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Stability of LTI Systems with Unstructured Uncertainty Using Quadratic Disc Criterion

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Abstract – This paper deals with robust stability of linear time-invariant (LTI) systems with unstructured uncertainties. A new relation between uncertainties and system poles perturbed by the uncertainties is derived from a graphical analysis. A stability criterion for LTI systems with uncertainties is proposed based on this result. The migration range of the poles in the proposed criterion is represented as the bound of uncertainties, the condition number of a system matrix, and the disc containing the poles of a given nominal system. Unlike the existing methods depending on the solutions of algebraic matrix equations, the proposed criterion provides a simpler way which does not involves algebraic matrix equations, and a more flexible root clustering approach by means of adjusting the center and the radius of the disc as well as the condition number.

Keywords: Root clustering, Unstructured uncertainty, Condition number, Quadratic disc criterion

1. Introduction

In the case that real systems are approximated to linear time-invariant (LTI) systems, it is impossible to describe exactly the behavior of the real systems because of uncertainties caused by the inaccuracy of modeling, the variation of coefficients, and the external disturbance, etc. These uncertainties may arouse unexpected behaviors and even instability in the control systems. Thus, it is a typical method to represent real systems as LTI systems with uncertainties and design a robust controller against uncertainties.

The pole placement is one of the most important methods to guarantee the robust stability of uncertain systems because, roughly speaking, the robustness against uncertainties is proportional to the distance between the dominant pole and the imaginary axis in the left complex half plane. There are many approaches based on the locations of system poles and they are divided into two categories according to the kinds of uncertainties so called structured and unstructured uncertainties. The uncertainties in a parametric description of a system are structured. Gershigorin circle theorem and Kharitonov theorem are well known methods to decide the stability of LTI systems with such uncertainties. Gershigorin approach provides the approximated bounds of system poles in the form of circles whose centers are the diagonal terms of a given system matrix and radii are the absolute summation of off-diagonal terms of the corresponding row [15]. Kharitanov approach provides several characteristic equations whose coefficients are the minimum and the maximum values of parametric variations and the stability is guaranteed only if all characteristic equations are stable [5, 13]. But, these methods need the prior knowledge of the variation range in parameters. Moreover, the structured uncertainties can not describe the uncertainty which is not related to the parametric variation such as an error caused by the approximation of high order systems [4]. On the other hand, the uncertainties affecting a system as a whole are unstructured. This kind of uncertainty can be treated as an additional system disturbing a nominal system. In practice, it is quite common that the uncertainties are given as a disturbance system whose elements in a disturbance matrix are not known exactly but only the bounds of the matrix are known [3]. Under this situation, many researches have been devoted to assigning the poles perturbed by the disturbance system into a desired region on the complex plane, which are known as D-stability or root clustering [1, 7, 11, 12]. These methods depend on the solutions of algebraic matrix equations such as Riccati equation and Lyapunov equation. However, several simplifying assumptions are required or numerical approaches such as linear matrix inequalities (LMI) are used to obtain the solution because sometimes it is not easy to solve the given algebraic matrix equations explicitly.

In this paper, we employ the condition number of a system matrix to avoid algebraic matrix equations. The condition number which is a criterion for the reliability of solutions of matrix equations can be used as a measure for the approximation of the migration range of system poles perturbed by unstructured uncertainties [9]. When the condition number of the system matrix is considerably

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large, the required location of the poles of the nominal system tends to be much biased toward the left complex half plane because of the overestimation of the migration range. In this case, to assign system poles perturbed by uncertainties into the desired region, one can not help using high feedback gains, which makes the system more sensitive to disturbance or noise and may cause the saturation in a control input [14]. This is the reason why the minimization of the condition number is an important issue. However, the explicit relation between the condition number and the location of the poles is still an open problem [2, 8]. We derive a new relation between unstructured uncertainties and the poles perturbed by the uncertainties. This relation named quadratic disc criterion (QDC) is described by the disc containing the poles of the nominal system as well as the bound of uncertainties and the condition number of the system matrix. Therefore, the criterion gives a chance of the more flexible root clustering by means of adjusting the center and the radius of the disc unlike the existing methods focusing on the minimization of the condition number only. Finally, numerical examples show that the proposed QDC can achieve the robust stability of LTI system with an arbitrary disturbance system whose norm bound is restricted.

