Module
2

AC to DC Converters

Lesson
9

Single Phase Uncontrolled Rectifier

Operation and Analysis of single phase uncontrolled rectifiers

Instructional Objectives

On completion the student will be able to

- Classify the rectifiers based on their number of phases and the type of devices used.

- Define and calculate the characteristic parameters of the voltage and current waveforms.

- Analyze the operation of single phase uncontrolled half wave and full wave rectifiers supplying resistive, inductive, capacitive and back emf type loads.

- Calculate the characteristic parameters of the input/output voltage/current waveforms associated with single phase uncontrolled rectifiers.

9.1   Introduction

One of the first and most widely used application of power electronic devices have been in rectification. Rectification refers to the process of converting an ac voltage or current source to dc voltage and current. Rectifiers specially refer to power electronic converters where the electrical power flows from the ac side to the dc side. In many situations the same converter circuit may carry electrical power from the dc side to the ac side where upon they are referred to as inverters. In this lesson and subsequent ones the working principle and analysis of several commonly used rectifier circuits supplying different types of loads (resistive, inductive, capacitive, back emf type) will be presented. Points of interest in the analysis will be.

- Waveforms and characteristic values (average, RMS etc) of the rectified voltage and current.
- Influence of the load type on the rectified voltage and current.
- Harmonic content in the output.
- Voltage and current ratings of the power electronic devices used in the rectifier circuit.
- Reaction of the rectifier circuit upon the ac network, reactive power requirement, power factor, harmonics etc.
- Rectifier control aspects (for controlled rectifiers only)

In the analysis, following simplifying assumptions will be made.

- The internal impedance of the ac source is zero.
- Power electronic devices used in the rectifier are ideal switches.

The first assumption will be relaxed in a latter module. However, unless specified otherwise, the second assumption will remain in force.

Rectifiers are used in a large variety of configurations and a method of classifying them into certain categories (based on common characteristics) will certainly help one to gain significant insight into their operation. Unfortunately, no consensus exists among experts regarding the criteria to be used for such classification. For the purpose of this lesson (and subsequent lessons) the classification shown in Fig 9.1 will be followed.
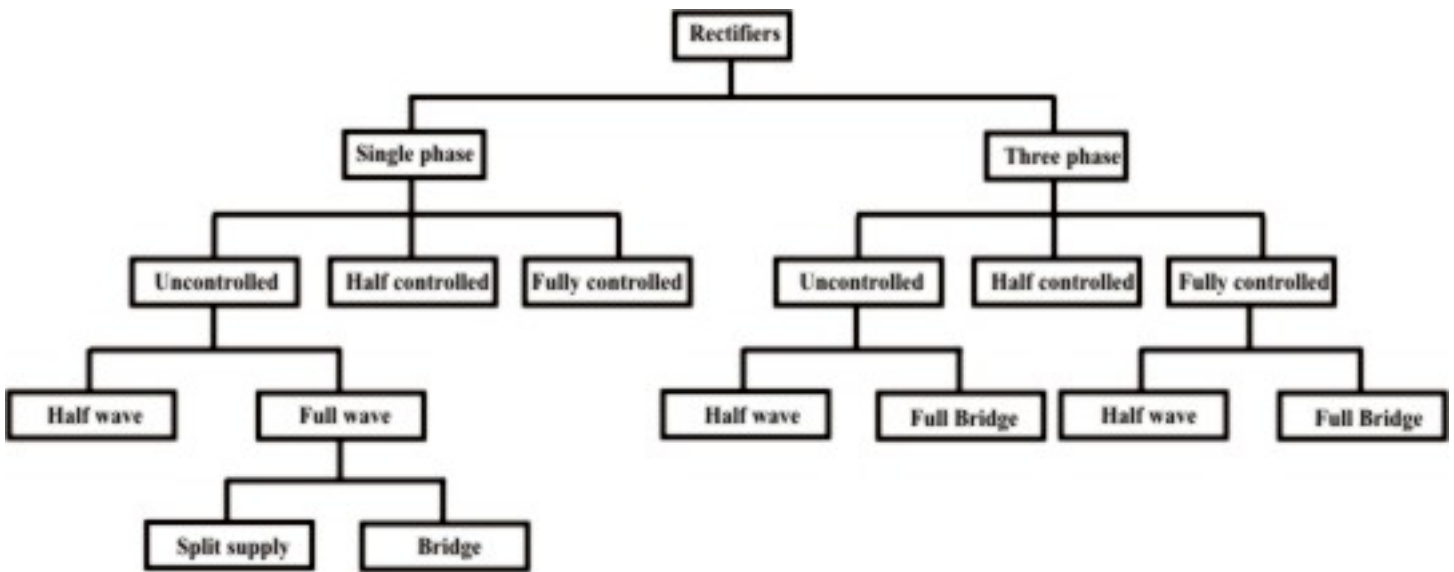
Fig 9.1: Classification of Rectifiers

This Lesson will be concerned with single phase uncontrolled rectifiers.

9.2 Terminologies

Certain terms will be frequently used in this lesson and subsequent lessons while characterizing different types of rectifiers. Such commonly used terms are defined in this section.

Let "f" be the instantaneous value of any voltage or current associated with a rectifier circuit, then the following terms, characterizing the properties of "f", can be defined.

Peak value of f $(\hat{f})$**:** As the name suggests $\hat{f} = |f|_{max}$ over all time.

Average (DC) value of f(F_{av}) : Assuming f to be periodic over the time period T

$$F_{av} = \frac{1}{T} \int_0^T f(t)dt \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.1)$$

RMS (effective) value of f(F_{RMS}) : For f , periodic over the time period T,

$$F_{RMS} = \sqrt{\frac{1}{T} \int_0^T f^2(t)dt} \dots\dots\dots\dots\dots\dots\dots\dots\dots(9.2)$$

Form factor of f(f_{FF}) : Form factor of 'f ' is defined as

$$f_{FF} = \frac{F_{RMS}}{F_{av}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \dots(9.3)$$

Ripple factor of f(f_{RF}) : Ripple factor of f is defined as

$$f_{RF} = \frac{\sqrt{F_{RMS}^2 - F_{av}^2}}{F_{av}} = \sqrt{f_{FF}^2 - 1} \dots\dots\dots\dots\dots\dots\dots(9.4)$$

Ripple factor can be used as a measure of the deviation of the output voltage and current of a rectifier from ideal dc.

Peak to peak ripple of $f\left(\hat{f}_{pp}\right)$: By definition

$$\hat{f}_{pp} = f_{max} - f_{min} \text{ Over period T.} \dots\dots\dots\dots \dots (9.5)$$

Fundamental component of f(F₁): It is the RMS value of the sinusoidal component in the Fourier series expression of f with frequency 1/T.

$$\therefore \quad F_1 = \sqrt{\frac{1}{2}\left(f_{A1}^2 + f_{B1}^2\right)} \dots\dots\dots\dots\dots\dots\dots\dots (9.6)$$

$$\text{where} \quad f_{A1} = \frac{2}{T}\int_0^T f(t)\cos 2\pi \, {}^{t}\!/_{T} \, dt \dots\dots\dots\dots\dots (9.7)$$

$$f_{B1} = \frac{2}{T}\int_0^T f(t)\sin \frac{2\pi t}{T} \, dt \dots\dots\dots\dots\dots (9.8)$$

Kᵗʰ harmonic component of f(F_K): It is the RMS value of the sinusoidal component in the Fourier series expression of f with frequency K/T.

$$\therefore \quad F_K = \sqrt{\frac{1}{2}\left(f_{AK}^2 + f_{BK}^2\right)} \dots\dots\dots\dots\dots\dots\dots (9.9)$$

$$\text{where} \quad f_{AK} = \frac{2}{T}\int_0^T f(t)\ \cos 2\pi K\, t/T\ dt \dots\dots\dots\dots\dots (9.10)$$

$$f_{BK} = \frac{2}{T}\int_0^T f(t)\ \sin 2\pi K\, t/T\ dt \dots\dots\dots\dots\dots (9.11)$$

Crest factor of f(C_f) : By definition

$$C_f = \frac{\hat{f}}{F_{RMS}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (9.12)$$

Distortion factor of f(DF_f) : By definition

$$DF_f = \frac{F_1}{F_{RMS}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (9.13)$$

Total Harmonic Distortion of f(THD_f): The amount of distortion in the waveform of f is quantified by means of the index Total Harmonic Distortion (THD). By definition

$$THD_f = \sqrt{\sum_{\substack{K=0 \\ K\neq 1}}^{\alpha}\left(\frac{F_k}{F_1}\right)^2} \dots\dots\dots\dots\dots\dots (9.14)$$

From which it can be shown that

$$THD_f = \frac{\sqrt{1-DF_f^2}}{DF_f} \dots\dots\dots\dots\dots\dots\dots\dots (9.15)$$

Displacement Factor of a Rectifier (DPF): If v_i and i_i are the per phase input voltage and input current of a rectifier respectively, then the Displacement Factor of a rectifier is defined as.

$$DPF = \cos\phi_i \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.16)$$

Where ϕ_i is the phase angle between the fundamental components of v_i and i_i.

Power factor of a rectifier (PF): As for any other equipment, the definition of the power factor of a rectifier is

$$PF = \frac{\text{Actual power input to the Rectifier}}{\text{Apparent power input to the Rectifier}} \quad \dots(9.17)$$

if the per phase input voltage and current of a rectifier are v_i and i_i respectively then

$$PF = \frac{V_{i1}I_{i1}\cos\varphi_i}{V_{iRMS}I_{iRMS}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.18)$$

If the rectifier is supplied from an ideal sinusoidal voltage source then

$$V_{i1} = V_{iRMS}$$

so,
$$PF = \frac{I_{i1}}{I_{iRMS}}\cos\varphi_i = DF_{i1} \times DPF \quad \dots\dots\dots\dots\dots\dots..(9.19)$$

In terms of THD_{ii}

$$PF = \frac{DPF}{\sqrt{1+THD_{ii}^2}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots...(9.20)$$

Majority of the rectifiers use either diodes or thyristors (or combination of both) in their circuits. While designing these components standard manufacturer's specifications will be referred to. However, certain terms are used in relation to the rectifier as a system. They are defined next.

Pulse number of a rectifier (p): Refers to the number of output voltage/current pulses in a single time period of the input ac supply voltage. Mathematically, pulse number of a rectifier is given by

$$p = \frac{\text{Time period of the input supply voltage}}{\text{Time period of the minium order harmonic in the output voltage/current.}}.$$

Classification of rectifiers can also be done in terms of their pulse numbers. Pulse number of a rectifier is always an integral multiple of the number of input supply phases.

Commutation in a rectifier: Refers to the process of transfer of current from one device (diode or thyristor) to the other in a rectifier. The device from which the current is transferred is called the "out going device" and the device to which the current is transferred is called the "incoming device". The incoming device turns on at the beginning of commutation while the out going device turns off at the end of commutation.

Commutation failure: Refers to the situation where the out going device fails to turn off at the end of commutation and continues to conduct current.

Firing angle of a rectifier (α): Used in connection with a controlled rectifier using thyristors. It refers to the time interval from the instant a thyristor is forward biased to the instant when a gate pulse is actually applied to it. This time interval is expressed in radians by multiplying it with

the input supply frequency in rad/sec. It should be noted that different thyristors in a rectifier circuit may have different firing angles. However, in the steady state operation, they are usually the same.

Extinction angle of a rectifier (γ): Also used in connection with a controlled rectifier. It refers to the time interval from the instant when the current through an outgoing thyristor becomes zero (and a negative voltage applied across it) to the instant when a positive voltage is reapplied. It is expressed in radians by multiplying the time interval with the input supply frequency (ω) in rad/sec. The extinction time (γ/ω) should be larger than the turn off time of the thyristor to avoid commutation failure.

Overlap angle of a rectifier (μ): The commutation process in a practical rectifier is not instantaneous. During the period of commutation, both the incoming and the outgoing devices conduct current simultaneously. This period, expressed in radians, is called the overlap angle "μ" of a rectifier. It is easily verified that $\alpha + \mu + \gamma = \pi$ radian.

Exercise 9.1

Fill in the blank(s) with the appropriate word(s).

    i)   In a rectifier, electrical power flows from the _____ side to the _____ side.
    ii)   Uncontrolled rectifiers employ _____ where as controlled rectifiers employ _____ in their circuits.
    iii)   For any waveform "Form factor" is always _____ than or equal to unity.
    iv)   The minimum frequency of the harmonic content in the Fourier series expression of the output voltage of a rectifier is equal to its _____.
    v)   "THD" is the specification used to describe the quality of _____ waveforms where as "Ripple factor" serves the same purpose for _____ for waveforms.
    vi)   Input "power factor" of a rectifier is given by the product of the _____ factor and the _____ factor.
    vii)   The sum of "firing angle", "Extinction angle" and "overlap angle" of a controlled rectifier is always equal to _____.

