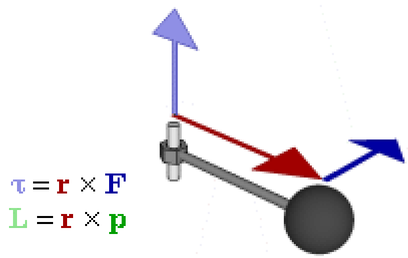
Quantum Mechanics_ moment of force



Relationship between Force **F**, torque **τ**, linear momentum **p**, and Angular momentum **L** in a system which has rotation constrained in one plane only (forces and moments due to gravityand Friction not considered).

Torque, **moment** or **moment of force** (see the terminology below), is the tendency of aForce to rotate an object about an axis,[1]fulcrum, or pivot. Just as a force is a push or a pull, a torque can be thought of as a twist to an object. Mathematically, torque is defined as thecross product of the lever-arm distance vectorand the Force vector, which tends to produce rotation.

Loosely speaking, torque is a measure of the turning force on an object such as a bolt or aflywheel. For example, pushing or pulling the handle of a wrench connected to a nut or bolt produces a torque (turning force) that loosens or tightens the nut or bolt. The symbol for torque is typically τ, the Greek letter *tau*. When it is called moment, it is commonly denoted *M*.

The magnitude of torque depends on three quantities: the Force applied, the length of the*lever arm*[2] connecting the axis to the point of force application, and the angle between the force vector and the lever arm. In symbols:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$
$$\tau = \|\mathbf{r}\| \, \|\mathbf{F}\| \sin\theta$$

where

- **τ** is the torque vector and τ is the magnitude of the torque,
- **r** is the displacement vector (a vector from the point from which torque is measured to the point where force is applied),
- **F** is the force vector,

$\times$ denotes the cross product,
θ is the angle between the force vector and the lever arm vector.

The length of the lever arm is particularly important; choosing this length appropriately lies behind the operation of levers, pulleys,gears, and most other simple machinesinvolving a mechanical advantage.

The SI unit for torque is the newton metre(N·m). For more on the units of torque, seebelow.

Terminology

See also: Couple (mechanics)

This article follows US physics terminology by using the word *torque*. In the UK and in US mechanical engineering,[3] this is called *moment of force*[4], usually shortened to *moment*. In US mechanical engineering, the term *torque* means "the resultant moment of a Couple,"[5] and (unlike in US physics), the terms *torque* and*moment* are not interchangeable.

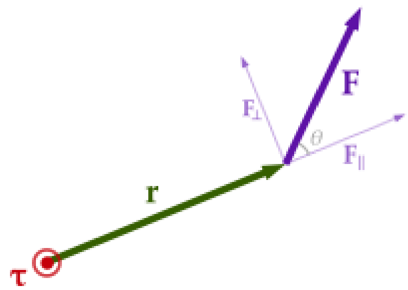
Torque is defined mathematically as the rate of change of Angular momentum of an object. The definition of torque states that one or both of the Angular velocityor the Moment of inertia of an object are changing. *Moment* is the general term used for the tendency of one or more applied forces to rotate an object about an axis, but not necessarily to change the angular momentum of the object (the concept which is called *torque* in physics).[5] For example, a rotational force applied to a shaft causing acceleration, such as a drill bit accelerating from rest, results in a moment called a *torque*. By contrast, a lateral force on a beam produces a moment (called a bending moment), but since the angular momentum of the beam is not changing, this bending moment is not called a *torque*. Similarly with any force couple on an object that has no change to its angular momentum, such moment is also not called a *torque*.

This article follows the US physics terminology by calling all moments by the term *torque*, whether or not they cause the angular momentum of an object to change.

History

The concept of torque, also called Moment or Couple, originated with the studies of Archimedes on levers. The rotational analogues of Force, Mass, andAcceleration are torque, Moment of inertia and Angular acceleration, respectively.

Definition and relation to angular momentum



A particle is located at position **r** relative to its axis of rotation. When a force **F** is applied to the particle, only the perpendicular component $\mathbf{F}_{\perp}$ produces a torque. This torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ has magnitude $\tau = |\mathbf{r}|\,|\mathbf{F}_{\perp}| = |\mathbf{r}|\,|\mathbf{F}|\sin\theta$ and is directed outward from the page.

