SIGNAL FLOW GRAPHS

An alternate to block diagram is the signal flow graph due to S. J. Mason. A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations. Each signal flow graph consists of a network in which nodes are connected by directed branches. Each node represents a system variable, and each branch acts as a signal multiplier. The signal flows in the direction indicated by the arrow.

Definitions:

Node: A node is a point representing a variable or signal.

Branch: A branch is a directed line segment joining two nodes.

Transmittance: It is the gain between two nodes.

Input node: A node that has only outgoing branche(s). It is also, called as source and corresponds to independent variable.

Output node: A node that has only incoming branches. This is also called as sink and corresponds to dependent variable.

Path: A path is a traversal of connected branches in the direction of branch arrow.

Loop: A loop is a closed path.

Self loop: It is a feedback loop consisting of single branch.

Loop gain: The loop gain is the product of branch transmittances of the loop.

Nontouching loops: Loops that do not posses a common node.

Forward path: A path from source to sink without traversing an node more than once.

Feedback path: A path which originates and terminates at the same node.

Forward path gain: Product of branch transmittances of a forward path.

Properties of Signal Flow Graphs:
1) Signal flow applies only to linear systems.
2) The equations based on which a signal flow graph is drawn must be algebraic equations in the form of effects as a function of causes. Nodes are used to represent variables. Normally the nodes are arranged left to right, following a succession of causes and effects through the system.
3) Signals travel along the branches only in the direction described by the arrows of the branches.
4) The branch directing from node X_k to X_j represents dependence of the variable X_j on X_k but not the reverse.
5) The signal traveling along the branch X_k and X_j is multiplied by branch gain a_{kj} and signal $a_{kj}X_k$ is delivered at node X_j.

Guidelines to Construct the Signal Flow Graphs:

The signal flow graph of a system is constructed from its describing equations, or by direct reference to block diagram of the system. Each variable of the block diagram becomes a node and each block becomes a branch. The general procedure is

1) Arrange the input to output nodes from left to right.
2) Connect the nodes by appropriate branches.
3) If the desired output node has outgoing branches, add a dummy node and a unity gain branch.
4) Rearrange the nodes and/or loops in the graph to achieve pictorial clarity.

Signal Flow Graph Algebra

Addtion rule

The value of the variable designated by a node is equal to the sum of all signals entering the node.

Transmission rule

The value of the variable designated by a node is transmitted on every branch leaving the node.

Multiplication rule

A cascaded connection of n-1 branches with transmission functions can be replaced by a single branch with new transmission function equal to the product of the old ones.

Masons Gain Formula

The relationship between an input variable and an output variable of a signal flow graph is given by the net gain between input and output nodes and is known as overall gain of the system. Masons gain formula is used to obtain the over all gain (transfer function) of signal flow graphs.

Gain P is given by

$$P = \frac{1}{\Delta} \sum_{k} P_k \Delta_k$$

Where, P_k is gain of kth forward path,
Δ is determinant of graph

Δ=1-(sum of all individual loop gains)+(sum of gain products of all possible combinations of two nontouching loops – sum of gain products of all possible combination of three nontouching loops) + $\cdots$

Δ_k is cofactor of kth forward path determinant of graph with loops touching k^{th} forward path. It is obtained from Δ by removing the loops touching the path P_k.