Answers: (i) ac, dc; (ii) diodes, thyristors; (iii) greater; (iv) pulse number; (v) ac, dc; (vi) displacement, distortion; (vii) π

9.3   Single phase uncontrolled half wave rectifier

This is the simplest and probably the most widely used rectifier circuit albeit at relatively small power levels. The output voltage and current of this rectifier are strongly influenced by the type of the load. In this section, operation of this rectifier with resistive, inductive and capacitive loads will be discussed.
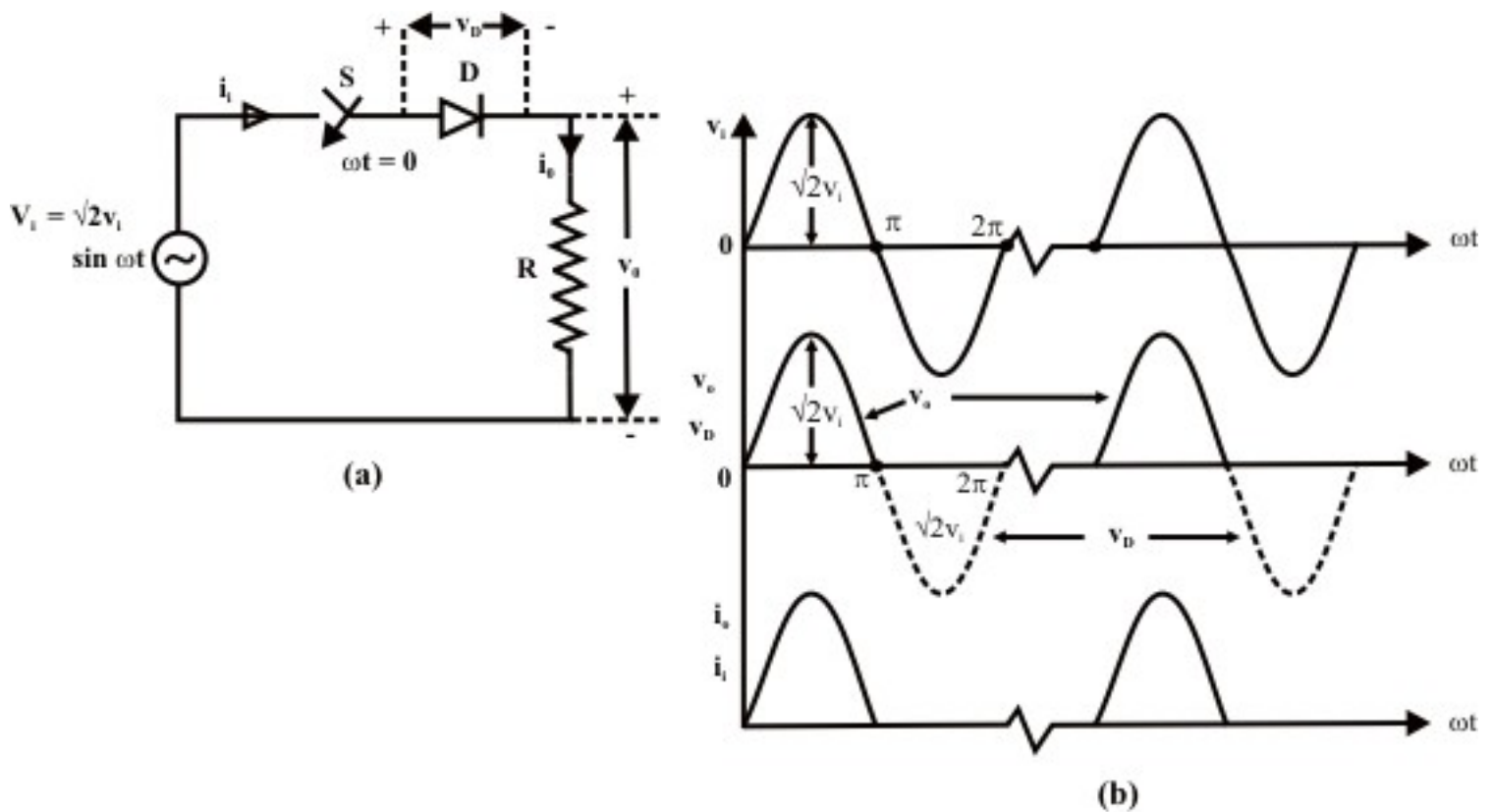
Fig. 9.2: Single phase uncontrolled half wave rectifier with resistive load.
(a) circuit diagram; (b) wave forms.

Fig 9.2 shows the circuit diagram and the waveforms of a single phase uncontrolled half wave rectifier. If the switch S is closed at at $t = 0$, the diode D becomes forward biased in the the interval $0 < \omega t \leq \pi$. If the diode is assumed to be ideal then

For $0 < \omega t \leq \pi$

$$v_0 = v_i = \sqrt{2} \, V_i \sin \omega t$$

$$v_D = v_i - v_0 = 0 \qquad \qquad \text{............................(9.21)}$$

Since the load is resistive

$$i_0 = v_0/R = \frac{\sqrt{2}V_0}{R} \sin\omega t \text{.......................(9.22)}$$

$$i_i = i_0$$

For $\omega t > \pi$, v_i becomes negative and D becomes reverse biased. So in the interval $\pi < \omega t \leq 2\pi$

$$i_i = i_0 = 0$$

$$v_0 = i_0 R = 0\text{.......................................(9.23)}$$

$$v_D = v_i - v_0 = v_i = \sqrt{2} \, V_i \sin\omega t$$

From these relationships

$$V_{0AV} = \frac{1}{2\pi}\int_0^{2\pi} v_0 d\omega t = \frac{1}{2\pi}\int_0^{\pi} \sqrt{2}V_i \sin\omega t d\omega t = \frac{\sqrt{2}V_i}{\pi} \text{..........(9.24)}$$

$$V_{DRMS} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} 2V_i^2 \sin^2\omega t d\omega t} = \frac{V_i}{\sqrt{2}} \text{.........................(9.25)}$$

It is evident from the waveforms of v_0 and i_0 in Fig 9.2 (b) that they contain significant amount of harmonics in addition to the dc component. Ripple factor of v_0 is given by

$$V_{ORF} = \frac{\sqrt{V_{DRM}^2 - V_{DAV}^2}}{V_{DAV}} = \frac{1}{2}\sqrt{\pi^2 - 4} \dots\dots\dots\dots\dots\dots\dots (9.26)$$

With a resistive load ripple factor of i_0 will also be same.

Because of such high ripple content in the output voltage and current this rectifier is seldom used with a pure resistive load.

The ripple factor of output current can be reduced to same extent by connecting an inductor in series with the load resistance as shown in Fig 9.3 (a). As in the previous case, the diode D is forward biased when the switch S is turned on. at $\omega t = 0$. However, due to the load inductance i_0 increases more slowly. Eventually at $\omega t = \pi$, v_0 becomes zero again. However, i_0 is still positive at this point. Therefore, D continues to conduct beyond $\omega t = \pi$ while the negative supply voltage is supported by the inductor till its current becomes zero at $\omega t = \beta$. Beyond this point, D becomes reverse biased. Both v_0 and i_0 remains zero till the beginning of the next cycle where upon the same process repeats.
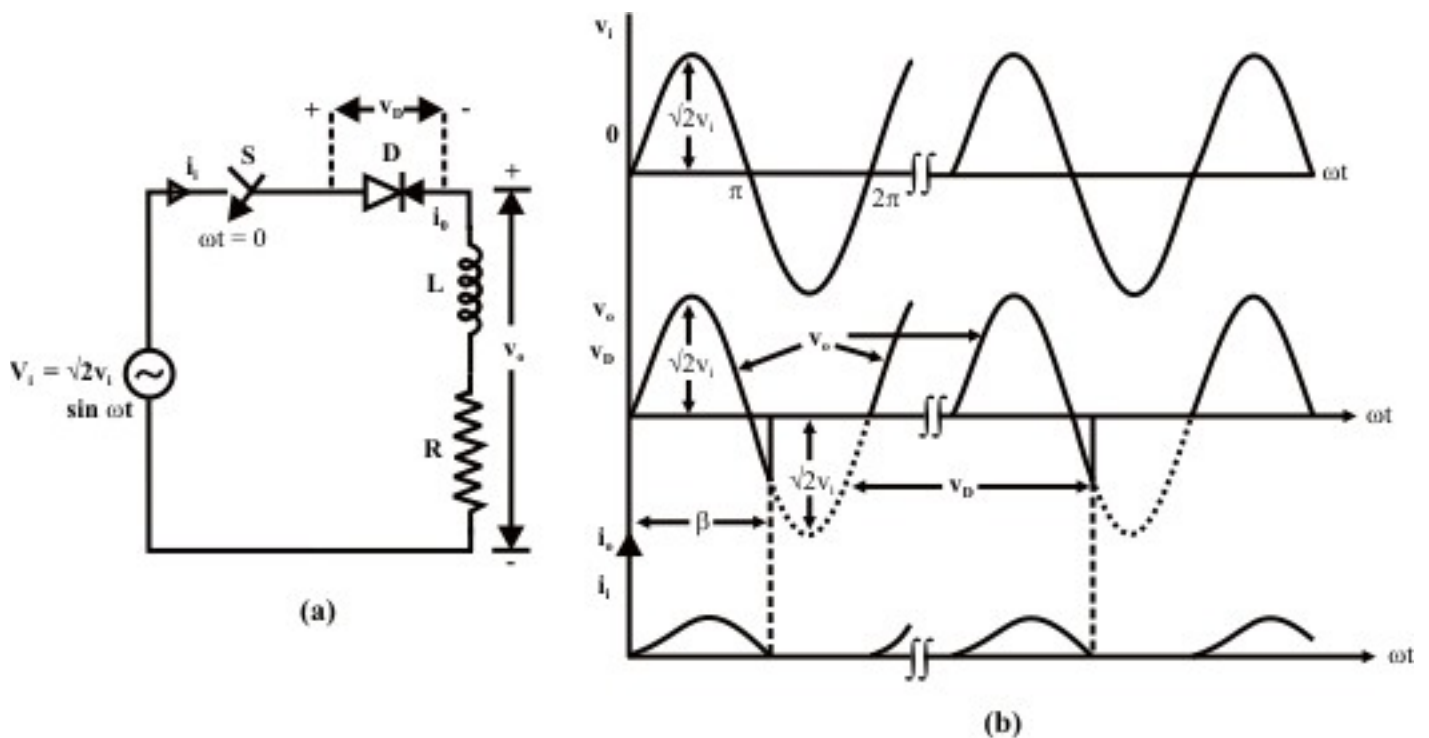

Fig. 9.3: Single phase uncontrolled half wave rectifier with inductive load.
(a) circuit diagram; (b) wave forms.

From the preceding discussion

For $0 \leq \omega t \leq \beta$
$v_D = 0$
$v_0 = v_i$
$i_0 = i_i \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.27)$
for $\beta \leq \omega t \leq 2\pi$

$$v_0 = 0$$
$$i_0 = i_i = 0$$
$$v_D = v_i - v_0 = v_i$$

$$V_{0AV} = \frac{1}{2\pi}\int_0^{2\pi} v_0 d\omega t = \frac{1}{2\pi}\int_0^{\beta} \sqrt{2}V_i \sin\omega t\, d\omega t \quad\text{.....................} \quad (9.28)$$

$$\text{or } V_{0AV} = \frac{\sqrt{2}V_i}{\pi}\left(\frac{1-\cos\beta}{2}\right) \text{...} \quad (9.29)$$

$$V_{0RMS} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} 2V_i^2 \sin^2\omega t\, d\omega t}$$

$$= \sqrt{\frac{V_i^2}{2\pi}\left(\beta - \frac{1}{2}\sin2\beta\right)} = \frac{V_i}{\sqrt{2}}\sqrt{\frac{2\beta - \sin2\beta}{2\pi}} \text{.........................}(9.30)$$

Form factor of the voltage waveform is

$$v_{OFF} = \frac{V_{0RMS}}{V_{0AV}} = \pi\sqrt{\frac{2\beta - \sin2\beta}{2\pi(1-\cos\beta)^2}} \text{......................................}(9.31)$$

The ripple factor.

$$V_{0RF} = \sqrt{v_{OFF}^2 - 1} = \sqrt{\frac{\pi(2\beta - \sin2\beta)}{2(1-\cos\beta)^2} - 1} \text{....................................}(9.32)$$

All these quantities are functions of β which can be found as follows.
For $0 \le \omega t \le \beta$

$$v_i = \sqrt{2}V_i \sin\omega t = L\frac{dio}{dt} + Ri_0 \text{...}(9.33)$$

$$i_0(\omega t = 0) = i_0(\omega t = \beta) = 0$$

The solution is given by

$$i_0 = I_0 e^{-\frac{\omega t}{\tan\varphi}} + \frac{\sqrt{2}V_i}{Z}\sin(\omega t - \varphi) \text{...}(9.34)$$

where $\tan\varphi = \dfrac{\omega L}{R}$

and $Z = \sqrt{R^2 + \omega^2 L^2}$ ....(9.35)

Putting the initial conditions of (9.33)

$$i_0 = \frac{\sqrt{2}V_i}{Z}\left[\sin\varphi\, e^{-\frac{\omega t}{\tan\varphi}} + \sin(\omega t - \varphi)\right] \text{...................................}(9.36)$$

$$i_0(\omega t = \beta) = \frac{\sqrt{2}V_i}{Z}\left[\sin\varphi\, e^{-\frac{\beta}{\tan\varphi}} + \sin(\beta - \varphi)\right] = 0$$

$$\text{or } \sin\varphi\, e^{-\frac{\beta}{\tan\varphi}} = \sin(\varphi - \beta) \text{...}(9.37)$$

β as a function of φ can be obtained by solving equation 9.37.

