GLOBAL CHAOS SYNCHRONIZATION OF UNCERTAIN SPROTT J AND K SYSTEMS BY ADAPTIVE CONTROL

Sundarapandian Vaidyanathan[1]

[1]Research and Development Centre, Vel Tech Dr. RR & Dr. SR Technical University
Avadi, Chennai-600 062, Tamil Nadu, INDIA

sundarvtu@gmail.com

ABSTRACT

In this paper, we apply adaptive control method to derive new results for the global chaos synchronization of identical Sprott J systems (Sprott, 1994), identical Sprott K systems (Sprott, 1994) and non-identical Sprott J and K systems. Our results are derived for the general case when the parameters of both master and slave systems are unknown and adaptive synchronizing schemes have been derived using the estimates of parameters for both master and slave systems. Our adaptive synchronization schemes derived in this paper are established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the adaptive control method is very effective and convenient to synchronize identical and non-identical Sprott J and K chaotic systems. Numerical simulations are presented to validate and illustrate the effectiveness of the proposed adaptive synchronization schemes for the uncertain Sprott J and K chaotic systems addressed in this paper.

KEYWORDS

Adaptive Control, Chaos, Synchronization, Sprott J System; Sprott K System.

1. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. The sensitive nature of chaotic systems is commonly called as the *butterfly effect* [1].
The first chaotic system was experimentally discovered by Lorenz ([2], 1963), when he was studying weather patterns. Since then, chaos has been extensively interesting study area for many scientists and many chaotic systems were introduced by them such as Rössler system (Rössler, [3], 1976), Chen system (Chen and Ueta, [4], 1999), Lü system (Lü and Chen, [5], 2002), Liu system (Liu *et al.,* [6], 2004), etc.

Synchronization of chaos is a phenomenon that may occur when two or more chaotic oscillators are coupled or one chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect, which causes the exponential divergence of two identical chaotic systems started with nearly the same initial conditions, having two chaotic systems evolving in synchrony is a challenging research problem. It has been found that synchronization of chaos has important applications in engineering such are secure communication, data encryption, etc.

Since the seminal work by Pecora and Carroll ([7], 1990), chaos synchronization problem has been studied extensively and intensively in the literature [8-34]. Chaos theory has been applied to a variety of fields such as physical systems [8], chemical systems [9], ecological systems [10], secure communications [11-12], etc.

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In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

In the last two decades, various schemes have been successfully applied for chaos synchronization such as PC method [7], OGY method [13], active control method [14-20], adaptive control method [21-25], time-delay feedback method [26], backstepping design method [27-29], sampled-data feedback method [30], sliding mode control method [31-34], etc.

In this paper, we apply adaptive control method to derive new results for the global chaos synchronization of identical Sprott J systems ([35], 1994), identical Sprott K systems ([35], 1994) and non-identical Sprott J and K systems. We assume that the parameters of the master and slave systems are unknown and we devise adaptive synchronizing schemes using the estimates of the parameters for both master and slave systems.

This paper has been organized as follows. In Section 2, we give a description of Sprott J and K chaotic systems. In Section 3, we discuss the adaptive synchronization of identical Sprott J systems. In Section 4, we discuss the adaptive synchronization of identical Sprott K systems. In Section 5, we discuss the adaptive synchronization of Sprott J and K systems. In Section 6, we summarize the main results obtained in this paper.

2. SYSTEMS DESCRIPTION

The Sprott J system ([35], 1994) is described by

$$\begin{aligned} \dot{x}_1 &= ax_3 \\ \dot{x}_2 &= -bx_2 + x_3 \\ \dot{x}_3 &= -cx_1 + x_2 + x_2^2 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the state variables and a, b, c are positive, constant parameters of the system.

The system (1) is chaotic when the parameter values are taken as $a = 2,$ $b = 2$ and $c = 1.$

The state orbits of the Sprott J chaotic system (1) are shown in Figure 1.

The Sprott K system ([35], 1994) is described by

$$\begin{aligned} \dot{x}_1 &= x_1x_2 - \alpha x_3 \\ \dot{x}_2 &= x_1 - \beta x_2 \\ \dot{x}_3 &= x_1 + \gamma x_3 \end{aligned} \tag{2}$$

where x_1, x_2, x_3 are the state variables and α, β, γ are positive, constant parameters of the system.

The system (2) is chaotic when the parameter values are taken as $\alpha = 1,$ $\beta = 1$ and $\gamma = 0.3.$

The state orbits of the Sprott K chaotic system (2) are shown in Figure 2.

