Uncertainty in measurements of power and energy on power networks

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Abstract. The problem on measurement of power and energy in single-phase and three-phase networks is always very important. The accuracy and precision of measurement is of equal importance for both manufacturers and consumers. The general problem is to determine the uncertainty of the energy produced in a specified power station and delivered to different consumers. Measuring instruments used for energy measurement are voltage and current instrument transformers and electricity meters. The approach proposed combines the information that can be derived from the nominal accuracy specifications of the instrument transformers with the information resulting from the calibration of the electricity meters. The combination allows achieving the evaluation of uncertainty in power and energy measurement. Both instrument transformers are subject to ratio error and phase error. In the paper an expression for maximum (positive and negative) errors as a function of the relative load current is derived. A method for calibration of electricity meters, measurement instruments used in the procedure, processing and presentation of the final results are presented as well. Having the information about the accuracy of both voltage and current instrument transformers and finding the uncertainty of electricity meter (according to the method outlined), it is possible to determine the total uncertainty of energy (power) measurement in conditions of load current variations and different modes of operation. The final accuracy of energy measurement may be presented by giving the specification limits in terms of measured value.

Key words

measurement, power, network, uncertainty, procedure.

1. Introduction

The problem on measurement of power and energy in mono-phase and three-phase networks is always very important. The accuracy and precision of measurement is of equal importance for both manufacturers and consumers, taking into account world globalization and increasing market demand and exchange of electrical energy. It should be mentioned that considering the great number of publications in the field of power measurement it is difficult to realize significant contribution to the above problem. On the other hand every scientific research leading to the improvement of metrology characteristics and features of power and energy measurement systems is valuable and. with practical application. Currently the large number of the power and energy measuring devices are sophisticated computer-based systems using digital conversion of input current and voltage and having various features for measurement, calculation and visualization of different parameters.

2. Statement of the problem

In the paper the general problem is to determine the uncertainty of the energy produced in a specified power station and delivered to different consumers. This problem is very important now for Bulgaria because the energy home trade within country has been recently introduced. The electrical energy area includes the generation of energy, the distribution of energy and finally the utilization of energy. It is clear that the generated energy should be measured before distribution for each one of the power station outputs. Measuring instruments used for energy measurement usually are:

- Voltage and current instrument transformers;
- Electricity meters

The above mentioned instruments should be with compatible characteristics.

3. Uncertainty Analysis

A. Voltage and current instrument transformers

It is well known that both instrument transformers are subject to ratio error and phase error. These errors are set by National and International Standards and are outlined into the transformer documentation. Both ratio errors and phase errors are very important when voltage and current instrument transformers are used to extend the range of watt meters and electricity meters (watt-hour meters).

The voltage ratio error (f_u) of the voltage instrument transformer is a result of the difference between nominal and real transformation coefficient. Phase error (δ_u) is defined by the phase angle difference between the vectors of the primary and secondary voltage. The difference is zero when transformer is ideal. Both ratio and phase errors depend on:

- Burden values;
- Power factor;
- Range of the input voltage deviation;

The current ratio error (f_i) of the current instrument transformer is a result of the difference between nominal

and real transformation coefficient. Phase error (δ_i) is defined by the phase angle difference between the vectors of the primary and secondary current. The difference is zero when transformer is ideal. Both ratio and phase errors depend on:

- Burden values;
- Power factor;
- Range of the primary current variation;

Maximum positive error f_{p+} for power measurement using both voltage and current instrument transformers is defined by the formula:

$$f_{p+} = \frac{(1+f_u)(1+f_i)\cos(\phi-\delta_u-\delta_i)}{\cos\phi}-1 \qquad (1)$$

Maximum negative error fp- for the above mentioned case is:

$$f_{p-} = \frac{(1-f_u)(1-f_i)\cos(\phi+\delta_u+\delta_i)}{\cos\phi}-1 \qquad (2)$$

Where: f_u & f_i are ratio errors of the voltage and current instrument transformers;

δ_u & δ_i are phase errors of the voltage and current instrument transformers;

$\cos\phi$ - power factor of the burden;

All errors are taken with a positive sign.