It can be shown that β increases with φ. From Equation (9.29), V_{0AV} decreases with increasing β while V_{0RMS} increases with β. Therefore, with increasing φ (and hence increasing L) the form factor and the ripple factor of v_0 worsens. However, the ripple factor of i_0 decreases with increasing L. Therefore, in certain applications, where a smooth dc current is of prime importance (e.g. the field supply of a dc motor) this configuration of the rectifier is preferred.
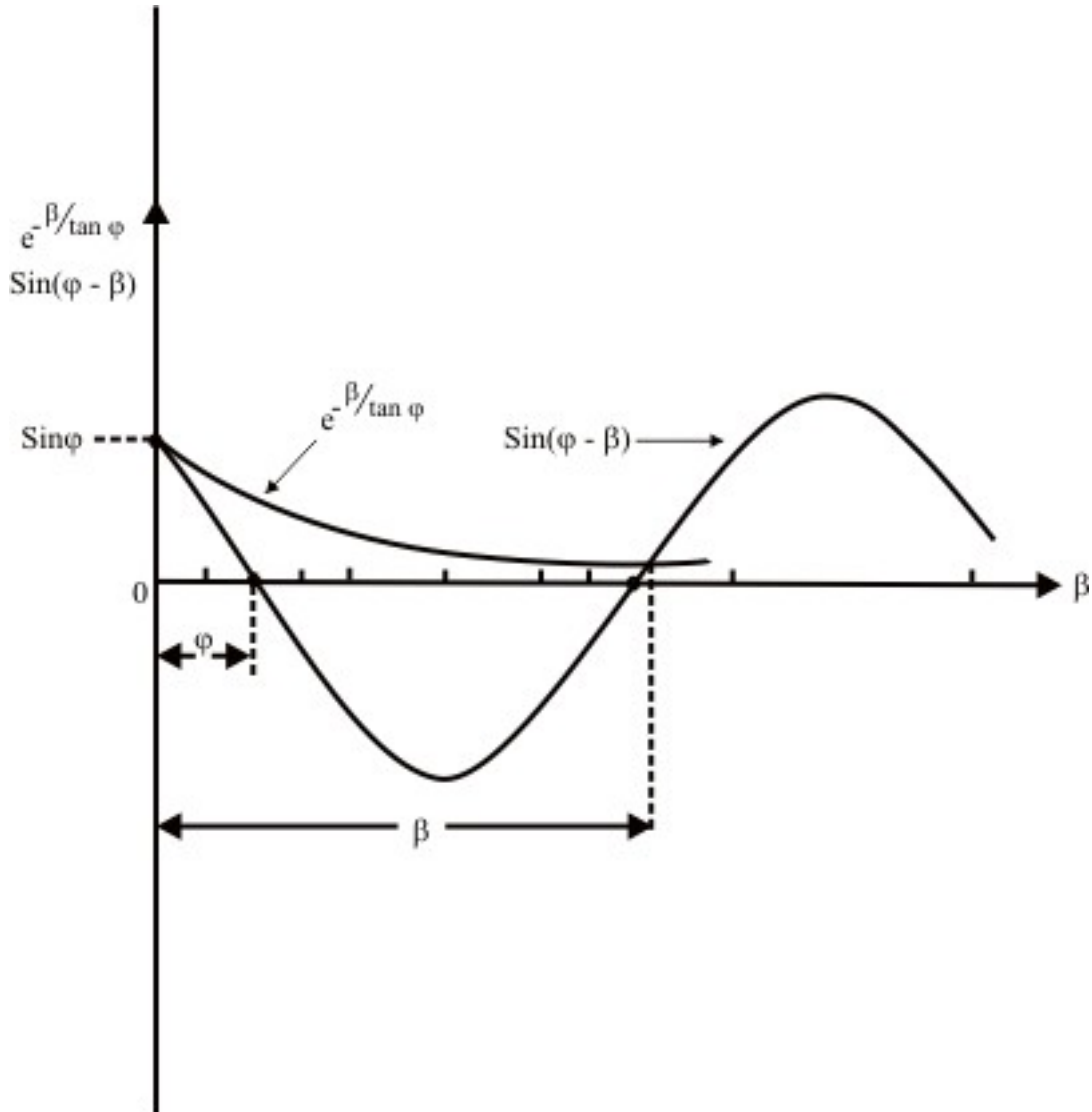

Fig. 9.4: Graphical solution of Equ. 9.37

The problem of poor form factor (ripple factor) of the output voltage can be solved to some extent by connecting a capacitor across the load resistance of Fig 9.2 (a). This single phase half wave rectifier supplying a capacitive load is shown in Fig 9.5 (a). Corresponding waveforms are shown in Fig 9.5 (b).

If the capacitor was initially discharged the diode "D" is forward biased when the switch S is turned on at $\omega t = 0$. The output voltage follows the input voltage. The diode D carries both the capacitor charging current and the load current. At $\omega t = \beta$ the sum of these two currents becomes zero and tends to grow in the negative direction. At this point the diode becomes

reverse biased and disconnects the load (along with the capacitor) from the supply. The capacitor then discharges with the load current. Diode D does not become forward biased till the input supply voltage becomes equal to the capacitor voltage in the next cycle at $\omega t = (2\pi + \varphi)$. The same process repeats thereafter.
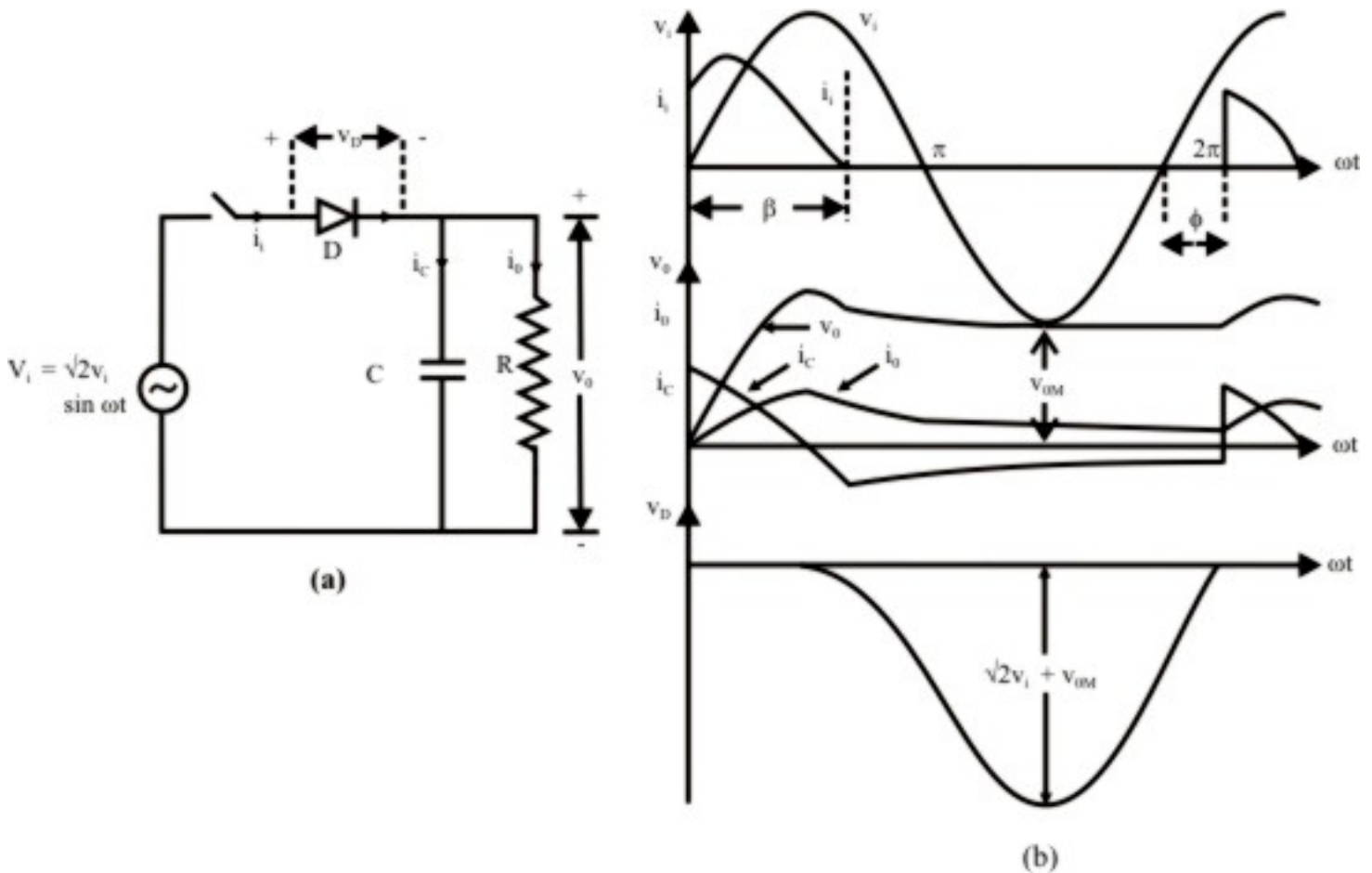

Fig 9.5: Single phase uncontrolled half wave rectifier with capacitive load.
(a) circuit diagram
(b) wave forms

From the preceding discussion

For $2\pi + \varphi \le \omega t \le 2\pi + \beta$

$$v_0 = v_i = \sqrt{2}V_i \sin\omega t \,\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.38)$$

$$i_i = i_c + i_0 = c\frac{dv_0}{dt} + \frac{v_0}{R}$$

or $\quad i_i = \dfrac{\sqrt{2}V_i}{R}[\omega RC\cos\omega t + \sin\omega t]$

$$= \frac{\sqrt{2}V_i}{R}\left(1 + \omega^2 R^2 C^2\right)^{\frac{1}{2}}\cos(\omega t - \theta) \,\dots\dots\dots\dots\dots\dots(9.39)$$

where $\theta = \tan^{-1}\dfrac{1}{\omega RC}$

At $\omega t = \beta + 2\pi$, $i_i = 0$ so $\beta - \theta = \pi/2$ or $\beta = \theta + \pi/2$

$$\text{or} \quad \beta = \frac{\pi}{2} + \tan^{-1}\frac{1}{\omega RC} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.40)$$

Again for $\beta \le \omega t \le 2\pi + \varphi$

$$i_i = 0, \ C\frac{dv_0}{dt} + \frac{v_0}{R} = 0, \ v_0(\omega t = \beta) = \sqrt{2}V_i\cos\theta.$$

$$\therefore \ v_0 = \sqrt{2}V_i\cos\theta \ e^{-(\omega t-\beta) \tan\theta} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(9.41)$$

$$\text{at} \quad \omega t = 2\pi + \varphi, \quad v_0 = \sqrt{2}V_i\sin\varphi$$

$$\sqrt{2}V_i\sin\varphi = \sqrt{2}V_i\cos\theta \ e^{-(2\pi+\varphi-\frac{\pi}{2}-\theta) \tan\theta}$$

$$\text{or} \ \sin\varphi = \cos\theta \ e^{-(\frac{3\pi}{2}+\varphi-\theta) \tan\theta}$$

$$\text{or} \ \sin\varphi = \left[\cos\theta \ e^{-(\frac{3\pi}{2}-\theta) \tan\theta}\right]e^{-\varphi \tan\theta} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots...(9.42)$$

From which φ can be solved. Peak to peak ripple in v_0 is

$$\hat{v}_{0pp} = \sqrt{2}V_i(1-\sin\varphi) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.43)$$

As $c \to \alpha$, $\theta \to 0$ and β and $\varphi \to \pi/2$ and $\hat{v}_{0pp} \to 0$

Therefore, a very large capacitor helps to improve the ripple factor of the output voltage of this rectifier. However, as indicated by Equation (9.39) the peak current through the diode increases proportionately. It is also interesting to observe that unlike the previous cases the peak reverse voltage appearing across D is given by.

$$v_D/\max = \sqrt{2}V_i + v_{0M} \approx 2\sqrt{2}V_i \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.44)$$

This is sometimes referred to as the peak inverse voltage rating (PIV) of the diode.