2. Preliminary

Consider an LTI system with additive uncertainties as follows:

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u, \tag{1}$$

where $x \in \mathbf{R}^{n\times 1}$ is a state variable, $A \in \mathbf{R}^{n\times n}$ and $B \in \mathbf{R}^{n\times 1}$ are system and input matrices, respectively, $\Delta A \in \mathbf{R}^{n\times n}$ and $\Delta B \in \mathbf{R}^{n\times 1}$ are system and input uncertainties, respectively, and $u \in \mathbf{R}$ is a control input. Assume that the system (A, B) is controllable. The control input $u = Fx$ by a state feedback gain $F \in \mathbf{R}^{1\times n}$ allows (1) to be rewritten as

$$\begin{aligned}\dot{x} &= (A + BF + \Delta A + \Delta BF)x \\ &= (G + \Delta G)x,\end{aligned}$$

where $G = A + BF$ is a nominal system and $\Delta G = \Delta A + \Delta BF$ is an additional disturbance system. We assume that the norm bound of the disturbance system is known because it is quite common that the uncertainties are given as a disturbance system whose elements in a disturbance matrix are not known exactly but only the bounds of the matrix are known as follows:

$$\|\Delta G\|_2 \le \delta \tag{2}$$

In case the nominal system $\dot{x} = Gx$ is stable, the whole system has some robust stability against uncertainties, which is proportional to the distance between the dominant pole of the nominal system and the imaginary axis on the left complex half plane. At first sight, the system is likely to be stable against uncertainties given in (2) if the poles of the nominal system are laid on the left of the size of uncertainties δ from the imaginary axis. But, the migration range of the poles perturbed by uncertainties is not determined by the size of uncertainties only. In fact, the range depends not only on the size of uncertainties but also on the condition number of a matrix related to the nominal system as follows [10]:

$$\|\Delta G\|_2 \inf k(Q_G), \tag{3}$$

where $\inf(\cdot)$ denotes the infimum, $k(\cdot)$ is the condition number of the corresponding matrix, and Q_G is a matrix consisting of the eigenvectors of the system matrix G. This result means that the poles of the uncertain system wander around the poles of the nominal system within the radius given in (3). The radius depends on the condition number more than the size of uncertainties because the condition number is much larger than 1 in general, which is the reason why the condition number is a critical factor in the robustness problem. And the migration range of the poles can be treated as an optimization problem through minimizing the condition number because the range is provided in the form of the infimum of the condition number. However, the relation between the condition number and the migration range of the system poles is not given explicitly, which makes researchers to depend on numerical methods in order to obtain a well-conditioned solution [11].

3. Quadratic Disc Criterion

In this paper, we derive a new relation between unstructured uncertainties and the system poles perturbed by the uncertainties. This relation is represented as the disc containing the poles of the nominal system, the bound of uncertainties, and the condition number of the system matrix. Unlike the existing methods focusing on the minimization of the condition number, we propose a flexible method by means of adjusting the center and the radius of the disc as well as the condition number of the system.

When given matrices are simple, that is, the eigenvalues of a matrix are distinct, the regions including the eigenvalues of the sum of these matrices are well known as the following theorem.

Lemma 1 [10]. Let $G, H \in \mathbf{R}^{n\times n}$. If G has the distinct eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ and ξ is an eigenvalue of $G + H$, then ξ lies in at least one of the discs

$$\left\{s : |s-\lambda_i| \le \|H\|_2 \inf k(Q_G)\right\},\ i = 1, 2, \cdots, n \qquad (4)$$

of the complex s-plane, where $\inf(\cdot)$ denotes the infimum, $k(\cdot)$ is the condition number of the matrix, and Q_G is any matrix which diagonalizes G into $Q_G \Lambda_G Q_G^{-1}$. ■

This result is mentioned already in (3), which means the migration ranges of the system poles depend on the size of uncertainties and the condition number of the matrix which diagonalizes the nominal system when the nominal system G is perturbed by the disturbance system H. In case the condition number has a large value, an unrealistic feedback gain is required for the migration range in (4) not to violate the right complex half plane. The condition number required in (4) is its infimum, but it is not easy to find the infimum because the condition number changes according to the location of poles.