A force applied at a right angle to a lever multiplied by its distance from thelever's fulcrum (the length of the lever arm) is its torque. A force of three newtonsapplied two metres from the fulcrum, for example, exerts the same torque as a force of one newton applied six metres from the fulcrum. The direction of the torque can be determined by using the right hand grip rule: if the fingers of the right hand are curled from the direction of the lever arm to the direction of the force, then the thumb points in the direction of the torque.[6]

More generally, the torque on a particle (which has the position **r** in some reference frame) can be defined as the cross product:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F},$$

where **r** is the particle's position vector relative to the fulcrum, and **F** is the force acting on the particle. The magnitude τ of the torque is given by

$$\tau = rF\sin\theta,$$

where r is the distance from the axis of rotation to the particle, F is the magnitude of the force applied, and θ is the angle between the position and force vectors. Alternatively,

$$\tau = rF_{\perp},$$

where $F_{\perp}$ is the amount of force directed perpendicularly to the position of the particle. Any force directed parallel to the particle's position vector does not produce a torque.[7]

It follows from the properties of the cross product that the torque vector is perpendicular to both the position and force vectors. It points along the axis of the rotation that this torque would initiate, starting from rest, and its direction is determined by the right-hand rule.[7]

The unbalanced torque on a body along axis of rotation determines the rate of change of the body's Angular momentum,

$$\boldsymbol{\tau} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}$$

where **L** is the angular momentum vector and t is time. If multiple torques are acting on the body, it is instead the net torque which determines the rate of change of the angular momentum:

$$\boldsymbol{\tau}_1 + \cdots + \boldsymbol{\tau}_n = \boldsymbol{\tau}_{\mathrm{net}} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}.$$

For rotation about a fixed axis,

$$\mathbf{L} = I\boldsymbol{\omega},$$

where I is the Moment of inertia and **ω** is the Angular velocity. It follows that

$$\boldsymbol{\tau}_{\mathrm{net}} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \frac{\mathrm{d}(I\boldsymbol{\omega})}{\mathrm{d}t} = I\frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} = I\boldsymbol{\alpha},$$

where **α** is the Angular acceleration of the body, measured in rad/s^2. This equation has the limitation that the torque equation is to be only written about instantaneous axis of rotation or center of mass for any type of motion - either motion is pure translation, pure rotation or mixed motion. *I* = Moment of inertia about point about which torque is written (either about instantaneous axis of rotation or center of mass only). If body is in translatory equilibrium then the torque equation is same about all points in the plane of motion.

A torque is not necessarily limited to rotation around a fixed axis, however. It may change the magnitude and/or direction of the angular momentum vector, depending on the angle between the velocity vector and the non-radial component of the force vector, as viewed in the pivot's frame of reference. A net torque on a spinning body therefore may result in a precession without necessarily causing a change in spin rate.

Proof of the equivalence of definitions

The definition of angular momentum for a single particle is:

$$\mathbf{L} = \mathbf{r} \times \boldsymbol{p}$$

where "×" indicates the vector cross product, **p** is the particle's linear momentum, and **r** is the displacement vector from the origin (the origin is assumed to be a fixed location anywhere in space). The time-derivative of this is:

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \times \frac{d\boldsymbol{p}}{dt} + \frac{d\mathbf{r}}{dt} \times \boldsymbol{p}.$$

This result can easily be proven by splitting the vectors into components and applying the product rule. Now using the definition of force $\mathbf{F} = \frac{d\boldsymbol{p}}{dt}$ (whether or not mass is constant) and the definition of velocity $\frac{d\mathbf{r}}{dt} = \mathbf{v}$

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \times \mathbf{F} + \mathbf{v} \times \boldsymbol{p}.$$

The cross product of momentum $\boldsymbol{p}$ with its associated velocity $\mathbf{v}$ is zero because velocity and momentum are parallel, so the second term vanishes.

By definition, torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$. Therefore torque on a particle is *equal* to the first derivative of its angular momentum with respect to time.

If multiple forces are applied, Newton's second law instead reads $\mathbf{F}_{net} = m\mathbf{a}$, *and it follows that*

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \times \mathbf{F}_{\text{net}} = \boldsymbol{\tau}_{\text{net}}.$$

This is a general proof.

Units

Torque has dimensions of force times distance. Official SI literature suggests using the unit *newton metre* (N·m) or the unit *joule per radian*.[8] The unit *newton metre* is properly denoted N·m or N m.[9] This avoids ambiguity with mN, millinewtons.