Exercise 9.2

1. Fill in the blank(s) with the appropriate word(s).

   i)   The ripple factor of the output voltage and current waveforms of a single phase uncontrolled half wave rectifier is _____ than unity.

   ii)   With an inductive load, the ripple factor of the output _____ of the half wave rectifier improves but that of the output _____ becomes poorer.

   iii) In both single phase half wave and full wave rectifiers the form factor of the output voltage approaches _____ with capacitive loads provided the capacitance is _____ enough.

   iv)   The PIV rating of the rectifier diode used in a single phase half wave rectifier supplying a capacitive load is approximately _____ the _____ input supply voltage.

   v)   The % THD of the input current of the rectifiers supplying capacitive loads is _____.

 Answers: (i) greater; (ii) current, voltage; (iii) unity, large; (iv) double, peak; (v) high.

2. An unregulated dc. power supply of average value 12 V and peak to peak ripple of 20% is to be designed using a single phase half wave rectifier. Find out the required input voltage, the output capacitance and the diode RMS current and PIV ratings. The equivalent load resistance is 50 ohms.

Answer: From equation 9.43.

$$\hat{v}_{opp} = \sqrt{2}V_i(1-\sin\varphi) = 0.2\times12 = 2.4 \text{ V}.$$

$$\therefore \quad V_{omax} = \sqrt{2}V_i = 12+2.4/2 = 13.2V \qquad \therefore \quad V_i = 9.33V$$

$$\therefore \quad \sin\varphi = 0.818 \quad \text{or} \quad \varphi = 54.9^o = 0.96 \text{ rad}.$$

Then from equation 9.42

$$0.818 = \cos\theta \, e^{-(3\pi/2+\varphi-\theta)\tan\theta}$$

$$\text{or} \quad 0.818 \, e^{(5.67-\theta)\tan\theta} = \cos\theta$$

From which $\quad \theta \approx 2.035^\circ$

$$\therefore \quad \tan\theta = \frac{1}{\omega RC} = 0.03553, \ R = 50\Omega, \ \therefore \ C = 1790\,\mu F$$

PIV of the diode $= 2\sqrt{2}V_i = 26.4V$.

$$\text{RMS. Diode current} = \sqrt{\frac{1}{2\pi}\int_\varphi^\beta i_i^2 \, d\omega t} = \frac{V_i}{R}\sqrt{2\left(1+\omega^2 R^2 C^2\right)}\sqrt{\frac{1}{2\pi}\int_{54.9^o}^{92.035}\cos^2(\omega t-\theta)d\omega t}$$

$$= 7.432\sqrt{\frac{1}{2\pi}\left[\frac{\beta-\varphi}{2}+\frac{1}{4}\sin2(\beta-\theta)-\frac{1}{4}\sin2(\varphi-\theta)\right]} = 0.8564 \text{ Amps}.$$

9.4   Single phase uncontrolled full wave rectifier

Single phase uncontrolled half wave rectifiers suffer from poor output voltage and/or input current ripple factor. In addition, the input current contains a dc component which may cause problem (e.g. Transformer saturation etc) in the power supply system. The output dc voltage is also relatively less. Some of these problems can be addressed using a full wave rectifier. They use more number of diodes but provide higher average and rms output voltage.

There are two types of full wave uncontrolled rectifiers commonly in use. If a split power supply is available (e.g. output from a split secondary transformer) only two diode will be required to produce a full wave rectifier. These are called split secondary rectifiers and are commonly used as the input stage of a linear dc voltage regulator. However, if no split supply is available the bridge configuration of the full wave rectifier is used. This is the more commonly used full wave uncontrolled rectifier configuration. Both these configurations are analyzed next.

9.4.1 Split supply single phase uncontrolled full wave rectifier.
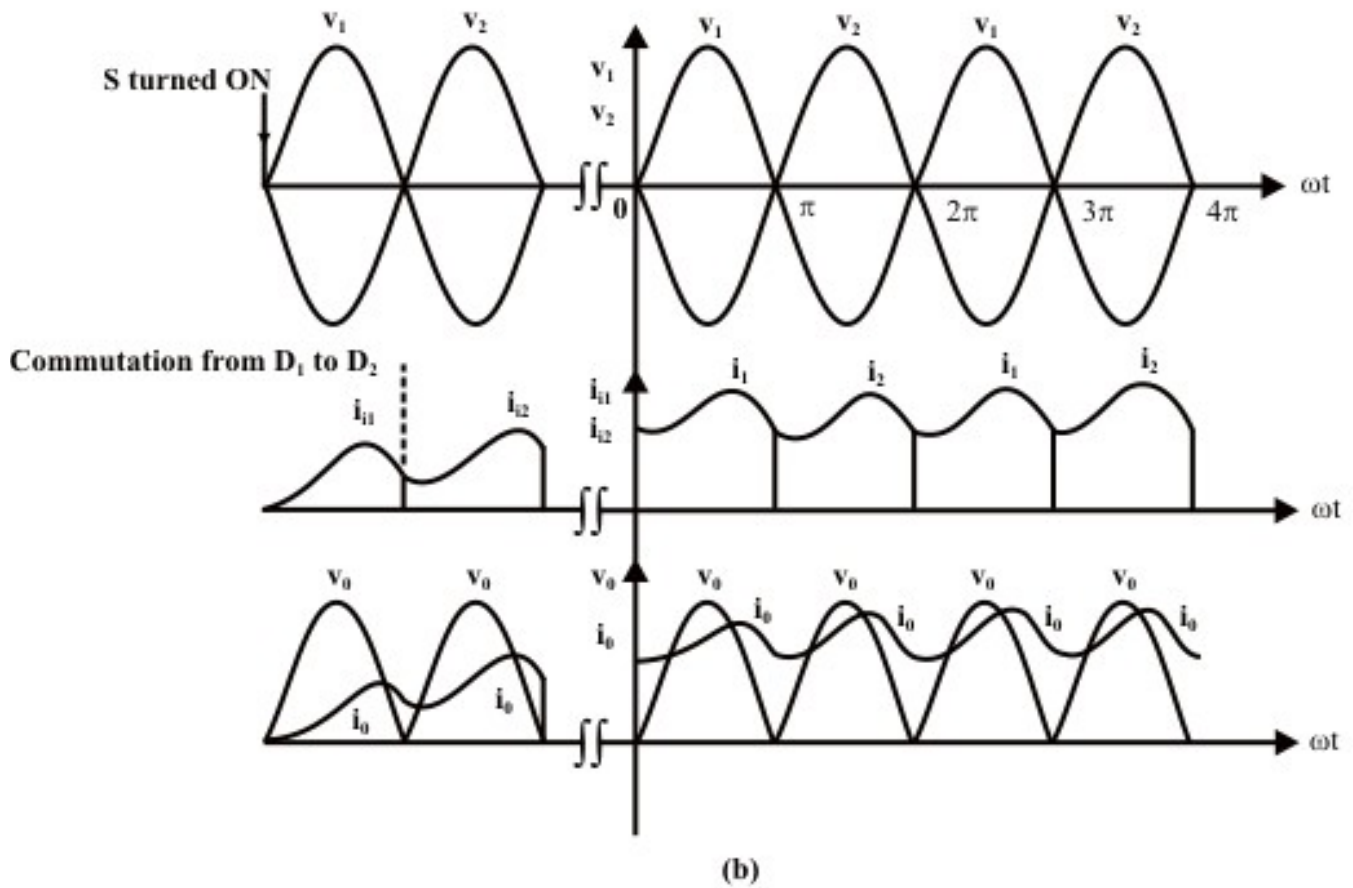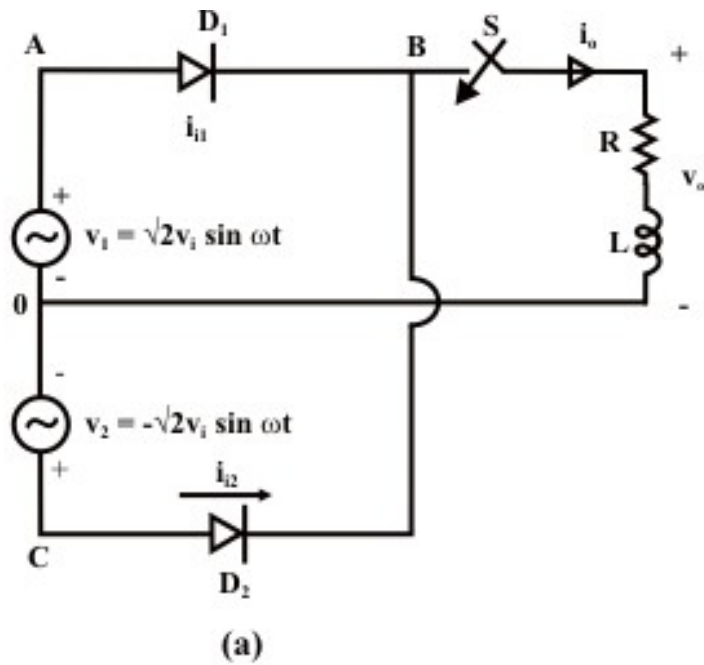

(a)

(b)

Fig. 9.6: Split supply single phase uncontrolled full wave rectifier supplying an R - L load, (a) circuit diagram, (b) wave forms

Fig 9.6 shows the circuit diagram and waveforms of a single phase split supply, uncontrolled full wave rectifier supplying an R – L load. The split power supply can be thought of to have been obtained from the secondary of a center tapped ideal transformer (i.e. no internal impedance).

When the switch is closed at the positive going zero crossing of v_1 the diode D_1 is forward biased and the load is connected to v_1. The currents i_0 and i_{i1} start rising through D_1. When v_1 reaches its negative going zero crossing both i_0 and i_{i1} are positive which keeps D_1 in conduction. Therefore, the voltage across D_2 is $v_{CB} = v_2 - v_1$. Beyond the negative going zero crossing of v_i, D_2 becomes forward biased and the current i_0 commutates to D_2 from D_1. The load voltage v_0 becomes equal to v_2 and D_1 starts blocking the voltage $v_{AB} = v_1 - v_2$. The current i_0 however continues to increase through D_2 till it reaches the steady state level after several cycles. Steady state waveforms of the variables are shown in Fig 9.6 (b) from $\omega t = 0$ onwards. It should be noted that the current i_0, once started, always remains positive. This mode of operation of the rectifier is called the "Continuous conduction mode" of operation. This should be compared with the i_0 waveform of Fig 9.3 (b) for the half wave rectifier where i_0 remains zero for some duration of the input supply waveform. This mode is called the " discontinuous conduction mode" of operation.