Both expressions define an area where the real error due to instrument transformers in power measurement is situated.

The graphical presentation of this area as a function of the relative load current is shown in Fig.1. For practical applications when the information about transformers is available (graphics for both ratio and phase error) these errors can be determined and considered as systematic.
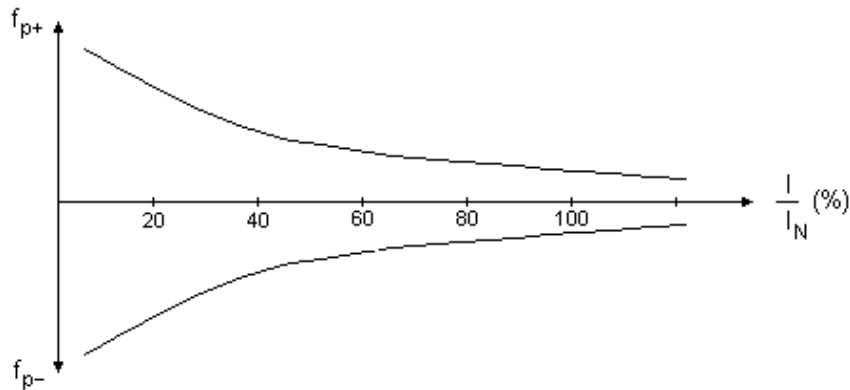

Fig.1. Error presentation

B. Calibration of single-phase and three-phase electricity meters

The following method defines the principle of calibration, measurement instruments used in the procedure, processing and presentation of the final results.

1) The principle of calibration

The calibration is based on direct comparison between measured energy values for both standard and electricity meter.

The relative error ε is given by the formula:

$$\varepsilon = \frac{N_c - N_r}{N_r}100 \;,[\%] \qquad (3)$$

where: $N_c = T.C_{pz}$

T- base time (time of measurement), in sec.

C_{pz}- frequency of the standard for 100 % P (Q), in p/s

(P – active power; Q – reactive power);

N_r – real value of the pulses proportional to the energy;

2) Measuring instrument (standard)

The standard used in calibration is a 3-Phase Precision Measuring Instrument TPZ 303. The following parameters are measured, calculated and displayed to provide a comprehensive analysis of a three phase system showing instantaneous and integrated values:

- True RMS values for each voltage and current input;
- Analysis of DC component and harmonic content;
- Phase angles between all voltages and currents;
- Active, reactive and apparent power;
- Power factor for each phase;
- Total active, reactive and apparent power;
- Phase rotation sequence and frequency.

It is possible to input external current and voltage transformers ratios, measured parameters will then be shown as primary values.

3) Processing of the results in calibration

The matematical model describing the relation between output value Δ (percentage error) and input values Δi is:

$$\Delta = \Delta_1 - \Delta_2 + \Delta_{pc} \qquad (4)$$

The estimate of the measured value Δ, denoted by δ results from equation (4) after substituting the estimates of all input values:

$$\delta = \delta_1 - \delta_2 + \delta_{pc}, \qquad (5)$$

where: δ – estimate of the electricity meter error;
δ_1 - estimate of the measured electricity meter error;
δ_2 - estimate of the standard error, presented in the calibration certificate;
δ_{pc} – correction for the resolution of the standard display;

All values are in percents.

The estimate of measured electricity meter error δ_1 is obtained by calculating the arithmetic mean of a set of n measurement results:

$$\delta_1 = \frac{1}{n}\sum_{\lambda=1}^{n} \varepsilon_\lambda, \qquad (6)$$

where : ε_λ – value of the error for λ result in the calibration point ,
n – number of the measurement results in the calibration point .