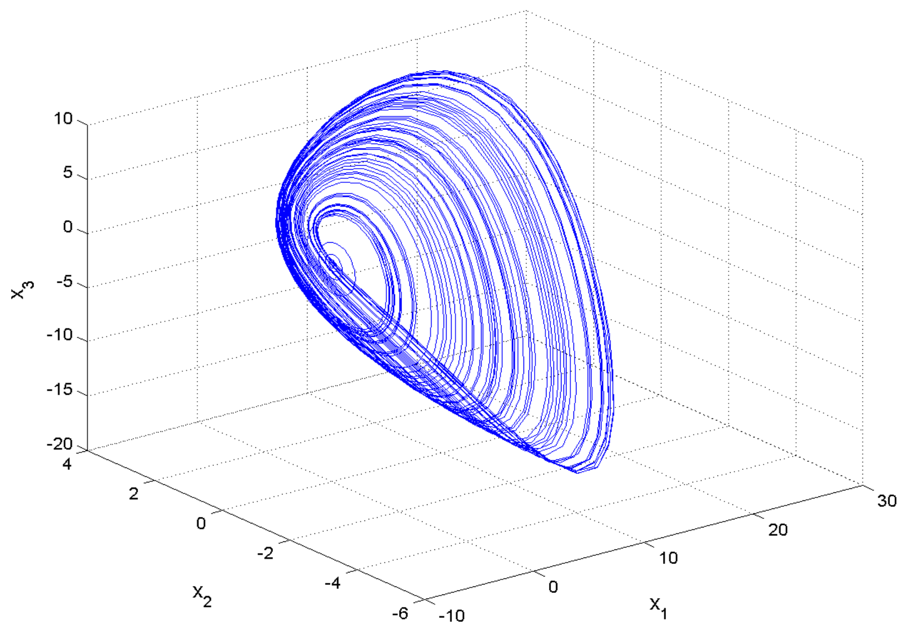

Figure 1. State Orbits of the Sprott J Chaotic System

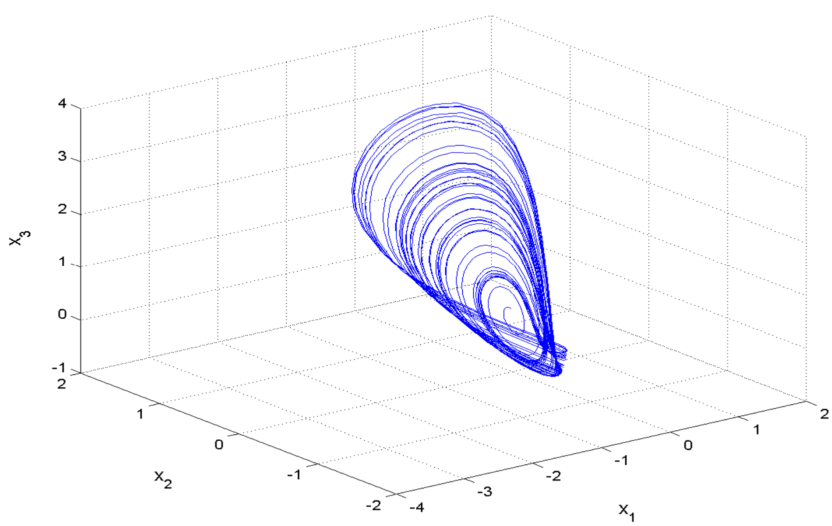

Figure 2. State Orbits of the Sprott K Chaotic System

3. ADAPTIVE SYNCHRONIZATION OF IDENTICAL SPROTT J SYSTEMS

3.1 Theoretical Results

In this section, we discuss the adaptive synchronization of identical Sprott J systems ([35], 1994), where the parameters of the master and slave systems are unknown.

As the master system, we consider the Sprott J dynamics described by

$$\begin{aligned} \dot{x}_1 &= ax_3 \\ \dot{x}_2 &= -bx_2 + x_3 \\ \dot{x}_3 &= -cx_1 + x_2 + x_2^2 \end{aligned} \tag{3}$$

where x_1, x_2, x_3 are the states and a, b, c are unknown real constant parameters of the system.

As the slave system, we consider the controlled Sprott J dynamics described by

$$\begin{aligned} \dot{y}_1 &= ay_3 + u_1 \\ \dot{y}_2 &= -by_2 + y_3 + u_2 \\ \dot{y}_3 &= -cy_1 + y_2 + y_2^2 + u_3 \end{aligned} \tag{4}$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are the nonlinear controllers to be designed.