From the above discussion

For $0 \le \omega t < \pi$

$$v_0 = v_1$$
$$i_0 = i_{i1} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.45)$$

for $\pi \le \omega t < 2\pi$

$$v_0 = v_2$$
$$i_0 = i_{i2} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.46)$$
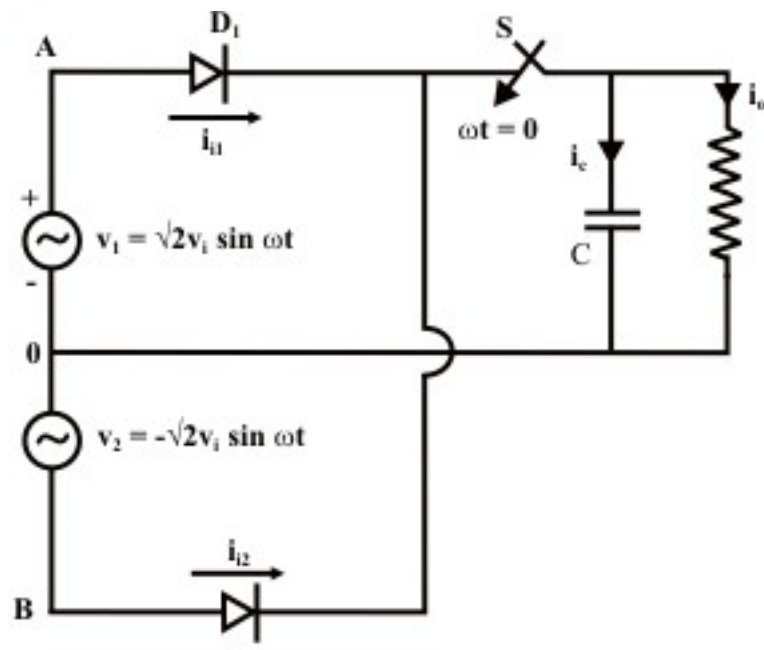
Since v_0 is periodic over an interval π

$$V_{0AV} = \frac{1}{\pi} \int_0^\pi v_0 d\omega t = \frac{\sqrt{2}V_i}{\pi} \int_0^\pi \sin\omega t d\omega t = \frac{2\sqrt{2}V_i}{\pi} \dots\dots\dots\dots(9.47)$$

$$V_{0RMS} = \sqrt{\frac{1}{\pi} \int_0^\pi 2V_i^2 \sin^2\omega t \, d\omega t} = V_i \dots\dots\dots\dots\dots\dots(9.48)$$

$$\therefore \quad V_{0FF} = \frac{V_{0RMS}}{V_{0AV}} = \frac{\pi}{2\sqrt{2}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.49)$$

$$V_{0RF} = \sqrt{V_{0FF}^2 - 1} = \frac{\sqrt{\pi^2 - 8}}{2\sqrt{2}} \dots\dots\dots\dots\dots\dots\dots\dots(9.50)$$

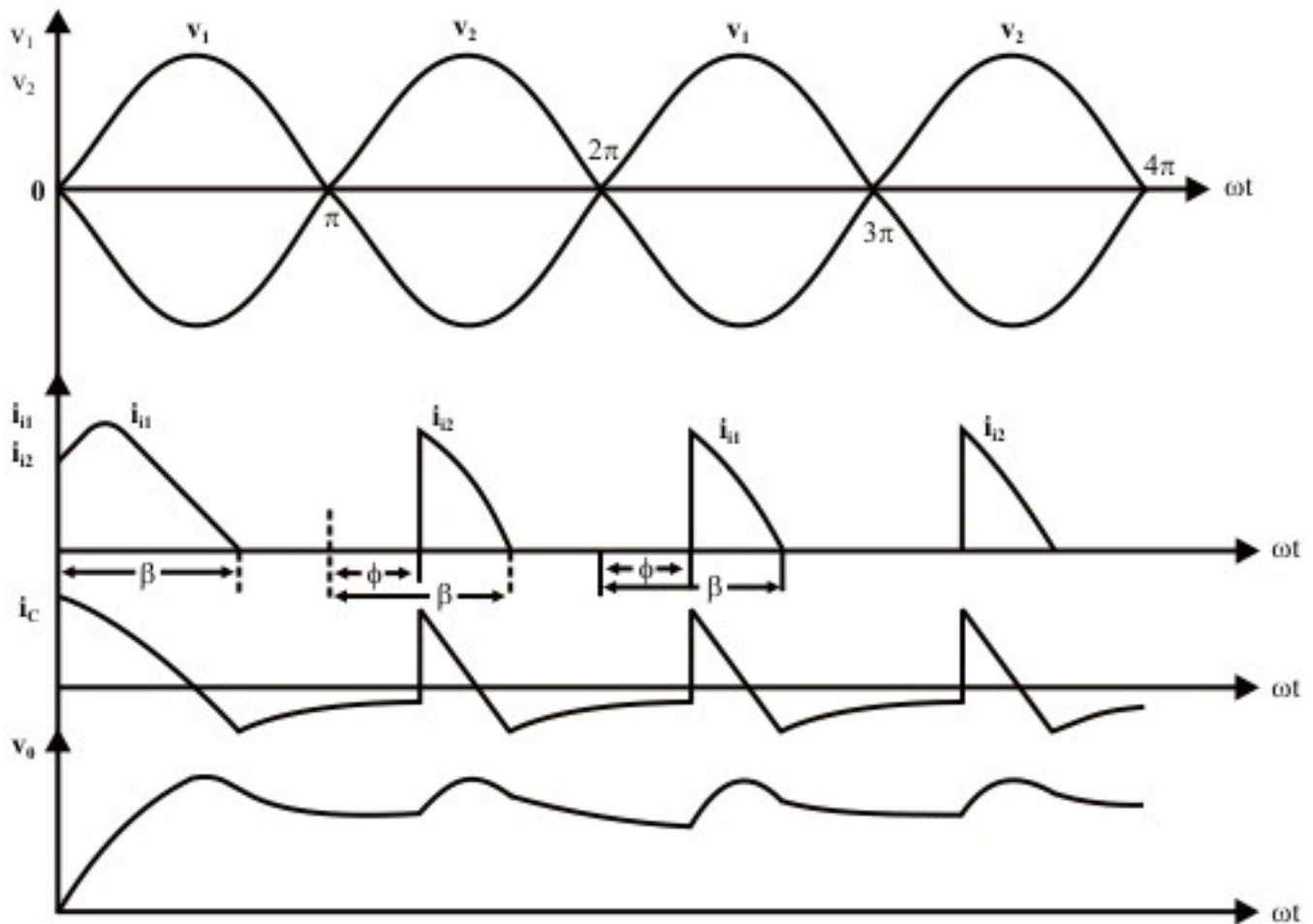Both the form factor and the ripple factor shows considerable improvement over their half wave counter parts.

Fig 9.7: Split supply single phase uncontrolled full wave rectifier supplying capacitive load
(a) circuit diagram (b) wave forms

The single phase full wave rectifier still does not offer a smooth dc voltage. With resistive load, considerable ripple current will flow into the load. This problem can be solved by connecting a capacitor across the load resistance just as in the case of a half wave rectifier.

If the capacitor was initially discharged, the diode D_1 is forward biased when the switch S is turned on at $\omega t = 0$. The diode D_2 remains reverse biased. The output voltage follows the input voltage. D_1 carries both the capacitor charging current and the load current. At $\omega t = \beta$ the sum of these two currents becomes zero and tends to grow in the negative direction. At this point the diode D_1 becomes reverse biased and disconnects the load along with the capacitor from the supply. The capacitor then discharges through the load until at $\omega t = \pi + \varphi$, v_2 becomes greater than v_0 and forward biases D_2. D_1 now remains reverse biased. D_2 conducts up to $\omega t = \pi + \beta$. The same process repeats thereafter.

From the discussion above

For $\pi + \varphi \le \omega t \le \pi + \beta$

$$v_0 = v_2 = -\sqrt{2}V_i \sin\omega t$$

$$i_{i2} = i_c + i_0 = C\frac{dv_0}{dt} + \frac{v_0}{R} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.51)$$

$$\text{or } i_{i2} = -\frac{\sqrt{2}V_i}{R}[\omega RC\cos\omega t + \sin\omega t]$$

$$= \frac{\sqrt{2}V_i}{R}\left(1 + \omega^2 R^2 C^2\right)^{\frac{1}{2}} \cos(\pi + \theta - \omega t) \text{ where } \theta = \tan^{-1}\frac{1}{\omega RC} \dots(9.52)$$

at $\omega t = \pi + \beta$, $i_{i1} = 0$ so $\beta - \theta = \frac{\pi}{2}$ or $\beta = \theta + \frac{\pi}{2}$ or $\beta = \frac{\pi}{2} + \tan^{-1}\frac{1}{\omega RC} \dots\dots\dots\dots(9.53)$

Again for $\beta \le \omega t \le \pi + \varphi$

$$i_{i1} = 0 \quad \therefore C\frac{dv_0}{dt} + \frac{v_0}{R} = 0 \quad v_0(\omega t = \beta) = \sqrt{2}V_i \sin\beta = \sqrt{2}V_i \cos\theta \dots\dots\dots(9.54)$$

$$\therefore v_0 = \sqrt{2}V_i \cos\theta \, e^{-(\omega t - \beta)\tan\theta} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.55)$$

at $\omega t = \pi + \varphi$, $\quad v_0 = \sqrt{2}\sin\varphi$

$$\sqrt{2}V_i \sin\varphi = \sqrt{2}V_i \cos\theta \, e^{\left(\frac{\pi}{2} + \theta - \pi - \varphi\right)\tan\theta}$$

$$\text{or } \sin\varphi = \cos\theta \, e^{-\left(\frac{\pi}{2} + \varphi - \theta\right)\tan\theta}$$

$$\text{or } \sin\varphi = \left[\cos\theta \, e^{-\left(\frac{\pi}{2} - \theta\right)\tan\theta}\right] e^{-\varphi\tan\theta} \dots\dots\dots\dots\dots\dots\dots\dots\dots(9.56)$$

From which φ can be solved. Peak to peak ripple in v_0 is

$$\hat{v}_{0pp} = \sqrt{2}V_i(1 - \sin\varphi) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9.57)$$

It can be shown that for the same R and C, $\hat{v}_{0pp}$ given by Equation (9.57) is smaller than that given by Equation (9.43) for the half wave rectifier. The diode PIV ratings remain equal to $2\sqrt{2}V_i$ however.

Exercise 9.3

1. Fill in the blank(s) with the appropriate word(s).

   i)   The output voltage form factor of a single phase full wave rectifier is _____ .
   ii)   The output voltage of a single phase full wave rectifier supplying an inductive load is _____ of the load parameters.
   iii)   The peak to peak output voltage ripple of a single phase split supply full wave rectifier supplying a capacitive load is _____ compared to an equivalent half wave rectifier.

Answers: (i) $\pi/2\sqrt{2}$; (ii) independent ; (iii) smaller.

2. An unregulated dc power supply is built around a single phase split supply full wave rectifier using the same input voltage and output capacitor found in the problem 2 of Exercise 9.2. The load resistance is 50 Ω. Find out the average output voltage, the peak to peak ripple in the output voltage and the RMS current ratings of the diodes.

Answer: From the given data $C = 1790\ \mu F,\ \ R = 50\ \Omega,\ \ \therefore\ \theta = 2.035°$

From equation 9.56

$$\text{Sin } \theta = \cos \theta\ e^{-(\pi/2 + \varphi - \theta)}\ \tan \theta$$

Or $\sin \theta = 0.99937\ e^{-0.03553(1.5353+\varphi)} = 0.946316\ e^{-0.03553\varphi}$

From which $\varphi = 65.33°$

$V_i = 9.33$ volts.
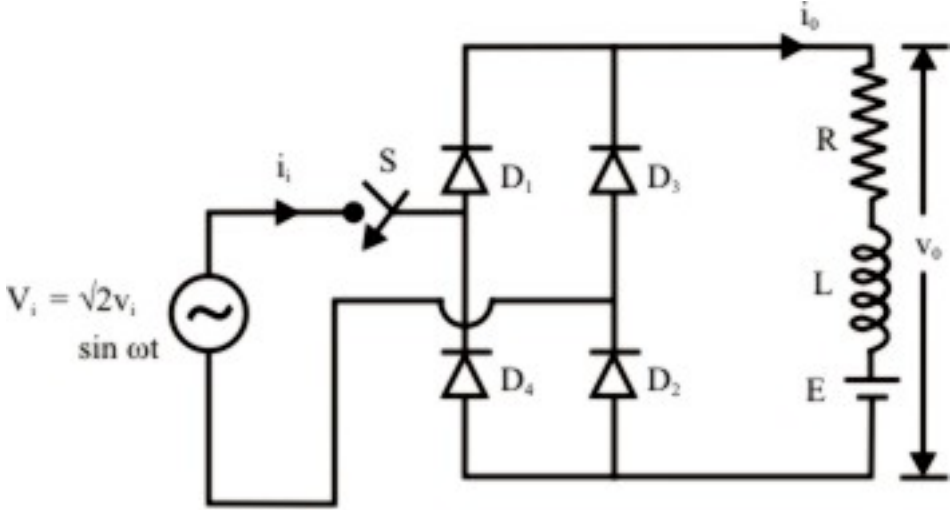
$$\therefore\ \ \hat{v}_{opp} = \sqrt{2}V_i(1-\sin\varphi) = 1.20\ \text{volts.}$$

$$V_{0AV} = V_{0Max} - \frac{\hat{v}_{opp}}{2} = \sqrt{2}V_i - 0.6V = 13.2 - 0.6V = 12.6V.$$

$$\therefore\ \ \%\ \text{ripple} = 9.5\%$$

$$\text{RMS diode current} = \sqrt{\frac{1}{2\pi}\int_{\varphi}^{\pi/2+\theta} i_i^2\,d\omega t} = \frac{V_i}{R}\sqrt{2\left(1+\omega^2 R^2 C^2\right)}\sqrt{\frac{1}{2\pi}\int_{\varphi}^{\beta}\cos^2(\omega t - \theta)d\omega t}$$

$$= 7.432\sqrt{\frac{1}{2\pi}\left[\frac{\beta-\varphi}{2} - \frac{1}{4}\sin2(\varphi-\theta)\right]} = 0.533\ \text{Amps.}$$