If the matrix H is not treated as a disturbance system but a special case of the nominal system, a new relation between uncertainties and the poles perturbed by the uncertainties can be derived. Consider the case when the eigenvalues of H is known.

Lemma 2 [10]. If, in addition to the hypothesis of Lemma 1, H has also the distinct eigenvalues $\mu_1, \mu_2, \cdots, \mu_n$, then ξ lies in at least one of the discs

$$\left\{s : |s-\lambda_i| \le \inf k\left(Q_G^{-1} Q_H\right) \max_{1\le j\le n} |\mu_j|\right\},\ i = 1, 2, \cdots, n,$$

where Q_H diagonalizes H into $Q_H \Lambda_H Q_H^{-1}$. ■

If H is the transpose matrix of G, the above result can be easily modified just by replacing Q_H and μ with Q_G and λ, respectively. In addition, when the eigenvalues are shifted as a constant value, we can obtain the following result.

Corollary 1 [16]. If, in addition to the hypothesis of Lemma 1 and Lemma 2, H is the transpose matrix of G, then ξ lies in at least one of the discs

$$\left\{s : \left|s-(\lambda_i-\alpha)\right| \le \inf k\left(Q_G Q_G'\right) \max_{1\le j\le n} |\lambda_j+\alpha|\right\},\ i = 1, 2, \cdots, n$$

for any constant α.

Proof. Because ξ is an eigenvalue of $G+G'$, there is a non-zero vector y such that

$$\left(G+G'+\alpha I-\alpha I\right) y = \xi y$$

for any constant α. Replacing G with $Q_G \Lambda_G Q_G^{-1}$, the equation can be rewritten as

$$\left(Q_G \Lambda_G Q_G^{-1} + \left(Q_G^{-1}\right)' \Lambda_G' Q_G' + \alpha I - \alpha I\right) y = \xi y .$$

In the left side, by taking Q_G and Q_G^{-1} out of the parenthesis,

$$Q_G\left(\Lambda_G + Q_G^{-1}\left(Q_G^{-1}\right)' \Lambda_G' Q_G' Q_G + \alpha I - \alpha I\right) Q_G^{-1} y = \xi y .$$

Setting $Q_G^{-1} y = z$ yields

$$Q_G\left(\Lambda_G + Q_G^{-1}\left(Q_G^{-1}\right)' \Lambda_G' Q_G' Q_G + \alpha I - \alpha I\right) z = \xi Q_G z .$$

Eliminate Q_G on the both sides, and rearrange the equation

$$\begin{aligned}\left[\xi I-(\Lambda_G-\alpha I)\right] z &= \left[Q_G^{-1}\left(Q_G^{-1}\right)' \Lambda_G Q_G' Q_G + \alpha I\right] z \\ &= \left[Q_G^{-1}\left(Q_G^{-1}\right)' (\Lambda_G+\alpha I) Q_G' Q_G\right] z\end{aligned}.$$

From this relation, we can obtain the inequality

$$\begin{aligned}\min \frac{\left\|\left[\xi I-(\Lambda_G-\alpha I)\right] z\right\|}{\|z\|} &\le \left\|\xi I-(\Lambda_G-\alpha I)\right\| \\ &\le \left\|Q_G^{-1}\left(Q_G^{-1}\right)'\right\| \left\|\Lambda_G+\alpha I\right\| \left\|Q_G' Q_G\right\| \\ &= k\left(Q_G' Q_G\right) \max_{1\le j\le n} |\lambda_j+\alpha|\end{aligned}.$$

Thus, $\min\left|\xi-(\lambda_i-\alpha)\right| \le k\left(Q_G' Q_G\right) \max|\lambda_j+\alpha|$. Since this is true for any possible Q_G which diagonalizes G, $\min\left|\xi-(\lambda_i-\alpha)\right| \le \inf k\left(Q_G' Q_G\right) \max|\lambda_j+\alpha|$. This result means that the eigenvalues ξ of $G+G'$ lie in the disc whose radius is $\inf k\left(Q_G' Q_G\right) \max|\lambda_j+\alpha|$ and its center $\lambda_i-\alpha$ is selected to minimize the distance from ξ. ■

Lemma 1 and Lemma 2 deal with the disc for each eigenvalue and its migration range. On the other hand, we can extend the discussion to the disc which includes all eigenvalues and their migration ranges by using Corollary 1.

