RESONANCE

Resonance is a phenomenon in which an oscillator responds most strongly to a driving force that matches its own natural frequency of vibration. For example, suppose a child is on a playground swing with a natural frequency of 1 Hz. That is, if you pull the child away from equilibrium, release her, and then stop doing anything for a while, she'll oscillate at 1 Hz. If there was no friction, as we assumed in section 2.5, then the sum of her gravitational and kinetic energy would remain constant, and the amplitude would be exactly the same from one oscillation to the next. However, friction is going to convert these forms of energy into heat, so her oscillations would gradually die out. To keep this from happening, you might give her a push once per cycle, i.e., the frequency of your pushes would be 1 Hz, which is the same as the swing's natural frequency. As long as you stay in rhythm, the swing responds quite well. If you start the swing from rest, and then give pushes at 1 Hz, the swing's amplitude rapidly builds up, as in figure a, until after a while it reaches a steady state in which friction removes just as much energy as you put in over the course of one cycle.

self-check:

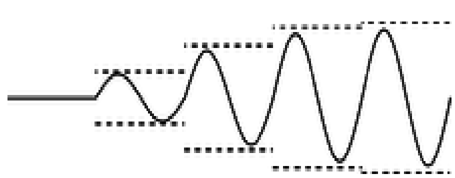

Figure a: An x-versus-t graph for a swing pushed at resonance.

In figure a, compare the amplitude of the cycle immediately following the first push to the amplitude after the second. Compare the energies as well. (answer in the back of the PDF version of the book)

What will happen if you try pushing at 2 Hz? Your first push puts in some momentum, p, but your second push happens after only half a cycle, when the swing is coming right back at you, with momentum −p! The momentum transfer from the second push is exactly enough to stop the swing. The result is a very weak, and not very sinusoidal, motion, b.

Figure b: A swing pushed at twice its resonant frequency.

Making the math easy

This is a simple and physically transparent example of resonance: the swing responds most strongly if you match its natural rhythm. However, it has some characteristics that are mathematically ugly and possibly unrealistic. The quick, hard pushes are known as *impulse* forces, c, and they lead to an x-t graph that has non differentiable kinks.

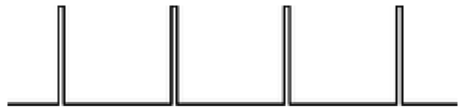

Figure c: The F-versus-t graph for an impulsive driving force.

Impulsive forces like this are not only badly behaved mathematically, they are usually undesirable in practical terms. In a car engine, for example, the engineers work very hard to make the force on the pistons change smoothly, to avoid excessive vibration. Throughout the rest of this section, we'll assume a driving force that is sinusoidal, d, i.e., one whose F-t graph is either a sine function or a function that differs from a sine wave in phase, such as a cosine. The force is positive for half of each cycle and negative for the other half, i.e., there is both pushing and pulling. Sinusoidal functions have many nice mathematical characteristics (we can differentiate and integrate them, and the sum of sinusoidal functions that have the same frequency is a sinusoidal function), and they are also used in many practical situations. For instance, my garage door zapper sends out a sinusoidal radio wave, and the receiver is tuned to resonance with it.

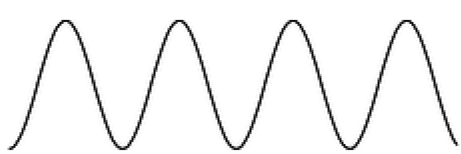

Figure d: A sinusoidal driving force.

A second mathematical issue that I glossed over in the swing example was how friction behaves. In section 3.2.4, about forces between solids, the empirical equation for kinetic friction was independent of velocity. Fluid friction, on the other hand, is velocity-dependent. For a child on a swing, fluid friction is the most important form of friction, and is approximately proportional to v2. In still other situations, e.g., with a low-density gas or friction between solid surfaces that have been lubricated with a fluid such as oil, we may find that the frictional force has some other dependence on velocity, perhaps being proportional to v, or having some other complicated velocity dependence that can't even be expressed with a simple equation. It would be extremely complicated to have to treat all of these different possibilities in complete generality, so for the rest of this section, we'll assume friction proportional to velocity

$$F=-bv,$$

simply because the resulting equations happen to be the easiest to solve. Even when the friction doesn't behave in exactly this way, many of our results may still be at least qualitatively correct.

Source: http://physwiki.ucdavis.edu/Fundamentals/03._Conservation_of_Momentum/3.3_Resonance