The chaos synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3) \tag{5}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= ae_3 + u_1 \\ \dot{e}_2 &= -be_2 + e_3 + u_2 \\ \dot{e}_3 &= -ce_1 + e_2 + y_2^2 - x_2^2 + u_3 \end{aligned} \tag{6}$$

Let us now define the adaptive control functions

$$\begin{aligned} u_1(t) &= -\hat{a}e_3 - k_1 e_1 \\ u_2(t) &= \hat{b}e_2 - e_3 - k_2 e_2 \\ u_3(t) &= \hat{c}e_1 - e_2 - y_2^2 + x_2^2 - k_3 e_3 \end{aligned} \tag{7}$$

where $\hat{a}, \hat{b}$ and $\hat{c}$ are estimates of a, b and c, respectively, and $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting (7) into (6), the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= (a-\hat{a})e_3 - k_1 e_1 \\ \dot{e}_2 &= -(b-\hat{b})e_2 - k_2 e_2 \\ \dot{e}_3 &= -(c-\hat{c})e_1 - k_3 e_3\end{aligned} \tag{8}$$

Let us now define the parameter estimation errors as

$$\begin{aligned}e_a &= a-\hat{a} \\ e_b &= b-\hat{b} \\ e_c &= c-\hat{c}\end{aligned} \tag{9}$$

Substituting (9) into (8), we obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= e_a e_3 - k_1 e_1 \\ \dot{e}_2 &= -e_b e_2 - k_2 e_2 \\ \dot{e}_3 &= -e_c e_1 - k_3 e_3\end{aligned} \tag{10}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used.

We consider the quadratic Lyapunov function defined by

$$V(e_1,e_2,e_3,e_a,e_b,e_c) = \frac{1}{2}\left(e_1^2+e_2^2+e_3^2+e_a^2+e_b^2+e_c^2\right) \tag{11}$$

which is a positive definite function on R^6.

We also note that

$$\dot{e}_a = -\dot{\hat{a}},\ \dot{e}_b = -\dot{\hat{b}} \quad \text{and} \quad \dot{e}_c = -\dot{\hat{c}} \tag{12}$$

Differentiating (11) along the trajectories of (10) and using (12), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[e_3 x_1 - \dot{\hat{a}}\right] + e_b\left[-e_2 x_2 - \dot{\hat{b}}\right] + e_c\left[-e_1 x_3 - \dot{\hat{c}}\right] \tag{13}$$

In view of Eq. (13), the estimated parameters are updated by the following law:

$$\begin{aligned}\dot{\hat{a}} &= e_3 x_1 + k_4 e_a \\ \dot{\hat{b}} &= -e_2 x_2 + k_5 e_b \\ \dot{\hat{c}} &= -e_1 x_3 + k_6 e_c\end{aligned} \tag{14}$$

where k_4, k_5 and k_6 are positive constants.

Substituting (14) into (12), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2 \tag{15}$$

which is a negative definite function on R^6.

Thus, by Lyapunov stability theory [36], it is immediate that the synchronization error $e_i, (i = 1, 2, 3)$ and the parameter estimation error e_a, e_b, e_c decay to zero exponentially with time.

This shows that the identical Sprott J uncertain chaotic systems are globally synchronized and the parameter estimation error also globally decays to zero exponentially with time.

Hence, we have proved the following result.

Theorem 1. The identical Sprott J chaotic systems (3) and (4) with unknown parameters are globally and exponentially synchronized by the adaptive control law (7), where the update law for the parameter estimates is given by (14) and $k_i, (i = 1, 2, \ldots, 6)$ are positive constants.

3.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic systems (3) and (4) with the adaptive control law (14) and the parameter update law (14) using MATLAB. We take $k_i = 3$ for $i = 1, 2, \ldots, 6$.

For the Sprott J chaotic systems (3) and (4), the parameter values are taken as

$$a = 2, \quad b = 2 \quad \text{and} \quad c = 1.$$

Suppose that the initial values of the parameter estimates are

$$\hat{a}(0) = 10, \quad \hat{b}(0) = 6, \quad \hat{c}(0) = 8.$$

The initial values of the master system (3) are taken as

$$x_1(0) = 12, \quad x_2(0) = 24, \quad x_3(0) = 18.$$

The initial values of the slave system (4) are taken as

$$y_1(0) = 8, \quad y_2(0) = 29, \quad y_3(0) = 30.$$

Figure 3 depicts the complete synchronization of the identical hyperchaotic Lorenz systems (3) and (4).

Figure 4 shows that the estimated values of the parameters, viz. $\hat{a}, \hat{b}$ and $\hat{c}$ converge to the system parameters