The SI unit for Energy or work is the joule. It is dimensionally equivalent to a force of one newton acting over a distance of one metre, but it is not used for torque. Energy and torque are entirely different concepts, so the practice of using different unit names (i.e., reserving newton metres for torque and using onlyjoules for energy) helps avoid mistakes and misunderstandings.[8] The dimensional equivalence of these units, of course, is not simply a coincidence: A torque of 1 N·m applied through a full revolution will require an Energy of exactly 2π joules. Mathematically,

$$E = \tau\theta$$

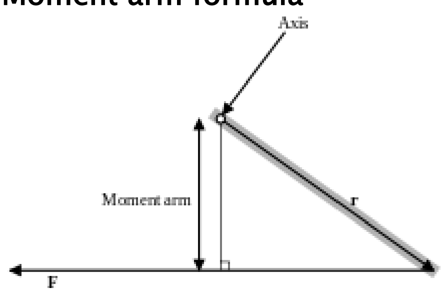
where E is the energy, τ is magnitude of the torque, and θ is the angle moved (inradians). This equation motivates the alternate unit name *joules per radian*.[8]

In Imperial units, "pound-force-feet" (lb·ft), "foot-pounds-force", "inch-pounds-force", "ounce-force-inches" (oz·in) are used, and other non-SI units of torque includes "metre-kilograms-force". For all these units, the word "force" is often left out,[10] for example abbreviating "pound-force-foot" to simply "pound-foot" (in this case, it would be implicit that the "pound" is pound-force and not pound-mass). This is an example of the confusion caused by the use of traditional units that may be avoided with SI units because of the careful distinction in SI between force (in newtons) and mass (in kilograms).

Sometimes one may see torque given units that do not dimensionally make sense. For example: gram centimetre. In these units, "gram" should be understood as the force given by the weight of 1 gram at the surface of the earth, i.e., 0.00980665 N. The surface of the earth is understood to have a standard acceleration of gravity(9.80665 m/s^2).

Special cases and other facts

Moment arm formula



Moment arm diagram

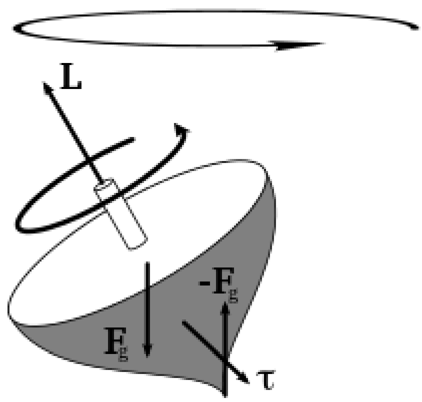
A very useful special case, often given as the definition of torque in fields other than physics, is as follows:

$$|\tau| = (\text{moment arm})(\text{force}).$$

The construction of the "moment arm" is shown in the figure to the right, along with the vectors **r** and **F** mentioned above. The problem with this definition is that it does not give the direction of the torque but only the magnitude, and hence it is difficult to use in three-dimensional cases. If the force is perpendicular to the displacement vector **r**, the moment arm will be equal to the distance to the centre, and torque will be a maximum for the given force. The equation for the magnitude of a torque, arising from a perpendicular force:

$$|\tau| = (\text{distance to centre})(\text{force}).$$

For example, if a person places a force of 10 N at the terminal end of a wrench which is 0.5 m long (or a force of 10 N exactly 0.5 m from the twist point of a wrench of any length), the torque will be 5 N-m – assuming that the person moves the wrench by applying force in the plane of movement of and perpendicular to the wrench.



The torque caused by the two opposing forces $\mathbf{F}_g$ and $-\mathbf{F}_g$ causes a change in the angular momentum **L** in the direction of that torque. This causes the top to process.

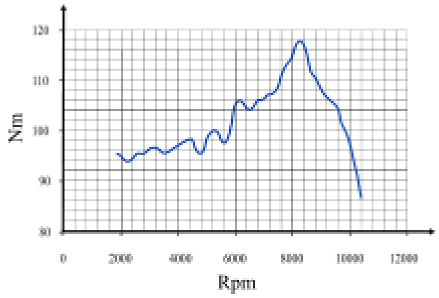
Static equilibrium

For an object to be in static equilibrium, not only must the sum of the forces be zero, but also the sum of the torques (moments) about any point. For a two-dimensional situation with horizontal and vertical forces, the sum of the forces requirement is two equations: $\Sigma H = 0$ and $\Sigma V = 0$, and the torque a third equation: $\Sigma\tau = 0$. That is, to solve statically determinate equilibrium problems in two-dimensions, three equations are used.