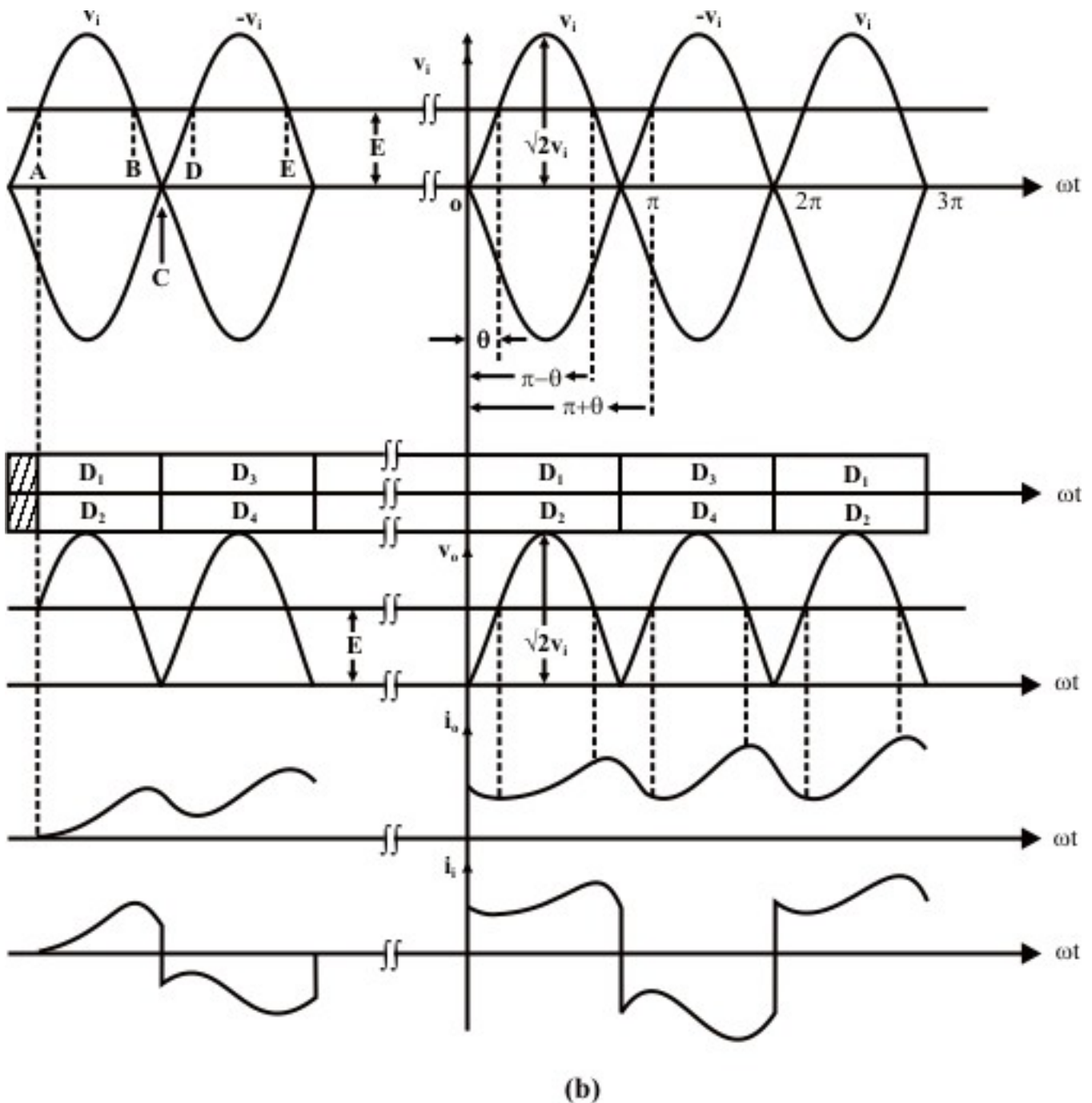# 9.4.2 Single phase uncontrolled full bridge rectifier

(a)

Fig 9.8: Single phase uncontrolled full bridge rectifier suppling R-L-E load. (a) circuit diagram; (b) waveforms.

The split supply full wave single phase rectifier offers as good performance as possible from a single phase rectifier in terms of the output voltage form factor and ripple factor. They have a few disadvantages however. These are

- They require a split power supply which is not always available.
- Each half of the split power supply carries current for only one half cycle. Hence they are underutilized.

- The ratio of the required diode PIV to the average out put voltage is rather high.

These problems can be mitigated by using a single phase full bridge rectifier as shown in Fig 9.8 (a). This is one of the most popular rectifier configuration and are used widely for applications requiring dc. power output from a few hundred watts to several kilo watts. Fig 9.8 (a) shows the rectifier supplying an R-L-E type load which may represent a dc. motor or a storage battery. These rectifiers are also very widely used with capacitive loads particularly as the front end of a variable frequency voltage source inverter. However, in this section analysis of this rectifier supplying an R-L-E load will be presented. Its operation with a capacitive load is very similar to that of a split supply rectifier and is left as an exercise.

When the switch S is turned on at the positive going zero crossing of vi no current flows in the circuit till vi crosses E at point A. Beyond this point, D_1 & D_2 are forward biased by vi and current starts increasing through them till the point B. After point B, vi falls below E and i_o starts decreasing. Now depending on the values of R, L & E one of the following situations may arise.

- i_o may become zero before the negative going zero crossing of vi at point C.
- i_o may continue to flow beyond C and become zero before the point D.
- i_o may still be non zero at point D.

It should be noted that if i_o > 0 either D_1D_2 or D_3D_4 must conduct. Fig 9.4 (b) shows the waveforms for the third situation.

If i_o > 0 at point C the negative going input voltage reverse biases D1 & D2. Current i_o commutates to D3 and D4 as shown in the associated "conduction Diagram" in Fig 9.8 (b). It shows pictorially the conduction interval of different devices. The current i_o continues to decrease up to the point D beyond which it again increases. It should be noted that in this mode of conduction i_o always remain greater than zero. Consequently, this is called the continuous conduction mode of operation of the rectifier. In the other two situations the mode of operation will be discontinuous.

The steady state waveforms of the rectifier under continuous conduction mode is shown to the right of the point ωt = 0 in Fig 9.4 (b).

From this figure and preceding discussion

For $\quad 0 < \omega t \le \pi$

$$v_o = v_i = \sqrt{2}V_i \sin \omega t \qquad\qquad (9.58)$$

$$i_i = i_o$$

for $\quad \pi < \omega t \le 2\pi$

$$v_o = - v_i = - \sqrt{2}V_i \sin \omega t \qquad\qquad (9.59)$$

$$i_i = - i_o$$

$$\therefore \; V_{oAV} = \frac{1}{\pi} \int_o^\pi \sqrt{2}V_i^2 \sin \omega t \, d\,\omega t \;\; = \;\; \frac{2\sqrt{2}}{\pi} \; V_i \qquad\qquad (9.60)$$

$$V_{oRMS} \;=\; \sqrt{\frac{1}{\pi} \int_o^\pi 2V_i^2 \sin^2\omega t \, d\,\omega t} = V_i \qquad\qquad (9.61)$$

$$\therefore \ v_{OFF} = \frac{V_{oRMS}}{V_{oAV}} = \frac{\pi}{2\sqrt{2}}$$

$$V_{oRF} = \sqrt{V_{OFF}^2 - 1} = \frac{\sqrt{\pi^2 - 8}}{2\sqrt{2}} \tag{9.62}$$

Finding out the characterizing quantities for i_i will be difficult owing to its complicated waveform. Considerable, simplification is achieved (without significant loss of accuracy) by replacing the actual i_o waveform by its average value $I_{oAV} = V_{oAV} / R$.

Fig 9.9 shows the approximate input current wave form and its fundamental component.
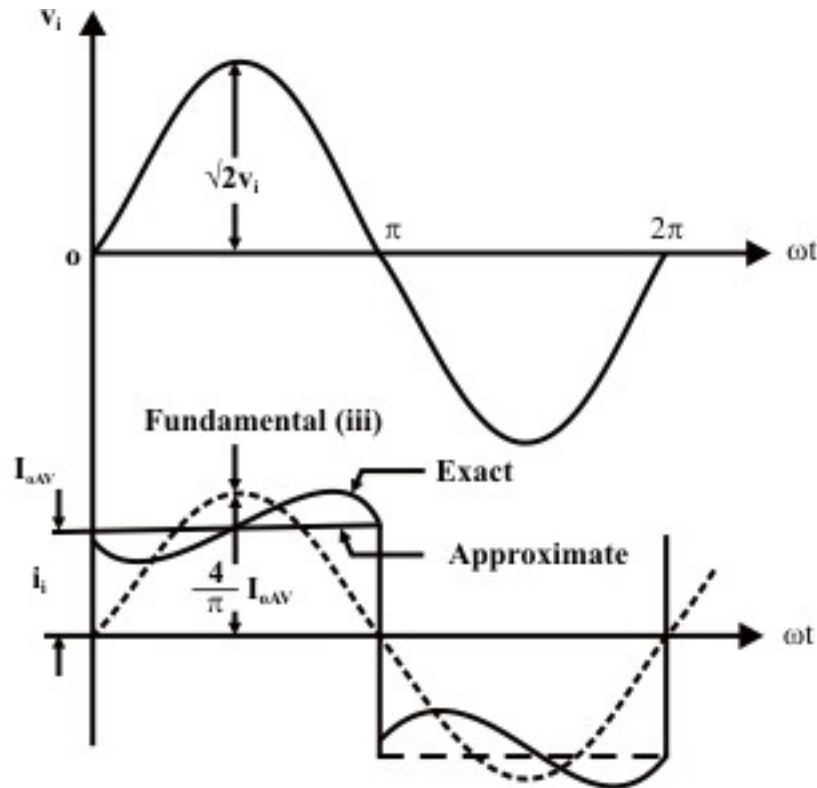
Fig. 9.9: Input current waveform in a single phase uncontrolled full bridge rectifier supplying a R-L-E load.

From Fig 9.9

Displacement angle $\phi_i = 0$

$\therefore$ Input displacement factor (DPF) = $\cos \varphi_i = 1$ $\tag{9.63}$

Distortion factor (DF$_{il}$) = $\dfrac{I_{il}}{I_{oAV}} = \dfrac{2\sqrt{2}}{\pi}$ $\tag{9.64}$

Power Factor (PF) = DPF $\times$ DF$_{il}$ = $\dfrac{2\sqrt{2}}{\pi}$ $\tag{9.65}$

% THD$_{i_i}$ = $100 \times \sqrt{\dfrac{1 - DFi_i^2}{DFi_i^2}}$ = $100 \times \dfrac{\sqrt{\pi^2 - 8}}{2\sqrt{2}}$ $\tag{9.66}$

The exact analytical expression for i_o (and hence i_i) can be obtained as follows.

for $0 < \omega t \leq \pi$

$$v_i = \sqrt{2}V_i \sin \omega t = Ri_o + \frac{Ldi_o}{dt} + E \qquad (9.67)$$

$$i_o \big|_{\omega t=0} = i_o \big|_{\omega t=\pi} \quad \text{(steady state periodic boundary cond.)}$$

The general solution can be written as

$$i_o = I_o e^{-\frac{\omega t}{\tan\varphi}} + \frac{\sqrt{2}V_i}{Z}\left[\sin(\omega t - \varphi) - \frac{\sin\theta}{\cos\varphi}\right] \qquad (9.68)$$

where $\tan\varphi = \dfrac{\omega L}{R}$; $Z = \sqrt{R^2 + \omega^2 L^2}$; $\sin\theta = \dfrac{E}{\sqrt{2}V_i}$

From the boundary condition

$$I_o - \frac{\sqrt{2}V_i}{Z}\left[\sin\varphi + \frac{\sin\theta}{\cos\varphi}\right] = I_o e^{-\frac{\pi}{\tan\varphi}} + \frac{\sqrt{2}V_i}{Z}\left[\sin\varphi - \frac{\sin\theta}{\cos\varphi}\right]$$

$$I_o = \frac{\sqrt{2}V_i}{Z} \frac{2\sin\varphi}{1 - e^{-\frac{\pi}{\tan\varphi}}} \qquad (9.69)$$

$$\therefore \ i_o = \frac{\sqrt{2}\,V_i}{Z}\left[\frac{2\sin\varphi}{1 - e^{-\frac{\pi}{\tan\varphi}}} e^{-\frac{\omega t}{\tan\varphi}} + \sin(\omega t - \varphi) - \frac{\sin\theta}{\cos\varphi}\right] \qquad (9.70)$$

From which the condition for continuous conduction can be obtained.

for continuous conduction $i_o \geq 0$ for all $0 < \omega t \leq \pi$

hence $i_o\big|_{Min} \geq 0$ or $i_o\big|_{\omega t=\theta} \geq 0$

$\therefore$ Condition for continuous conduction is

$$\frac{2\sin\varphi}{1 - e^{-\frac{\pi}{\tan\varphi}}} e^{-\frac{\theta}{\tan\varphi}} = \sin(\varphi - \theta) + \frac{\sin\theta}{\cos\varphi} \qquad (9.71)$$

