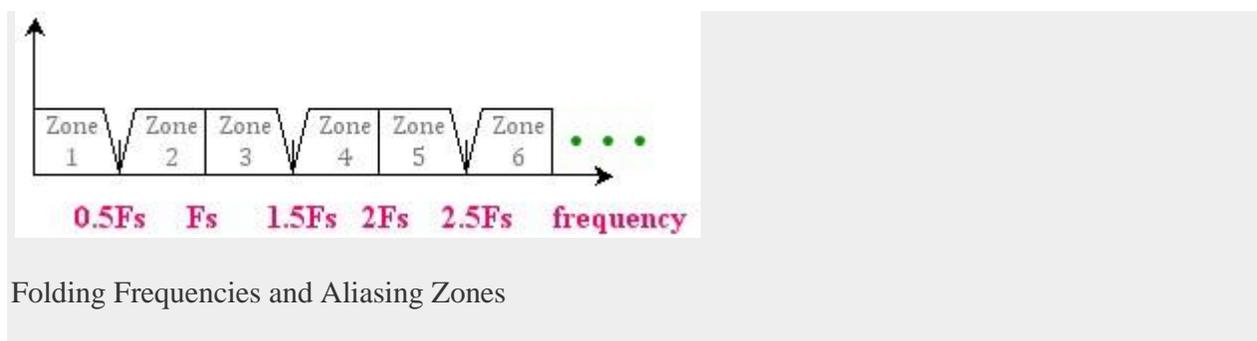
ALIASING AND ANTI-ALIASING

Consider a signal with two frequency components $f_1=10\text{Hz}$ – which is our desired signal and $f_2=20\text{Hz}$ – which is a noise. Lets say we sample the signal at 30Hz . The first frequency component $f_1=10\text{Hz}$ will generate following frequency components at the output of the multiplier (sampler) – $10\text{Hz}, 20\text{Hz}, 40\text{Hz}, 50\text{Hz}, 70\text{Hz}$ and so on. The second frequency component $f_2=20\text{Hz}$ will generate the following frequency components at the output of the multiplier – $20\text{Hz}, 10\text{Hz}, 50\text{Hz}, 40\text{Hz}, 80\text{Hz}$ and so on...