 Standard uncertainty $u(\delta_1)$ is determined by the standard deviation of the estimated error value according to the expression:

$$u(\delta_1) = \sqrt{\frac{1}{n(n-1)}\sum_{\lambda=1}^{n}(\varepsilon_\lambda - \delta_1)^2}. \qquad (7)$$

Standard uncertainty of the standard is given in its calibration certificate. If the expanded uncertainty is U for a coverage probability P = 95 % it can be written:

$$u(\delta_2) = \frac{U}{2}. \qquad (8)$$

The correction δ_{pc} due to the finite resolution of the standard display is with rectangular probability distribution. The estimate is zero and margins δ'_{pc}, being a half of the least significant digit displayd by the standard.. Based on the above information the standard uncertainty is:

$$u(\delta_{pc}) = \frac{\delta_{pc}'}{\sqrt{3}}. \qquad (9)$$

Uncertainty contributions $u_i(\delta)$ related to the output estimate δ, are calculated by the following expression:

$$u_i(\delta) = c_i.u(\delta_i), \qquad (10)$$

where c_i are sensitivity coefficients related to input estimate δ_i, i.e. fractional derivatives   of the model function with a respect to Δ_i and estimates δ_i of the

input values: $c_1 = \dfrac{\partial\delta}{\partial\delta_1} = \dfrac{\partial\Delta}{\partial\Delta_1} = 1;$

$$c_2 = \frac{\partial\delta}{\partial\delta_2} = \frac{\partial\Delta}{\partial\Delta_2} = -1; \quad c_3 = \frac{\partial\delta}{\partial\delta_{pc}} = \frac{\partial\Delta}{\partial\Delta_{pc}} = 1$$

and $u(\delta_i)$ are standard uncertainty of the input values.
The standard uncertainty associated with output estimate is:

$$u(\delta) = \sqrt{\sum_{i=1}^{3} u_i^{\,2}(\delta)}, \qquad (11)$$

where: $u_1(\delta) = u(\delta_1)$; $u_2(\delta) = -u(\delta_2)$;
$u_3(\delta) = u(\delta_{pc})$.

The expanded measurement uncertainty U is found as a product of standard uncertainty $u(\delta)$ of the output estimate and coverage factor k :

$$U = k.u(\delta). \qquad (12)$$

For a normal distribution of measured quantity and a coverage probability   95 %,
k may be assumed $= 2$ and formula (12) is modified:

$$U = 2.u(\delta). \qquad (13)$$

4) Presentation of calibrations results

The first part of the presented results includs information about the calibrated electricity meter and conditions of calibration-TABLE1. The second part is Uncertainty budget presented in the TABLE 2.

TABLE I. – Measurement results for different load currents (calibration points).

Current value	Power factor	Error value, %	Aritmetic mean of the error, %	Uncertainty, %
10 % $I_н$	1	$\varepsilon_1...\varepsilon_\lambda$	δ_1	$u(\delta_1)$
	0,5 lagging	$\varepsilon_1...\varepsilon_\lambda$	δ_1	$u(\delta_1)$
20 % $I_н$	1	$\varepsilon_1...\varepsilon_\lambda$	δ_1	$u(\delta_1)$
	0,5 lagging	$\varepsilon_1...\varepsilon_\lambda$	δ_1	$u(\delta_1)$
50 % $I_н$	1	$\varepsilon_1...\varepsilon_\lambda$	δ_1	$u(\delta_1)$
	0,5 lagging	$\varepsilon_1...\varepsilon_\lambda$	δ_1	$u(\delta_1)$
100 % $I_н$	1	$\varepsilon_1...\varepsilon_\lambda$	δ_1	$u(\delta_1)$
	0,5 lagging	$\varepsilon_1...\varepsilon_\lambda$	δ_1	$u(\delta_1)$