$$a = 2, \quad b = 2 \quad \text{and} \quad c = 1.$$

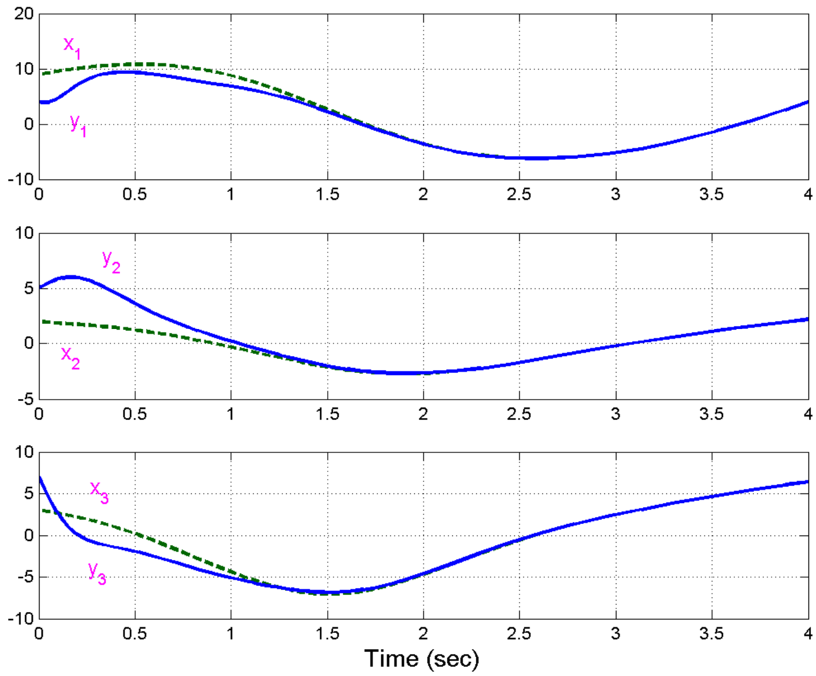

Figure 3. Complete Synchronization of the Sprott J Systems

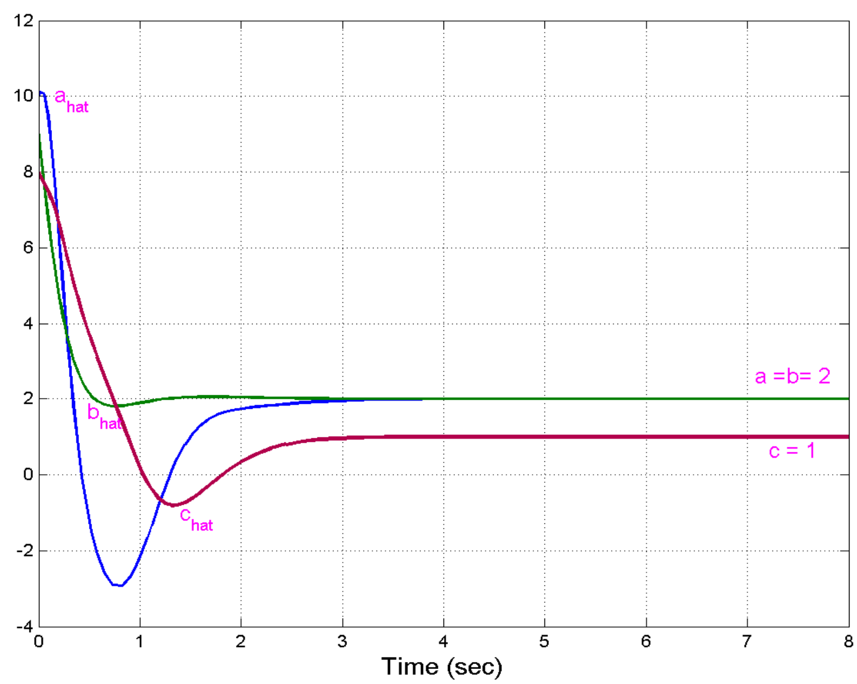

Figure 4. Parameter Estimates $\hat{a}, \hat{b}, \hat{c}$

4. ADAPTIVE SYNCHRONIZATION OF IDENTICAL SPROTT K SYSTEMS

4.1 Theoretical Results

In this section, we discuss the adaptive synchronization of identical Sprott K systems ([35], 1994), where the parameters of the master and slave systems are unknown.

As the master system, we consider the Sprott K dynamics described by

$$\begin{aligned} \dot{x}_1 &= x_1 x_2 - \alpha x_3 \\ \dot{x}_2 &= x_1 - \beta x_2 \\ \dot{x}_3 &= x_1 + \gamma x_3 \end{aligned} \tag{16}$$

where x_1, x_2, x_3 are the states and α, β, γ are unknown real constant parameters of the system.

As the slave system, we consider the controlled Sprott K dynamics described by

$$\begin{aligned} \dot{y}_1 &= y_1 y_2 - \alpha y_3 + u_1 \\ \dot{y}_2 &= y_1 - \beta y_2 + u_2 \\ \dot{y}_3 &= y_1 + \gamma y_3 + u_3 \end{aligned} \tag{17}$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are the nonlinear controllers to be designed.