If the parameters of the load (i.e, R, L &E) are such that the left hand side of equation 9.71 is less than the right hand side conduction of the rectifier becomes discontinuous i.e, the load current becomes zero for a part of the input cycle. Discontinuous conduction mode of operation of this rectifier is discussed next.
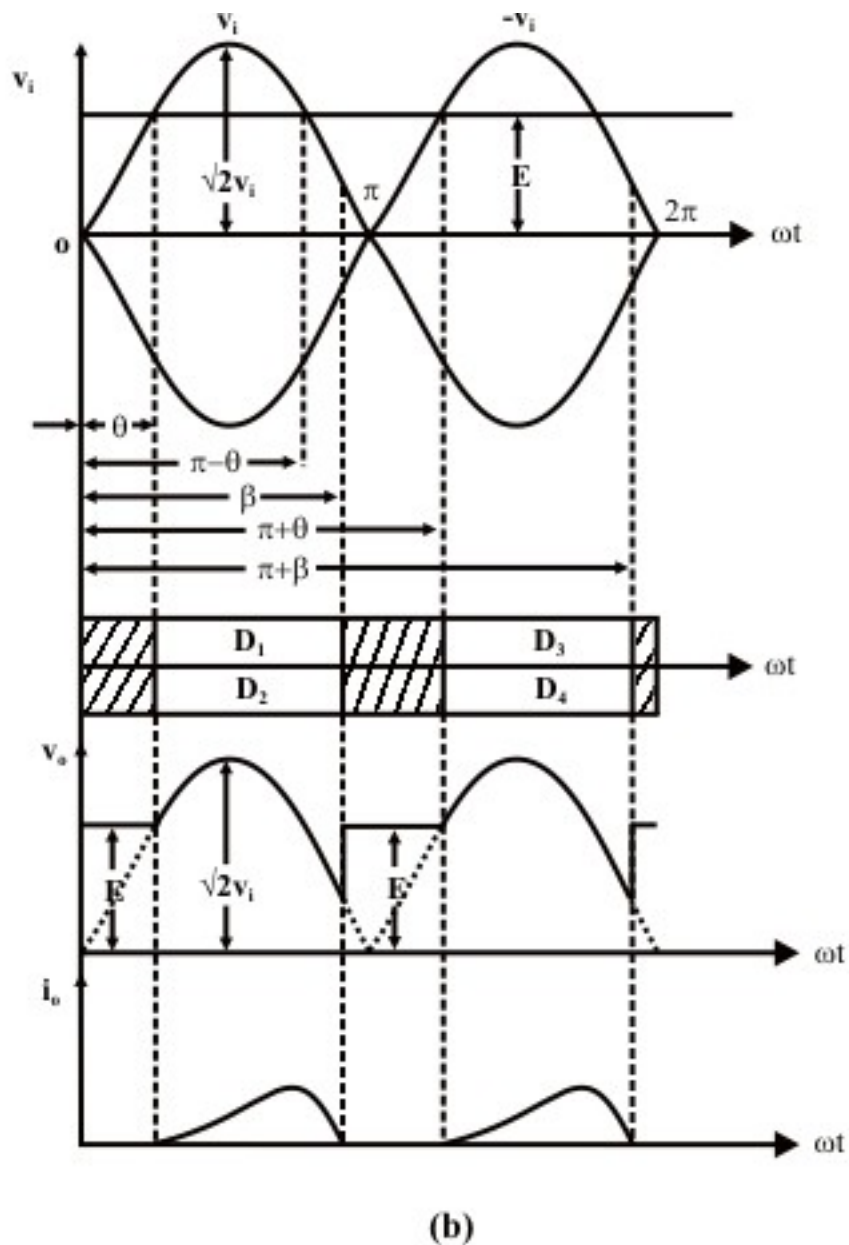
(a)
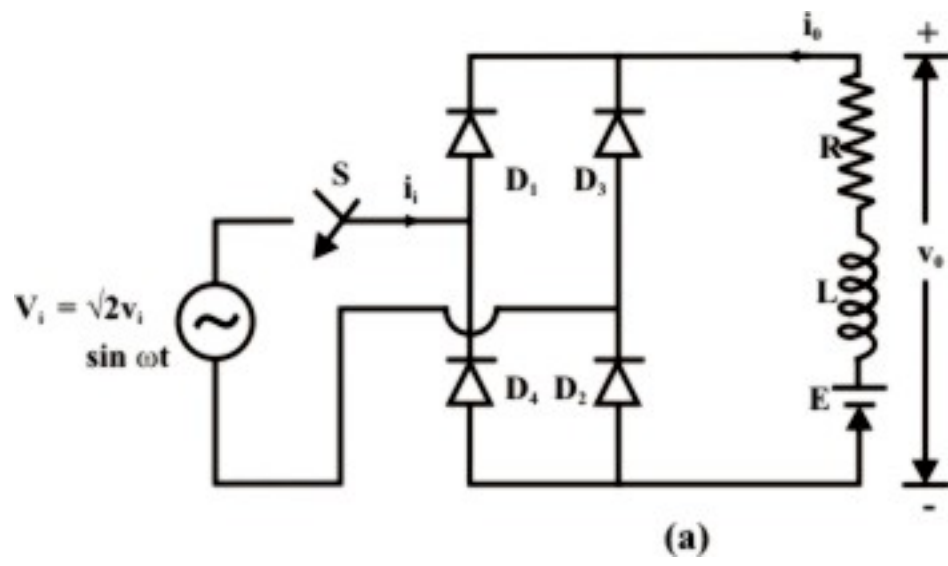
Fig 9.10: Single phase uncontrolled full bridge rectifier with R-L-E load operating in the discontinuous conduction mode (a) circuit diagram; (b) waveforms.

Fig. 9.10(b) shows the waveforms of different variables under discontinuous conduction mode of operation. In this mode of operation D_1 D_2 are not forward biased till vi exceed E at $\omega t = \theta$. Consequently, no current flows into the load till this time. After $\omega t = \theta$, the load is connected to the input source through D_1 D_2 and i_o starts building up. Beyond $\omega t = \pi - \theta$, i_o starts decreasing and becomes zero at $\omega t = \beta < \pi$. D_1 D_2 are reverse biased at this point. D_3 D_4 are forward biased at $\omega t = \pi + \theta$ when i_o starts increasing again. Thus none of the diodes conduct during the interval $\beta < \omega + \leq \pi + \theta$ and i_o remains zero during this period.

Form the preceding discussion

for $\theta < \omega t \le \beta < \pi$

$$v_o = v_i = \sqrt{2}V_i \sin \omega t \qquad (9.72)$$
$$i_i = i_o$$

for $\pi < \pi + \theta < \omega + \le \pi + \beta < 2\pi$

$$v_o = -v_i = -\sqrt{2}V_i \sin \omega t$$
$$i_i = -i_o \qquad (9.73)$$
$$v_o = E$$
$$i_i = i_o = 0 \qquad \text{other wise} \qquad (9.74)$$

$$V_{oAV} = \frac{1}{\pi} \int_\theta^{\pi+\theta} v_o d\,\omega t = \frac{1}{\pi} \left[\int_\theta^\beta \sqrt{2}V_i \sin \omega t + \int_\beta^{\pi+\theta} E\,d\,\omega t \right]$$

$$\text{OR } V_{oAV} = \frac{\sqrt{2}V_i}{\pi} \left[\cos \theta - \cos \beta + (\pi + \theta - \beta)\sin \theta \right] \qquad (9.75)$$

β can be found in the following manner
    for $\theta < \omega t \le \beta$

$$v = \sqrt{2} \sin \omega t = R\,i_o + \frac{L d i_o}{dt} + E \qquad (9.76)$$
$$i_o \big|_{\omega t=\theta} = i_o \big|_{\omega t=\beta} = 0$$

The general solution is

$$i_o = I_o\, e^{-\frac{\omega t-\theta}{\tan\varphi}} + \frac{\sqrt{2}V_i}{Z} \left[\sin(\omega t - \varphi) - \frac{\sin\theta}{\cos\varphi} \right] \qquad (9.77)$$

where $\tan\varphi = \omega L / R$, $Z = \sqrt{R^2 + \omega^2 L^2}$, $\sin\theta = E / \sqrt{2}V_i$

From the initial condition $i_o \big|_{\omega t=\theta} = 0$

$$I_o = \frac{\sqrt{2}V_i}{Z} \left[\sin(\varphi - \theta) + \sin\theta / \cos\varphi \right] \qquad (9.78)$$

$$\therefore \; i_o = \frac{\sqrt{2}V_i}{Z} \left[\sin(\varphi - \theta)\, e^{\frac{\omega t-\theta}{\tan\varphi}} - \frac{\sin\theta}{\cos\varphi} \left(1 - e^{\frac{-\omega t-\theta}{\tan\varphi}} \right) + \sin(\omega t - \varphi) \right] \qquad (9.79)$$

Putting $i_o \big|_{\omega t=\beta} = 0$ in Equation 9.79.

$$\sin(\beta - \varphi) = \frac{\sin\theta}{\cos\varphi} \left[1 - e^{\frac{\theta-\beta}{\tan\varphi}} \right] - \sin(\varphi - \theta)\, e^{\frac{\theta-\beta}{\tan\varphi}} \qquad (9.80)$$

Form which β can be solved.

Exercise 9.4

1. Fill in the blank(s) with the appropriate word(s).

    i)   The average output voltage of a full wave bridge rectifier and a split supply full wave
          rectifier are _____ provided the input voltages are _____.

ii)   For the same input voltage the bridge rectifier uses _____ the number of diodes used in a split supply rectifier with _____ the PIV rating.

iii)   For continuous conduction, the load impedance of a bridge rectifier should be _____.

iv)   In the _____ conduction mode the output voltage of a bridge rectifier is _____ of load parameters.

Answers: (i) equal, equal; (ii) double, half; (iii) inductive; (iv) continuous, independent.

2. A battery is to be charged using a full bridge single phase uncontrolled rectifier. On full discharge the battery voltage is 10.2 V. and on full charge it is 12.7 volts. The battery internal resistance is 0.1Ω. Find out the input voltage to the rectifier so that the battery charging current under full charge condition is 10% of the charging current under fully discharged condition. Assume continuous conduction under all charging condition and find out the inductance to be connected in series with the battery for this condition.

Answer: Let the rectifier input voltage be V_i and the charging current under fully discharged condition be I.

Then assuming continuous conduction

$$\frac{2\sqrt{2}V_i}{\pi} - 0.1I = 10.2 \quad \text{and} \quad \frac{2\sqrt{2}}{\pi}V_i - 0.01I = 12.7$$

$$\therefore \ 0.09I = 2.5 \text{ V} \ \therefore \ I = 27.78 \text{ Amps and } V_i = 14.415 \text{ volts.}$$

If conduction is continuous at full charge condition it will be continuous for all other charging conditions.

For continuous conduction

$$\frac{2\sin\varphi}{1 - e^{-\pi/\tan\varphi}} \, e^{-\theta/\tan\varphi} = \sin(\varphi - \theta) + \frac{\sin\theta}{\cos\varphi}$$

From given data $\sin\theta = \dfrac{E}{\sqrt{2}V_i} = 0.623, \quad \theta = 38.535°$

From which $\varphi = 86.5°$

$\therefore \ \tan\varphi = \dfrac{\omega L}{R} = 16.35 \ \text{ or } \ \omega L = 1.635 \text{ ohms}$

$\therefore \ L = 5.2 \text{ mH.}$

References

[1]   P.C. Sen, "Power Electronics", Tata McGraw –Hill Publishing Company Limited. 1995

[2]   Muhammad H. Rashid, "Power Electronics, circuits, Devices and applications" Prentice – Hall of India Private Limited, Second Edition, 1994

Module Summary

- A rectifier is a power electronic converter which converts ac voltage or current sources to dc voltage and current.

- In a rectifier, electrical power flows from the ac input to the dc output.

- In many rectifier circuits, power can also flow from the dc side to the ac side, where upon, the rectifier is said to be operating in the "inverter mode".

- Rectifiers can be classified based on the type of device they use, the converter circuit topology, number of phases and the control mechanism.

- All rectifiers produce unwanted harmonies both at the out put and the input. Performance of a rectifier is judged by the relative magnitudes of these harmonies with respect to the desired output.

- For a given input voltage and load, the output voltage (current) of an uncontrolled rectifier can not be varied. However, the output voltage may vary considerably with load.

- Single phase uncontrolled half wave rectifier with resistive or inductive load have low average output voltage, high from factor and poor ripple factor of the output voltage waveform.

- Single phase uncontrolled full wave rectifier have higher average output voltage and improved ripple factor compared to a half wave rectifier with resistive and inductive load.

- With highly inductive load the output voltage waveform of a full wave rectifier may be independent of the load parameters.