TABLE II. – Uncertainty budget

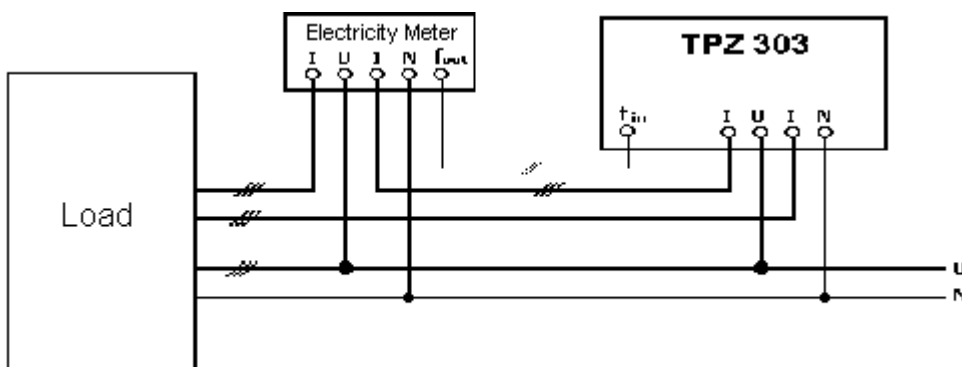| Quantity Δ_I	Estimate δ_i,%	Probability Distribution	Standard uncertainty $u(\delta_i)$, %	Sensitivity Coefficient c_I	Uncertainty Contribution $u_i(\delta)=c_i.u(\delta_i)$ %
Error of the measured Energy	δ_1	Normal	$u(\delta_1)$	1	$u_1(\delta)$
Error of the Standard	δ_2	Normal	$u(\delta_2)$	-1	$-u_2(\delta)$
Correction due to the display resolution	0	Rectangular	$u(\delta_3)$	1	$u_3(\delta)$
Δ	Δ	Normal			$U(\delta)$

4. Electrical circuits
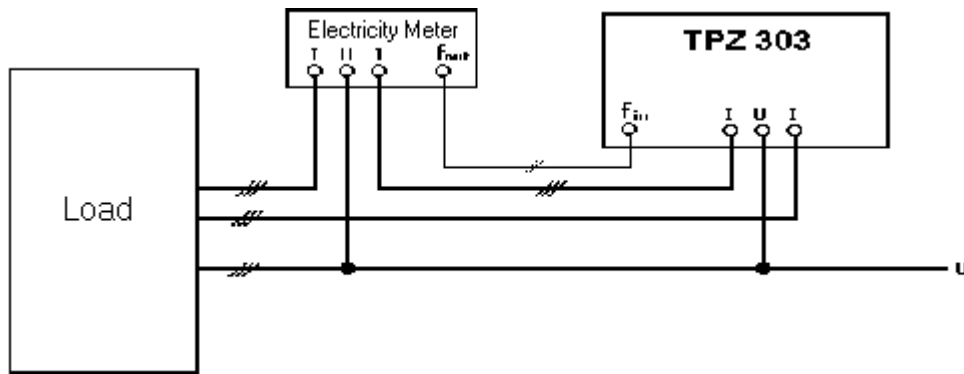

Fig. 2 – Four-Wire Energy Measurement
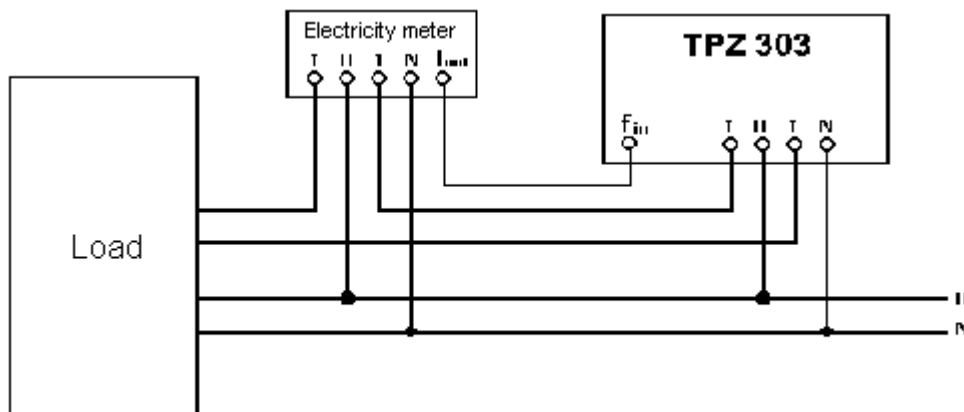
Fig.3. – Three-Wire Energy Measurement

Fig. 4. – Two-Wire Energy Measurement

5. Conclusion

Having the information about the accuracy of both voltage and current instrument transformers and finding the uncertainty of electricity meter (according to the method just outlined), it is possible to determine the total uncertainty of energy (power) measurement in conditions of load current variations and different modes of operation. The final accuracy of energy measurement may be presented by giving the specification limits in terms of measured value.

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