Theorem 1 [16]. If all eigenvalues of G lie in the disc

$$\left\{s : |s-c| \le r\right\},$$

then all eigenvalues of $G+G'$ lie in the disc

$$\left\{s : |s-2c| \le r\left[1+\inf k\left(Q_G Q_G'\right)\right]\right\},$$

where $c, r \in \mathbf{R}$.

Proof. Move the disc including all eigenvalues of G as $\pm c$ along the real axis, and then one is located at the origin and the other is located at $2c$ with the same radius r. By Corollary 1, the eigenvalues of $G+G'$ lie in at least one of the discs

$$\left\{s : \left|s-(\lambda_i+c)\right| \le \inf k\left(Q_G Q_G'\right) \max_{1\le j\le n}\left|\lambda_j - c\right|\right\},\ i=1,2,\cdots,n.$$

As shown in Fig. 1, all eigenvalues moved to the right lie in the disc at the origin with the radius r, which implies $\max\left|\lambda_j - c\right| = r$. Each eigenvalue moved to the left lies in at least one of the above discs and the whole disc including these individual discs is located at $2c$ with the radius $r[1+\inf k\left(Q_G Q_G'\right)]$. ■

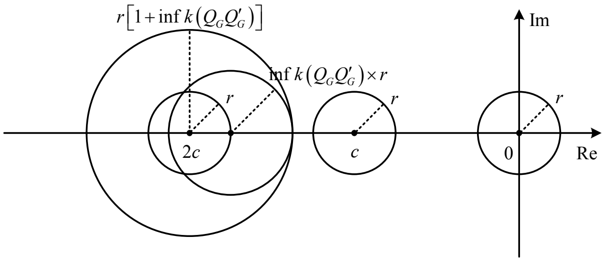

Fig. 1. Disc containing all eigenvalues

Most of approaches for the stability of uncertain systems base on the existence of the solution of an algebraic matrix equation such as Riccati equation and Lyapunov equation. Sometimes, it is not easy to obtain the explicit solution of a given algebraic matrix equation. In this paper, we avoid finding the solution of such equations by using the simplest energy function which plays the role of the solution of Lyapunov equation. The stability of LTI system with norm-bounded uncertainties can be shown by using the result of Theorem 1 as follows:

Theorem 2. If, in addition to the hypothesis of Theorem 1, the disturbance is bounded as $\|\Delta G\|_2 \le \delta$ and $2c + r[1+\inf k\left(Q_G Q_G'\right)] + 2\delta < 0$, then the uncertain system $\dot{x}(t) = (G+\Delta G)x(t)$ is stable.

Proof. Set a Lyapunov function for the uncertain system $\dot{x}(t) = (G+\Delta G)x(t)$ as

$$V = x'x.$$

The time derivative of the function is

$$\dot{V} = x'\left(G+G'+\Delta G+\Delta G'\right)x.$$

Let $G+G'=X$ and $\Delta G+\Delta G'=Y$. By Wely inequality (See Appendix 1), the eigenvalues of the sum of symmetric matrices satisfy

$$\lambda_{i+j-1}(X+Y) \le \lambda_i(X) + \lambda_j(Y),\ \ i+j \le n+1.$$

In the case of the largest eigenvalue, i.e. $i=j=1$,

$$\lambda_{\max}\left(X+Y\right) \le \lambda_{\max}\left(X\right) + \lambda_{\max}\left(Y\right),$$

which means

$$\begin{aligned} &\lambda_{\max}\left(G+G'+\Delta G+\Delta G'\right) \\ &\le \lambda_{\max}\left(G+G'\right) + \lambda_{\max}\left(\Delta G+\Delta G'\right) \end{aligned}. \tag{5}$$

By Theorem 1,

$$\lambda_{\max}\left(G+G'\right) \le 2c + r\left[1+\inf k\left(Q_G Q_G'\right)\right]. \tag{6}$$