- With a capacitive load the output voltage form factor approaches unity with increasing capacitance value for both the half wave and the full wave rectifiers. However, THD of the input current also increases.

- A full wave bridge rectifier generates higher average dc voltage compared to a split supply full wave rectifier. However it also uses more number of diodes.

Practice Problems and Answers

Q1.   What will be the load voltage and current waveform when a single phase half wave uncontrolled rectifier supplies a purely inductive load? Explain your answer with waveforms.

Q2.   The split supply of a single phase full wave rectifier is obtained from a single phase transformer with a single primary and a center tapped secondary. The rectifier supplies a purely resistive load. Assuming the transformer to be ideal find out the, displacement factor, distortion factor and the power factor at the primary side of the transformer.

Q3.   A single phase split supply full wave rectifier is designed to supply an inductive load. The average load current is 20 A, and the ripple current is negligible. Can the same rectifier be used with a capacitive load drawing the same 20 Amps average current? Justify your answer.

Q4.   A 200V, 15 Amps, 1500 rpm separately excited dc motor has an armature resistance of 1 Ω and inductance of 50 mH. The motor is supplied from a single phase full wave bridge rectifier with input voltage of 230 V, 50 HZ. Neglecting all no load losses, find out the no load speed of the machine. Also find out the torque and speed at the boundary between continuous and discontinuous conduction.
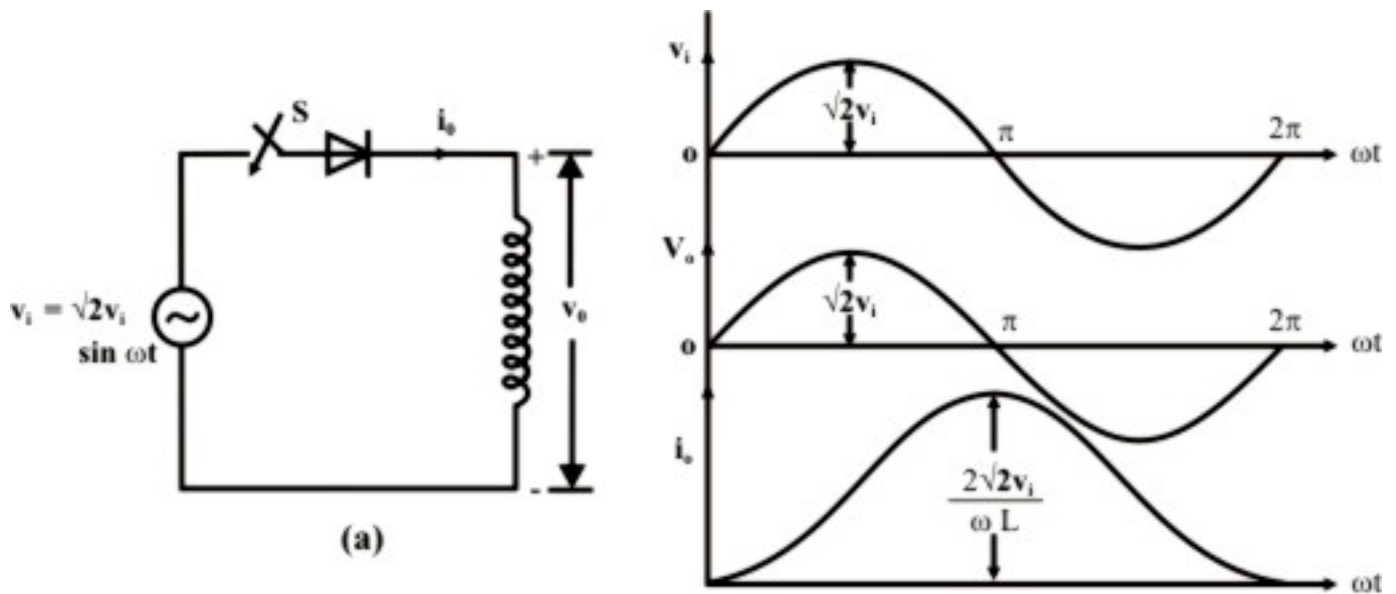
Answer 1

Fig PPM 9.1: Single phase half wave rectifier with purely inductive load

Without loss of generality it can be assumed that S is turned ON at $\omega t = 0$. Since, if it is turned ON anytime after $\omega t = 0$, the volt-sec. across the inductor will dictate that the current through it becomes zero before the next positive going zero crossing of v_i .

In the region $0 \le \omega t < \pi$
D is forward biased and $v_0 = v_i$

$$\therefore \quad L\frac{di_0}{dt} = \sqrt{2}V_i \sin\omega t \qquad i_0(0) = 0$$

$$\text{or} \quad \omega L\frac{di_0}{d\omega t} = \sqrt{2}V_i \sin\omega t \qquad i_0 \big/_{\omega t = 0} = 0$$

$$\therefore \quad i_0 = I_0 - \frac{\sqrt{2}V_i}{\omega L}\cos\omega t \qquad I_0 = \frac{\sqrt{2}V_i}{\omega L}$$

$$\therefore \quad i_0 = \frac{\sqrt{2}V_i}{\omega L}(1 - \cos\omega t)$$

$$\text{at} \quad \omega t = \pi, \qquad i_0 = \frac{2\sqrt{2}V_i}{\omega L} > 0$$

$\therefore$ D conducts beyond $\omega t = \pi$ until i_0 is zero again.
Let the extinction angle be $\omega t = \beta > \pi$.

Then for $0 \le \omega t \le \beta$

$$i_0 = \frac{\sqrt{2}V_i}{\omega L}(1 - \cos\omega t)$$

$$i_0 \big/_{\omega t = \beta} = \frac{\sqrt{2}V_i}{\omega L}(1 - \cos\beta) \quad \text{for } \pi \le \beta \le 2\pi \text{ the only solution is } \beta = 2\pi$$

$$\therefore \quad v_0 = v_i \quad \text{for } 0 \le \omega t \le 2\pi \quad \text{and} \quad i_0 = \frac{\sqrt{2}V_i}{\omega L}(1 - \cos\omega t) \quad 0 \le \omega t \le 2\pi$$
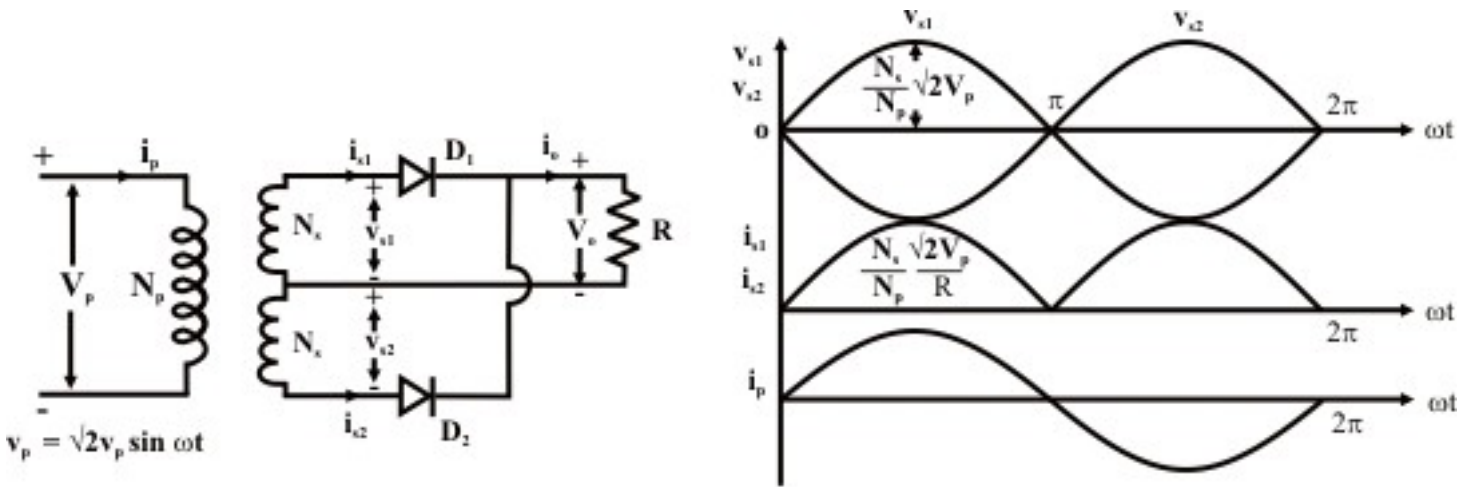
Answer 2

Fig PPM 9.2: Full wave rectifier supplied from Transformer with center taped secondary

Figure shows the secondary voltage and current waveforms of the rectifier.

From the given data

$$v_{S1} = \frac{N_S}{N_P}\sqrt{2}V_P\sin\omega t$$

$$i_{S1} = \frac{N_S}{N_P}\sqrt{2}\frac{V_P}{R}\sin\omega t \qquad \text{for } 0 \le \omega t \le \pi$$

$i_{S1} = 0$ otherwise.

$$v_{S2} = -\frac{N_S}{N_P}\sqrt{2}V_P\sin\omega t$$

$$i_{S2} = -\frac{N_S}{N_P}\sqrt{2}\frac{V_P}{R}\sin\omega t \qquad \text{for } \pi \le \omega t \le 2\pi$$

$i_{S2} = 0$ otherwise

From the MMF balance of an ideal transformer

$$N_P i_P - N_S i_{S1} + N_S i_{S2} = 0$$

$$\text{or} \quad i_P = \frac{N_S}{N_P}(i_{S1} - i_{S2}) = \frac{\sqrt{2}V_P}{\left(\dfrac{N_P}{N_S}\right)^2 R}\sin\omega t$$

$\therefore$ At the input

Displacement factor = Distortion factor = Power factor = 1.0

Answer 3

If the load current is 20A with negligible ripple. The required RMS current rating of the rectifier diode, with reference to Fig 9.6 (b) will be

$$I_{D1RMS} = I_{D2RMS} = \frac{20}{\sqrt{2}} \text{ Amps}.$$

However from Fig 9.7 (b) and Problem 2 of Exercise 9.3 the required RMS current for a capacitive load will be much larger than 20 Amps. Therefore the same rectifier can not be used.

Answer 4

Since all no load losses are neglected the developed power at no load and hence the no load torque will be zero. Therefore, the average armature current will also be zero. However, since a diode rectifier can not conduct instantaneous negative load current, zero average current will imply that the instantaneous value of the armature current at all time will be zero at no load. With reference to Fig 9.10 this condition will require the rectifier diodes to remain reverse biased at all time. Hence at no load

$$E \geq \sqrt{2}V_i$$

However E will not exceed $\sqrt{2}V_i$, since once i_a becomes zero when $E = \sqrt{2}V_i$ there will be no developed torque to accelerate the motor. Hence the motor speed and E will not increase any further.

Thus at no load $\quad E = \sqrt{2}V_i = 325.27$ volts .

Under the rated condition at 1500 rpm

$E_{rated} = 200 - 15 \times 1.0 = 185$ volts.

Now $\quad \dfrac{E}{E_{rated}} = \dfrac{N}{N_{rated}}$

$\therefore \quad N = N_{rated} \times \dfrac{E}{E_{rated}} = 1500 \times \dfrac{325.27}{185} = 2637$ rpm .

At the boundary between the continuous and discontinuous mode of conduction.

$$\frac{2\sin\varphi}{1 - e^{-\pi/\tan\varphi}} e^{-\theta/\tan\varphi} = \sin(\varphi - \theta) + \frac{\sin\theta}{\cos\varphi}$$

or $\quad \dfrac{2\sin\varphi}{1 - e^{-\pi/\tan\varphi}} = \left[\cos\varphi \, \sin(\varphi - \theta) + \sin\theta\right] e^{\theta/\tan\varphi}$

where $\quad \tan\varphi = \dfrac{\omega L}{R} = \dfrac{100\pi \times 50 \times 10^{-3}}{1} = 15.708$

$\cos\varphi = 0.0635 \qquad \varphi = 1.507$ rad. $\qquad$ and $\sin2\varphi = 0.1268$

$\therefore \quad 0.6995 = \left[0.0635 \sin(1.507 - \theta) + \sin\theta\right] e^{0.06366\,\theta}$

from which $\quad \theta = \sin^{-1} \dfrac{E}{\sqrt{2}V_i} = 38.5^{\circ} \qquad \therefore E = 202.48$ V

but E at 1500 RPM = 185 volts.

$\therefore$ Speed at the junction of continuous and discontinuous condition is

$1500 \times \dfrac{202.48}{185} = 1642$ RPM.

Average armature current is $\left(\dfrac{2\sqrt{2}V_i}{\pi}\right)\Big/R = 4.593$ Amps.

$\therefore$ Torque $= \dfrac{4.593}{15}\times100 = 30.62\%$ of rated torque.