Net force versus torque

When the net force on the system is zero, the torque measured from any point in space is the same. For example, the torque on a current-carrying loop in a uniform magnetic field is the same regardless of your point of reference. If the net force $\mathbf{F}$ is not zero, and $\boldsymbol{\tau}_1$ is the torque measured from $\mathbf{r}_1$, then the torque measured from $\mathbf{r}_2$ is ... $\boldsymbol{\tau}_2 = \boldsymbol{\tau}_1 + (\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{F}$

Machine torque



Torque curve of a motorcycle ("BMW K 1200 R 2005"). The horizontal axis is the speed (in rpm) that the crankshaft is turning, and the vertical axis is the torque (in Newton metres) that the engine is capable of providing at that speed.

Torque is part of the basic specification of an engine: the power output of an engine is expressed as its torque multiplied by its rotational speed of the axis.Internal-combustion engines produce useful torque only over a limited range of rotational speeds (typically from around 1,000–6,000 rpm for a small car). The varying torque output over that range can be measured with a dynamometer, and shown as a torque curve.

Steam engines and electric motors tend to produce maximum torque close to zero rpm, with the torque diminishing as rotational speed rises (due to increasing friction and other constraints). Reciprocating steam engines can start heavy loads from zero RPM without a clutch.

Relationship between torque, power, and energy

If a Force is allowed to act through a distance, it is doing mechanical work. Similarly, if torque is allowed to act through a rotational distance, it is doing work. Mathematically, for rotation about a fixed axis through the center of mass,

$$W = \int_{\theta_1}^{\theta_2} \tau \; \mathrm{d}\theta,$$

where W is work, τ is torque, and θ_1 and θ_2 represent (respectively) the initial and final angular positions of the body.[11] It follows from the work-energy theoremthat W also represents the change in the rotational kinetic energy E_r of the body, given by

$$E_\mathrm{r} = \tfrac{1}{2}I\omega^2,$$

where I is the Moment of inertia of the body and ω is its angular speed.[11]
power is the work per unit Time, given by

$$P = \boldsymbol{\tau} \cdot \boldsymbol{\omega},$$

where P is power, **τ** is torque, **ω** is the Angular velocity, and · represents the scalar product.

Mathematically, the equation may be rearranged to compute torque for a given power output. Note that the power injected by the torque depends only on the instantaneous angular speed – not on whether the angular speed increases, decreases, or remains constant while the torque is being applied (this is equivalent to the linear case where the power injected by a force depends only on the instantaneous speed – not on the resulting acceleration, if any).

In practice, this relationship can be observed in power stations which are connected to a large electrical power grid. In such an arrangement, the generator's angular speed is fixed by the grid's frequency, and the power output of the plant is determined by the torque applied to the generator's axis of rotation.

Consistent units must be used. For metric SI units power is watts, torque isNewton metres and angular speed is radians per second (not rpm and not revolutions per second).

Also, the unit newton metre is dimensionally equivalent to the joule, which is the unit of energy. However, in the case of torque, the unit is assigned to a vector, whereas for Energy, it is assigned to a scalar.

Conversion to other units

A conversion factor may be necessary when using different units of power, torque, or angular speed. For example, if Rotational speed (revolutions per time) is used in place of angular speed (radians per time), we multiply by a factor of 2π radians per revolution. In the following formulas, *P* is power, τ is torque and ω is rotational speed.

$$P = \tau \times 2\pi \times \omega$$

Adding units:

$$P/\mathrm{W} = \tau/(\mathrm{N \cdot m}) \times 2\pi(\mathrm{rad/rev}) \times \omega/(\mathrm{rev/sec})$$

Dividing on the left by 60 seconds per minute gives us the following.

$$P/\mathrm{W} = \frac{\tau/(\mathrm{N \cdot m}) \times 2\pi(\mathrm{rad/rev}) \times \omega/(\mathrm{rpm})}{60}$$

where rotational speed is in revolutions per minute (rpm).

Some people (e.g. American automotive engineers) use horsepower (imperial mechanical) for power, foot-pounds (lbf·ft) for torque and rpm for rotational speed. This results in the formula changing to:

$$P/\mathrm{hp} = \frac{\tau/(\mathrm{lbf \cdot ft}) \times 2\pi(\mathrm{rad/rev}) \times \omega/\mathrm{rpm}}{33{,}000}.$$

The constant below (in foot pounds per minute) changes with the definition of the horsepower; for example, using metric horsepower, it becomes approximately 32,550. Use of other units (e.g. BTU per hour for power) would require a different custom conversion factor.