And, by Fan-Hoffman inequality (See Appendix 2),

$$\lambda_i\left(\frac{Y+Y'}{2}\right) \le \sigma_i(Y),\ 1 \le i \le n,$$

which can be rewritten as follows in the case of the largest value

$$\lambda_{\max}\left(\Delta G+\Delta G'\right) \le \sigma_{\max}\left(\Delta G+\Delta G'\right). \tag{7}$$

Because Euclidean norm $\|\cdot\|_2$ is defined as the largest singular value,

$$\begin{aligned} \sigma_{\max}\left(\Delta G+\Delta G'\right) &= \left\|\Delta G+\Delta G'\right\|_2 \\ &\le \left\|\Delta G\right\|_2 + \left\|\Delta G'\right\|_2. \\ &= 2\left\|\Delta G\right\|_2 \end{aligned} \tag{8}$$

These inequalities of (5)-(8) mean

$$\lambda_{\max}\left(G+G'+\Delta G+\Delta G'\right) \le 2c + r\left[1+\inf k\left(Q_G Q_G'\right)\right] + 2\delta.$$

By the assumptions, the time derivative of the Lyapunov function is less than 0. ■

The migration range of all poles perturbed by uncertainties is represented as the size of uncertainties, the infimum of the condition number, and the disc including the eigenvalues of the nominal system. In the ideal case that the condition number is 1, the sufficient condition for the stability of the uncertain system in Theorem 2 becomes $c+r+\delta<0$. This implies all poles of the nominal system lie on the more left side than the size of uncertainties. However, in the actual situation, the condition number is much larger than 1, which makes the migration range depend on the condition number more than the size of uncertainties. This arouses the optimization problem of minimizing the condition number by adjusting the poles of the nominal system. In this paper, we focus the stability of LTI systems with an arbitrary disturbance system whose only norm bound is known by using the disc in Theorem 2 rather than minimizing the condition number. To do this, the approximation of the infimum of the condition number replaces the minimization of that.

Fact 1. The values of $\inf k\left(Q_G Q_G'\right)$ in the assumption of Theorem 2 can be approximated as

$$k(G)\left|\frac{\lambda_{\min}(G)}{\lambda_{\max}(G)}\right|.$$

Proof. The norms of G and G^{-1} can be rewritten as follows using the diagonal matrices and the definition of the condition number.

$$\begin{aligned}\|G\| &= \left\|Q_G \Lambda_G Q_G^{-1}\right\| \le \|Q_G\|\|\Lambda_G\|\left\|Q_G^{-1}\right\| = k(Q_G)\|\Lambda_G\|,\\ \left\|G^{-1}\right\| &= \left\|Q_G \Lambda_G^{-1} Q_G^{-1}\right\| \le \|Q_G\|\left\|\Lambda_G^{-1}\right\|\left\|Q_G^{-1}\right\| = k(Q_G)\left\|\Lambda_G^{-1}\right\|.\end{aligned}$$

The multiplication of these inequalities yields

$$k(G) \le k^2(Q_G)\, k(\Lambda_G). \tag{9}$$

Meanwhile,

$$\begin{aligned}k\left(Q_G Q_G'\right) &= \left\|Q_G Q_G'\right\|\left\|\left(Q_G'\right)^{-1} Q_G^{-1}\right\|\\ &= \|Q_G\|\left\|Q_G'\right\|\left\|\left(Q_G'\right)^{-1}\right\|\left\|Q_G^{-1}\right\|\end{aligned} \tag{10}$$

because the equality in $\|XY\| \le \|X\|\|Y\|$ holds when $Y = X'$. And

$$\begin{aligned}\|Q_G\|\left\|Q_G'\right\|\left\|\left(Q_G'\right)^{-1}\right\|\left\|Q_G^{-1}\right\| &= k(Q_G)\, k\left(Q_G'\right)\\ &= k^2(Q_G)\end{aligned} \tag{11}$$

because the condition numbers of a matrix and its transpose are equivalent to each other. In addition,

$$k(\Lambda_G) = \left|\frac{\lambda_{\max}(\Lambda_G)}{\lambda_{\min}(\Lambda_G)}\right| = \left|\frac{\lambda_{\max}(G)}{\lambda_{\min}(G)}\right| \tag{12}$$

because the condition number of a normal matrix, $\Lambda_G \Lambda_G' = \Lambda_G' \Lambda_G$, can be represented as the ratio of the maximum and the minimum eigenvalue, and the eigenvalues of Λ_G and G are equivalent to each other. These relations of (9)-(12) imply

$$k(G) \le k\left(Q_G Q_G'\right)\left|\frac{\lambda_{\max}(G)}{\lambda_{\min}(G)}\right|.$$