The chaos synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3) \tag{18}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= -\alpha e_3 + y_1 y_2 - x_1 x_2 + u_1 \\ \dot{e}_2 &= e_1 - \beta e_2 + u_2 \\ \dot{e}_3 &= e_1 + \gamma e_3 + u_3 \end{aligned} \tag{19}$$

Let us now define the adaptive control functions

$$\begin{aligned} u_1(t) &= \hat{\alpha} e_3 - y_1 y_2 + x_1 x_2 - k_1 e_1 \\ u_2(t) &= -e_1 + \hat{\beta} e_2 - k_2 e_2 \\ u_3(t) &= -e_1 - \hat{\gamma} e_3 - k_3 e_3 \end{aligned} \tag{20}$$

where $\hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ are estimates of α, β and γ respectively, and $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting (20) into (19), the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= -(\alpha - \hat{\alpha})e_3 - k_1 e_1 \\ \dot{e}_2 &= -(\beta - \hat{\beta})e_2 - k_2 e_2 \\ \dot{e}_3 &= (\gamma - \hat{\gamma})e_3 - k_3 e_3\end{aligned} \tag{21}$$

Let us now define the parameter estimation errors as

$$e_\alpha = \alpha - \hat{\alpha}, \quad e_\beta = \beta - \hat{\beta} \quad \text{and} \quad e_\gamma = \gamma - \hat{\gamma} \tag{22}$$

Substituting (22) into (21), we obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= -e_\alpha e_3 - k_1 e_1 \\ \dot{e}_2 &= -e_\beta e_2 - k_2 e_2 \\ \dot{e}_3 &= e_\gamma e_3 - k_3 e_3\end{aligned} \tag{23}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used.

We consider the quadratic Lyapunov function defined by

$$V(e_1, e_2, e_3, e_\alpha, e_\beta, e_\gamma) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2\right) \tag{24}$$

which is a positive definite function on R^6.

We also note that

$$\dot{e}_\alpha = -\dot{\hat{\alpha}}, \quad \dot{e}_\beta = -\dot{\hat{\beta}} \quad \text{and} \quad \dot{e}_\gamma = -\dot{\hat{\gamma}} \tag{25}$$

Differentiating (24) along the trajectories of (23) and using (25), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_\alpha\left[-e_1 e_3 - \dot{\hat{\alpha}}\right] + e_\beta\left[-e_2^2 - \dot{\hat{\beta}}\right] + e_\gamma\left[e_3^2 - \dot{\hat{\gamma}}\right] \tag{26}$$

In view of Eq. (26), the estimated parameters are updated by the following law:

$$\begin{aligned}\dot{\hat{\alpha}} &= -e_1 e_3 + k_4 e_\alpha \\ \dot{\hat{\beta}} &= -e_2^2 + k_5 e_\beta \\ \dot{\hat{\gamma}} &= e_3^2 + k_6 e_\gamma\end{aligned} \tag{27}$$

where k_4, k_5 and k_6 are positive constants.

Substituting (27) into (26), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_\alpha^2 - k_5 e_\beta^2 - k_6 e_\gamma^2 \tag{28}$$

which is a negative definite function on R^6.

Thus, by Lyapunov stability theory [30], it is immediate that the synchronization error $e_i, (i = 1, 2, 3)$ and the parameter estimation error $e_\alpha, e_\beta, e_\gamma$ decay to zero exponentially with time.

Hence, we have proved the following result.

Theorem 2. The identical Sprott K systems (16) and (17) with unknown parameters are globally and exponentially synchronized by the adaptive control law (20), where the update law for the parameter estimates is given by (27) and $k_i, (i = 1, 2, \ldots, 6)$ are positive constants.

4.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic systems (16) and (17) with the adaptive control law (14) and the parameter update law (27) using MATLAB. We take $k_i = 3$ for $i = 1, 2, \ldots, 6$.

For the Sprott K systems (16) and (17), the parameter values are taken as

$$\alpha = 1, \quad \beta = 1, \quad \gamma = 0.3.$$

Suppose that the initial values of the parameter estimates are

$$\hat{\alpha}(0) = 2, \quad \hat{\beta}(0) = 5, \quad \hat{\gamma}(0) = 8.$$

The initial values of the master system (16) are taken as

$$x_1(0) = 1, \quad x_2(0) = 2, \quad x_3(0) = 5.$$

The initial values of the slave system (17) are taken as

$$y_1(0) = 4, \quad y_2(0) = 6, \quad y_3(0) = 3$$

Figure 5 depicts the complete synchronization of the identical Sprott K systems (16) and (17). Figure 6 shows that the estimated values of the parameters, viz. $\hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ converge to the system parameters