Derivation

For a rotating object, the *linear distance* covered at the circumference of rotation is the product of the radius with the angle covered. That is: linear distance = radius ×

angular distance. And by definition, linear distance = linear speed × time = radius × angular speed × time.

By the definition of torque: torque = radius × force. We can rearrange this to determine force = torque ÷ radius. These two values can be substituted into the definition of power:

$$\text{power} = \frac{\text{force} \times \text{linear distance}}{\text{time}} = \frac{\left(\frac{\text{torque}}{r}\right) \times (r \times \text{angular speed} \times t)}{t} = \text{torque} \times \text{angular speed}.$$

The radius *r* and time *t* have dropped out of the equation. However, angular speed must be in radians, by the assumed direct relationship between linear speed and angular speed at the beginning of the derivation. If the rotational speed is measured in revolutions per unit of time, the linear speed and distance are increased proportionately by 2π in the above derivation to give:

$$\text{power} = \text{torque} \times 2\pi \times \text{rotational speed}.$$

If torque is in newton metres and rotational speed in revolutions per second, the above equation gives power in newton metres per second or watts. If Imperial units are used, and if torque is in pounds-force feet and rotational speed in revolutions per minute, the above equation gives power in foot pounds-force per minute. The horsepower form of the equation is then derived by applying the conversion factor 33,000 ft·lbf/min per horsepower:

$$\text{power} = \text{torque} \times 2\pi \times \text{rotational speed} \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{min}} \times \frac{\text{horsepower}}{33,000 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{min}}} \approx \frac{\text{torque} \times \text{RPM}}{5,252}$$

because $5252.113122 \approx \frac{33,000}{2\pi}.$

Principle of moments

The Principle of Moments, also known as Varignon's theorem (not to be confused with the geometrical theorem of the same name) states that the sum of torques due to several forces applied to *a single* point is equal to the torque due to the sum (resultant) of the forces. Mathematically, this follows from:

$$(\mathbf{r} \times \mathbf{F}_1) + (\mathbf{r} \times \mathbf{F}_2) + \cdots = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \cdots).$$

Torque multiplier

A torque multiplier is a gear box with reduction ratios greater than 1. The given torque at the input gets multiplied as per the reduction ratio and transmitted to the output, thereby achieving greater torque, but with reduced rotational speed.

References

1. ^ Serway, R. A. and Jewett, Jr. J. W. (2003). *Physics for Scientists and Engineers*. 6th Ed. Brooks Cole. ISBN 0-534-40842-7.
2. ^ Tipler, Paul (2004). *Physics for Scientists and Engineers: Mechanics, Oscillations and Waves, Thermodynamics (5th ed.)*. W. H. Freeman. ISBN 0-7167-0809-4.
3. ^ *Physics for Engineering* by Hendricks, Subramony, and Van Blerk, Chinappi page 148, Web link
4. ^ SI brochure
5. ^ *a b* *Dynamics, Theory and Applications* by T.R. Kane and D.A. Levinson, 1985, pp. 90-99: Free download
6. ^ "Right Hand Rule for Torque". Retrieved 2007-09-08.
7. ^ *a b* Halliday, David; Resnick, Robert (1970). *Fundamentals of Physics*. John Wiley & Sons, Inc. pp. 184-85.
8. ^ *a b c* From the official SI website: "...For example, the quantity torque may be thought of as the cross product of force and distance, suggesting the unit newton metre, or it may be thought of as energy per angle, suggesting the unit joule per radian."
9. ^ "SI brochure Ed. 8, Section 5.1". Bureau International des Poids et Mesures. 2006. Retrieved 2007-04-01.
10. ^ See, for example: "CNC Cookbook: Dictionary: N-Code to PWM". Retrieved 2008-12-17.
11. ^ *a b* Kleppner, Daniel; Kolenkow, Robert (1973). *An Introduction to Mechanics*. McGraw-Hill. pp. 267-68.

External links

- Power and Torque Explained A clear explanation of the relationship between Power and Torque, and how they relate to engine performance.
- "Horsepower and Torque" An article showing how power, torque, and gearing affect a vehicle's performance.
- "Torque vs. Horsepower: Yet Another Argument" An automotive perspective
- a discussion of torque and angular momentum in an online textbook
- *Torque and Angular Momentum in Circular Motion* on Project PHYSNET.
- An interactive simulation of torque
- Torque Unit Converter