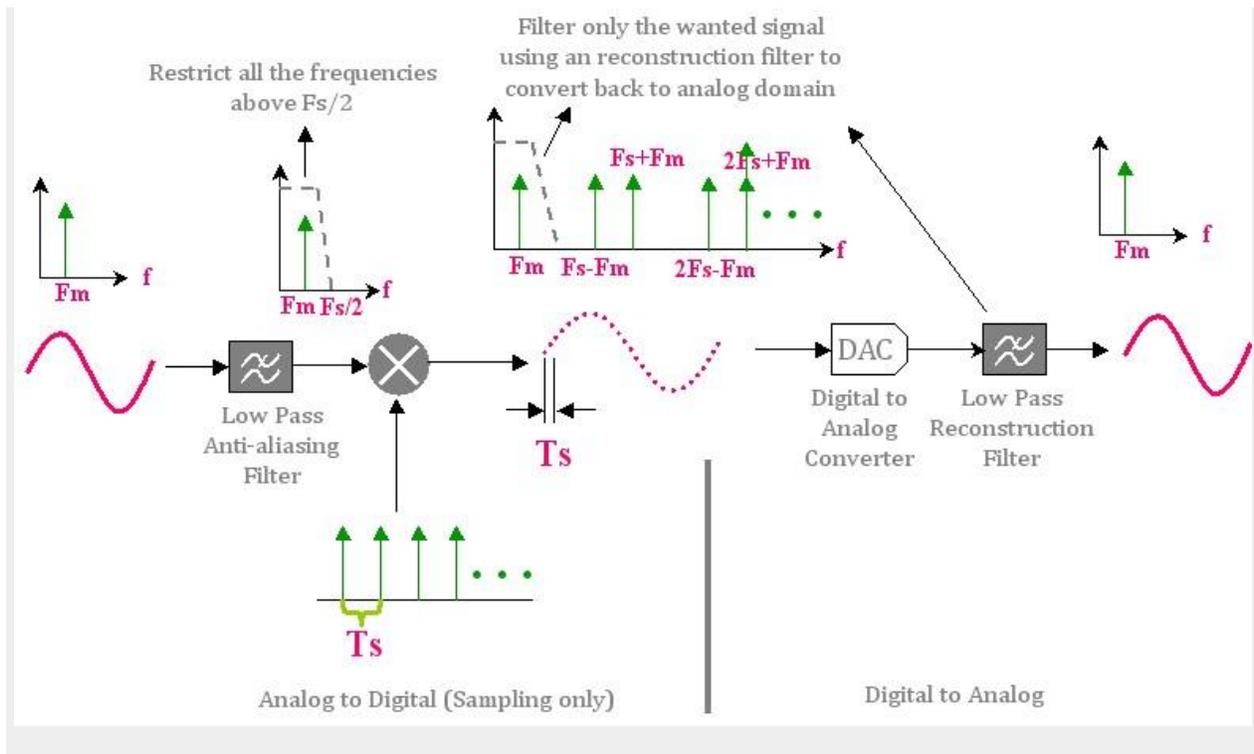


Aliasing and Anti-aliasing

Note the 10Hz component that is generated by $f_2=20\text{Hz}$. This 10Hz component (which is a manifestation of noisy component $f_2=20\text{Hz}$) will interfere with our original $f_1=10\text{Hz}$ component and are indistinguishable. This 10Hz component is called “alias” of the original component $f_2=20\text{Hz}$ (noise). Similarly the 20Hz component generated by $f_1=10\text{Hz}$ component is an “alias” of $f_1=10\text{Hz}$ component. This 20Hz alias of $f_1=10\text{Hz}$ will interfere with our original component $f_2=20\text{Hz}$ and are indistinguishable. We do not need to care about the interference that occurs at 20Hz since it is a noise and any way it has to be eliminated. But we need to do something about the aliasing component generated by the $f_2=20\text{Hz}$. Since this is a noise component, the aliasing component generated by this noise will interfere with our original $f_1=10\text{Hz}$ component and will corrupt it. Aliasing depends on the sampling frequency and its relationship with the frequency components. If we sample a signal at F_s , all the frequency components from $F_s/2$ to F_s will be alias of frequency components from 0 to $F_s/2$ and vice versa. This frequency – $F_s/2$ is called “Folding frequency” since the frequency components from $F_s/2$ to F_s folds back itself and interferes with the components from 0Hz to $F_s/2$ Hz and vice versa. Actually the aliasing zones occur on the either sides of $0.5F_s$, $1.5F_s$, $2.5F_s$, $3.5F_s$ etc... All these frequencies are also called “Folding Frequencies” that causes frequency reversal. Similarly aliasing also occurs on either side of F_s , $2F_s$, $3F_s$, $4F_s$... without frequency reversals. The following figure illustrates the concept of aliasing zones.

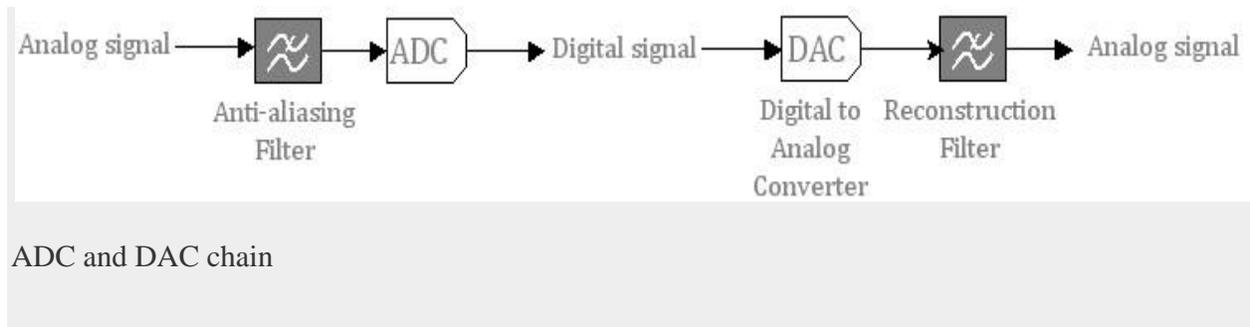


In the above figure, zone 2 is just a mirror image of zone 1 with frequency reversal. Similarly zone 2 will create aliases in zone 3 (without frequency reversal), zone 3 creates mirror image in zone 4 with frequency reversal and so on... In the example above, the folding frequency was at $F_s/2=15\text{Hz}$, so all the components from 15Hz to 30Hz will be the alias of the components from 0Hz to 15Hz. Once the aliasing components enter our band of interest, it is impossible to distinguish between original components and aliased components and as a result, the original content of the signal will be lost. In order to prevent aliasing, it is necessary to remove those frequencies that are above $F_s/2$ before sampling the signal. This is achieved by using an “anti-aliasing” filter that precedes the analog to digital converter. An anti-aliasing filter is designed to restrict all the frequencies above the folding frequency $F_s/2$ and therefore avoids aliasing that may occur at the output of the multiplier otherwise.



A complete design of ADC and DAC

Thus, a complete design of analog to digital conversion contains an anti-aliasing filter preceding the ADC and the complete design of digital to analog conversion contains a reconstruction filter succeeding the DAC.



Note: Remember that both the anti-aliasing and reconstruction filters are analog filters since they operate on analog signal. So it is imperative that the sampling rate has to be chosen carefully to relax the requirements for the anti-aliasing and reconstruction filters.

Source: <http://www.gaussianwaves.com/2011/07/sampling-theorem-baseband-sampling/>