$$\alpha = 1, \ \beta = 1 \text{ and } \gamma = 0.3.$$

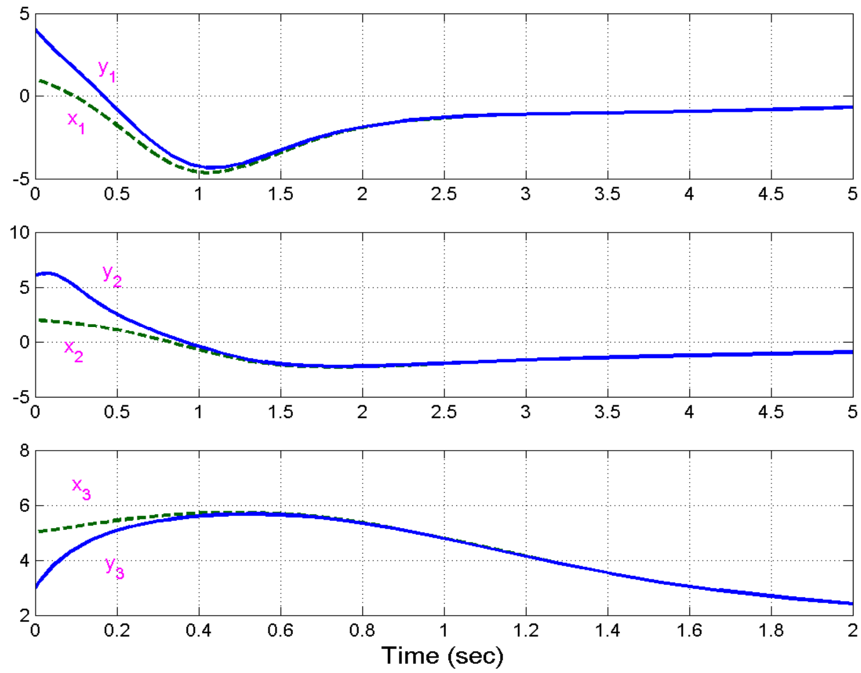

Figure 5. Complete Synchronization of the Sprott K Systems

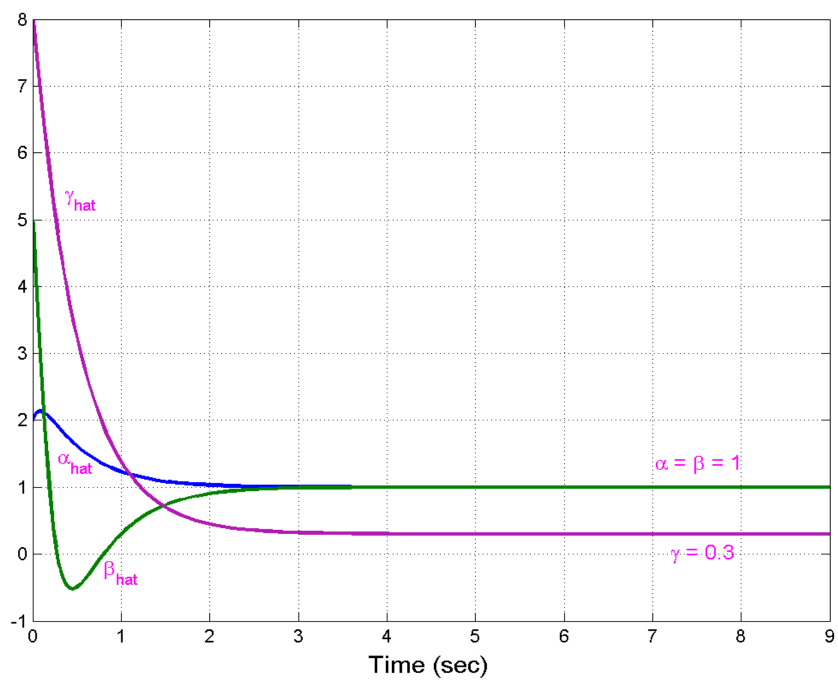

Figure 6. Parameter Estimates $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$

5. ADAPTIVE SYNCHRONIZATION OF NON-IDENTICAL SPROTT J AND K SYSTEMS

5.1 Theoretical Results

In this section, we discuss the adaptive synchronization of non-identical Sprott J and K systems, where the parameters of the master and slave systems are unknown.

As the master system, we consider the Sprott J dynamics described by

$$\begin{aligned} \dot{x}_1 &= ax_3 \\ \dot{x}_2 &= -bx_2 + x_3 \\ \dot{x}_3 &= -cx_1 + x_2 + x_2^2 \end{aligned} \tag{29}$$

where x_1, x_2, x_3 are the states and a, b, c are unknown real constant parameters of the system.

As the slave system, we consider the controlled Sprott K dynamics described by

$$\begin{aligned} \dot{y}_1 &= y_1 y_2 - \alpha y_3 + u_1 \\ \dot{y}_2 &= y_1 - \beta y_2 + u_2 \\ \dot{y}_3 &= y_1 + \gamma y_3 + u_3 \end{aligned} \tag{30}$$

where y_1, y_2, y_3 are the states, α, β, γ are unknown real constant parameters of the system and u_1, u_2, u_3 are the nonlinear controllers to be designed.