Multiplying the reciprocal of the ratio of eigenvalues on the both sides yields

$$k(G)\left|\frac{\lambda_{\min}(G)}{\lambda_{\max}(G)}\right| \le k\left(Q_G Q_G'\right).$$

Thus, the infimum of $k\left(Q_G Q_G'\right)$ can be approximated as

$$k(G)\left|\frac{\lambda_{\min}(G)}{\lambda_{\max}(G)}\right|. \qquad \blacksquare$$

This result provides an approximation of the infimum of the condition number using the nominal system itself, which is effective because one does not need to check all possible diagonal matrices of the nominal system.

4. Numerical Example

A common actuator in control systems is the DC motor. The electric circuit of the armature and the free body diagram of the rotor are shown in Fig. 2.

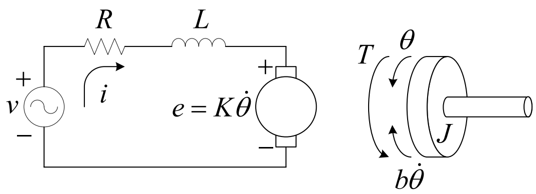

Fig. 2. Diagram of DC motor

The motor torque T is related to the armature current i by a constant factor K. The back electromagnetic force e is related to the rotational velocity $\dot{\theta}$ and the angular position θ of the shaft. The state representation of DC motor is

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot{\theta}\\ i\end{bmatrix} \dot{x} = \begin{bmatrix}0 & 1 & 0\\ 0 & -b/J & K/J\\ 0 & -K/L & -R/L\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ i\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1/L\end{bmatrix} v,$$

where J is the moment of inertia of the rotor, b is the damping ratio of the mechanical system, R and L are the electric resistance and the electric inductance, respectively. Assigning the state $x = [\theta\ \ \dot{\theta}\ \ i]^T$, $J = 3.2$, $b = 3.5$, $K = 2.7$, $R = 4$, $L = 1.5$ and the control input $u = v$ yields

$$\dot{x}=\begin{bmatrix}0 & 1 & 0\\ 0 & -1.0938 & 0.8438\\ 0 & -1.8000 & -2.6667\end{bmatrix}x+\begin{bmatrix}0\\ 0\\ 0.6667\end{bmatrix}u$$

This system can be stabilized by the conventional pole assignment such as $p_G=\{-1,-0.5\pm j0.4\}$. The resulting nominal system is

$$G=\begin{bmatrix}0 & 1 & 0\\ 0 & -1.0938 & 0.8438\\ -0.4859 & -0.4963 & -0.9062\end{bmatrix}. \qquad (13)$$

Suppose there is any uncertainty ΔG which is restricted to 0.5 in Euclidian norm for the system (13). The stability check for this case using Theorem 2 and Fact 1 named QDC yields

$$2c+r\left[1+k(G)\left|\frac{\lambda_{\min}(G)}{\lambda_{\max}(G)}\right|\right]+2\delta=2.3217>0$$

Therefore, the stability of this uncertain system is not guaranteed. The proposed QDC is a sufficient condition, so one can not assert that the uncertain system is unstable just because QDC is not satisfied. But, when an arbitrary uncertainty ΔG restricted under 0.5 in its norm bound is applied to (13), the poles marked by blue crosses migrate as shown in Fig. 3, where red dots are the poles of the nominal system. This result supports the stability of the uncertain system is not guaranteed.