The chaos synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3) \tag{31}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= -\alpha y_3 - ax_3 + y_1 y_2 + u_1 \\ \dot{e}_2 &= y_1 - \beta y_2 + bx_2 - x_3 + u_2 \\ \dot{e}_3 &= y_1 + cx_1 + \gamma y_3 - x_2 - x_2^2 + u_3 \end{aligned} \tag{32}$$

Let us now define the adaptive control functions

$$\begin{aligned} u_1(t) &= \hat{\alpha} y_3 + \hat{a} x_3 - y_1 y_2 - k_1 e_1 \\ u_2(t) &= -y_1 + \hat{\beta} y_2 - \hat{b} x_2 + x_3 - k_2 e_2 \\ u_3(t) &= -y_1 - \hat{c} x_1 - \hat{\gamma} x_3 + x_2 + x_2^2 - k_3 e_3 \end{aligned} \tag{33}$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ are estimates of a, b, c, α, β and γ, respectively, and $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting (33) into (32), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -(\alpha - \hat{\alpha}) y_3 - (a - \hat{a}) x_3 - k_1 e_1 \\ \dot{e}_2 &= -(\beta - \hat{\beta}) y_2 + (b - \hat{b}) x_2 - k_2 e_2 \\ \dot{e}_3 &= (c - \hat{c}) x_1 + (\gamma - \hat{\gamma}) y_3 - k_3 e_3 \end{aligned} \tag{34}$$

Let us now define the parameter estimation errors as

$$\begin{aligned} & e_a = a - \hat{a}, \;\; e_b = b - \hat{b}, \;\; e_c = c - \hat{c} \\ & e_\alpha = \alpha - \hat{\alpha}, \;\; e_\beta = \beta - \hat{\beta}, \;\; e_\gamma = \gamma - \hat{\gamma} \end{aligned} \tag{35}$$

Substituting (35) into (32), we obtain the error dynamics as

$$\begin{aligned} \dot{e}_1 &= -e_\alpha y_3 - e_a x_3 - k_1 e_1 \\ \dot{e}_2 &= -e_\beta y_2 + e_b x_2 - k_2 e_2 \\ \dot{e}_3 &= e_c x_1 + e_\gamma y_3 - k_3 e_3 \end{aligned} \tag{36}$$

We consider the quadratic Lyapunov function defined by

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2 \right) \tag{37}$$

which is a positive definite function on R^9.

We also note that

$$\dot{e}_a = -\dot{\hat{a}}, \; \dot{e}_b = -\dot{\hat{b}}, \;\; \dot{e}_c = -\dot{\hat{c}}, \;\; \dot{e}_\alpha = -\dot{\hat{\alpha}}, \;\; \dot{e}_\beta = -\dot{\hat{\beta}}, \;\; \dot{e}_\gamma = -\dot{\hat{\gamma}} \tag{38}$$

Differentiating (37) along the trajectories of (36) and using (38), we obtain

$$\begin{aligned} \dot{V} = & -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a \left[-e_1 x_3 - \dot{\hat{a}} \right] + e_b \left[e_2 x_2 - \dot{\hat{b}} \right] \\ & + e_c \left[e_3 x_1 - \dot{\hat{c}} \right] + e_\alpha \left[-e_1 y_3 - \dot{\hat{\alpha}} \right] + e_\beta \left[-e_2 y_2 - \dot{\hat{\beta}} \right] + e_\gamma \left[e_3 y_3 - \dot{\hat{\gamma}} \right] \end{aligned} \tag{39}$$

In view of Eq. (39), the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= -e_1 x_3 + k_4 e_a, & \dot{\hat{\alpha}} &= -e_1 y_3 + k_7 e_\alpha \\ \dot{\hat{b}} &= e_2 x_2 + k_5 e_b, & \dot{\hat{\beta}} &= -e_2 y_2 + k_8 e_\beta \\ \dot{\hat{c}} &= e_3 x_1 + k_6 e_c, & \dot{\hat{\gamma}} &= e_3 y_3 + k_9 e_\gamma \end{aligned} \tag{40}$$

where k_5, k_6, k_7 and k_8 are positive constants.

Substituting (40) into (39), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2 - k_7 e_\alpha^2 - k_8 e_\beta^2 - k_9 e_\gamma^2 \tag{41}$$

which is a negative definite function on R^{12}.

Thus, by Lyapunov stability theory [36], it is immediate that the synchronization error $e_i, (i = 1, 2, 3)$ and the parameter estimation error decay to zero exponentially with time.

Hence, we have proved the following result.

Theorem 3. The non-identical Sprott J system (29) and Sprott K system (30) with unknown parameters are globally and exponentially synchronized by the adaptive control law (33), where the update law for the parameter estimates is given by (40) and $k_i, (i = 1, 2, \ldots, 9)$ are positive constants.

5.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the chaotic systems (29) and (30) with the adaptive control law (27) and the parameter update law (40) using MATLAB. We take $k_i = 3$ for $i = 1, 2, \ldots, 9$.

For the Sprott J system, the parameter values are taken as

$$a = 2, \;\; b = 2 \;\; \text{and} \;\; c = 1. \tag{42}$$

For the Sprott K system, the parameter values are taken as

$$\alpha = 1, \;\; \beta = 1 \;\; \text{and} \;\; \gamma = 0.3. \tag{43}$$

Suppose that the initial values of the parameter estimates are

$$\hat{a}(0) = 3, \; \hat{b}(0) = 2, \; \hat{c}(0) = 7, \;\; \hat{\alpha}(0) = 4, \; \hat{\beta}(0) = 2, \; \hat{\gamma}(0) = 3 \; .$$

The initial values of the master system (29) are taken as

$$x_1(0) = 8, \;\; x_2(0) = 2, \;\; x_3(0) = 3$$

The initial values of the slave system (30) are taken as

$$y_1(0) = 5, \;\; y_2(0) = 6, \;\; y_3(0) = 1$$

Figure 7 depicts the complete synchronization of the non-identical Sprott J and K systems.

Figure 8 shows that the estimated values of the parameters, viz. $\hat{a}, \hat{b}, \hat{c}, \hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ converge to the original values of the parameters given in (42) and (43).

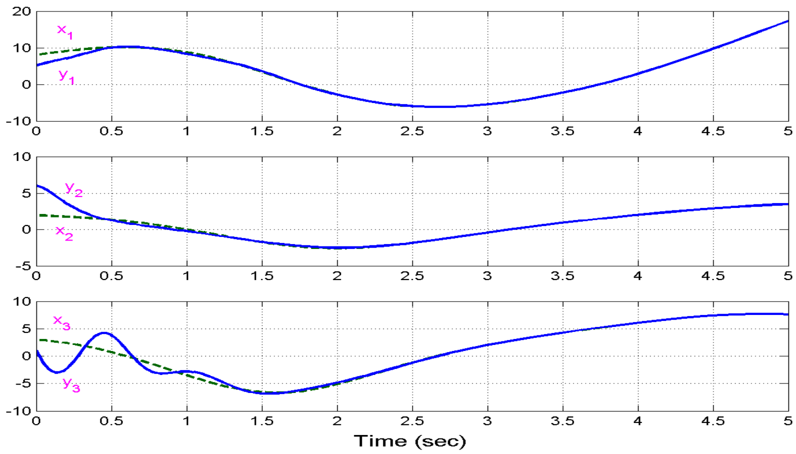

Figure 7. Complete Synchronization of the Sprott J and K Systems

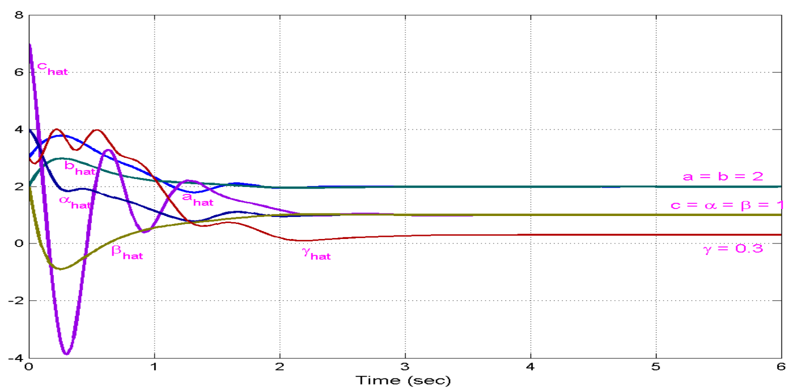

Figure 8. Parameter Estimates $\hat{a},\hat{b},\hat{c},\hat{\alpha},\hat{\beta},\hat{\gamma}$

5. CONCLUSIONS

In this paper, we have applied adaptive control method for the global chaos synchronization of identical Sprott J systems (1994), identical Sprott K systems (1994) and non-identical Sprott J and K systems with unknown parameters. The adaptive synchronization results derived in this paper are established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the adaptive control method is a very effective and convenient for achieving chaos synchronization for the uncertain chaotic systems discussed in this paper. Numerical simulations are shown to demonstrate the effectiveness of the adaptive synchronization schemes derived in this paper for the synchronization of identical and non-identical uncertain Sprott J and K chaotic systems.

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Author

Dr. V. Sundarapandian is a Professor (Systems and Control Engineering), Research and Development Centre at Vel Tech Dr. RR & Dr. SR Technical University, Chennai, India. His current research areas are: Linear and Nonlinear Control Systems, Chaos Theory, Dynamical Systems and Stability Theory, etc. He is the Editor-in-Chief of the AIRCC journals – International Journ al of Instrumentation and Control Systems, International Journal of Control Systems and Computer Modeling, and International Journal of Information Technology, Control and Automation. He is an Associate Editor of several International journals on Control Engineering, Computer Science, Electrical Engineering, Mathematical Modeling, Scientific Computing, etc. He has published over 190 research articles in international journals and two text-books with Prentice-Hall of India, New Delhi, India. He has published over 50 papers in International Conferences and100 papers in National Conferences. He has delivered several Key Note Lectures on Control Systems, Chaos Theory, Scientific Computing, MATLAB, SCILAB, etc.