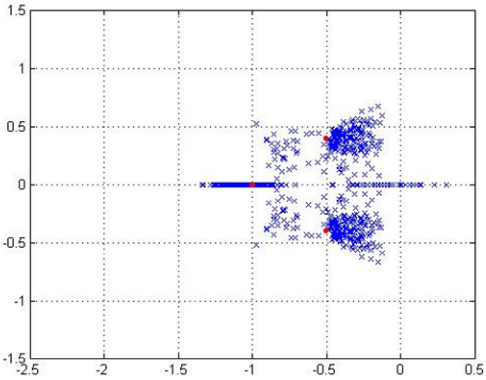

Fig. 3. The location of poles of the uncertain system

To guarantee the stability of the system with the uncertainties, one should check QDC with other poles in the disc with the center c and the radius r. In this case, the disc center should be moved to the left a litter more and the disc radius should be smaller. By some trial and error the poles are assigned at $p_G=\{-1.4,-1.3\pm j0.05\}$. Then the result of QDC for this case is

$$2c+r\left[1+k(G)\left|\frac{\lambda_{\min}(G)}{\lambda_{\max}(G)}\right|\right]+2\delta=-0.7139<0.$$

Therefore, the stability of the system with any uncertainties bounded under 0.5 in Euclidean norm is guaranteed. In actual, under an arbitrary uncertainty ΔG with norm bound as 0.5, the poles marked by blue crosses remains in the left half plane as shown in Fig. 4.

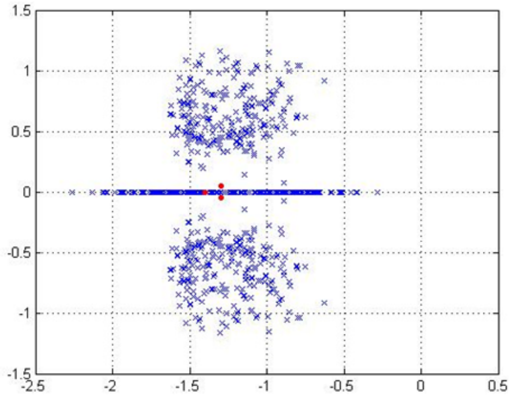

Fig. 4. The location of poles of the uncertain system

5. Conclusions

In this paper, we proposed a method for LTI systems with arbitrary uncertainties to be stabilized by using a new stability criterion named QDC. When LTI systems are disturbed by uncertainties, the migration range of the poles perturbed by uncertainties is proportional to the condition number of the system matrix. Thus, the minimization of the condition number is one of the main streams of the robust stability, which is usually carried out by numerical methods because the explicit relation between the condition number and the pole location is not clear. In case the system has a large condition number, the migration range of the poles sometimes has a tendency to be overestimated and a high feedback gain is required to guarantee the stability of the uncertain system. We have derived a new relation which represented the migration range of the poles of the uncertain system as the disc containing the poles as well as condition number of the nominal system. The proposed criterion provides a more flexible choice in root clustering by adjusting the center and the radius of the disc, and a simpler way which does not require the solution of an algebraic matrix equation.

Appendix 1. Wely inequality [6]

Let $X, Y\in\mathbf{R}^{n\times n}$ be symmetric matrices, then the eigenvalues of the sum of the matrices satisfy

$$\lambda_{i+j-n}(X+Y) \geq \lambda_i(X)+\lambda_j(Y),\ i+j \geq n+1$$
$$\lambda_{i+j-1}(X+Y) \leq \lambda_i(X)+\lambda_j(Y),\ i+j \leq n+1,$$

where $\lambda_1 > \lambda_2 > \cdots > \lambda_n$ are the eigenvalues arranged in order of size, and $i, j = 1, 2, \cdots, n$.

Appendix 2. Fan-Hoffman inequality [6]
For any matrix $Y \in \mathbf{R}^{n\times n}$, the following holds

$$\lambda_i\left(\frac{Y+Y'}{2}\right) \leq \sigma_i(Y),\ 1 \leq i \leq n\ ,$$

where $\lambda_1 > \lambda_2 > \cdots \lambda_n$ and $\sigma_1 > \sigma_2 > \cdots \sigma_n$ denote the eigenvalues and the singular values arranged in order of size, respectively.

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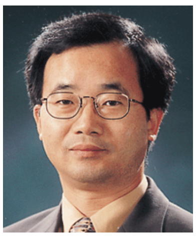

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