Module
8

Three-phase Induction Motor

Lesson
31

Equivalent Circuit and Power Flow Diagram of IM

Instructional Objectives

- determine the induced emfs, and the currents, per phase in the stator and rotor windings,

- find the relation between the rotor input, rotor copper loss and rotor output, in terms of slip,

- calculate the various losses – iron, copper and mechanical, in an IM, and

- find where do the losses occur.

Introduction

In the previous, i.e. second, lesson of this module, the construction (the stator and two types of rotor − squirrel cage and wound (slip-ring) one), of the three-phase induction motor (IM), has been described. Then, the principle of operation, i.e. production of torque, with the stator supplied from balanced three-phase supply and the rotor winding terminals short-circuited, is discussed. In this lesson, the equivalent circuit per phase of IM will be derived first, to be followed by the presentation of power flow diagram, wherein the various losses, and also where do they occur, are described.

Keywords: The induced voltages (emfs) in stator winding and rotor bar/winding, the equivalent circuit per phase of IM, the fictitious resistance in the circuit, input power to stator and rotor, losses – iron loss, stator and rotor copper loss, mechanical losses, output power.

The induced emf per phase, in the motor windings

The induced emf per phase in the stator winding is nearly equal to the input voltage per phase, and is given by
$$V_s \approx E_s = 2.22 \cdot k_{ws} \cdot f \cdot \phi \cdot Z_s = 4.44 \cdot k_{ws} \cdot f \cdot \phi \cdot T_s$$
where,

V_s = Input voltage (V) per phase, to the stator winding

E_s = Induced emf (V) per phase, in the stator winding

ϕ = Flux (Wb) per pole

f = Frequency (Hz or c/s) of the input voltage

$Z_s = 2 \cdot T_s$ = No. of stator conductors in series per phase

T_s = No. of stator turns or coils in series per phase

$k_{ws} = k_{ds} \cdot k_{ps}$ = Stator winding factor

k_{ds} = Distribution factor of stator winding

k_{ps} = Pitch factor of stator winding

The above formula, not derived here, is a standard one available in any textbook. It is similar to the formula for the induced emf in a transformer, derived in the earlier module. As the winding in this case is stationary, with the rotating magnetic field rotating at a

speed of $n_s = f / (p/2) = (2 \cdot f) / p$ (also termed as the synchronous speed), the frequency of the induced emf in the stator winding is same as that of input frequency ($f = (p/2) \cdot n_s$), where,

n_s = Speed of the rotating magnetic field (rev/sec)

p = No. of poles for which the stator winding of IM is designed

The induced emf per phase at standstill ($\omega_r = 0.0$, or $s = 1.0$) in the rotor winding (assuming a wound or slip-ring one), is given by

$$E_r = 2.22 \cdot k_{wr} \cdot f_r \cdot \phi \cdot Z_r = 4.44 \cdot k_{wr} \cdot f_r \cdot \phi \cdot T_r$$

where,

E_r = Induced emf (V) per phase, in the rotor winding at standstill ($s = 1.0$)

$f_r = s \cdot f = f$ = Frequency (Hz) in the rotor winding at standstill ($s = 1.0$)

$Z_r = 2 \cdot T_r$ = No. of rotor conductors in series per phase

T_r = No. of rotor turns or coils in series per phase

$k_{wr} = k_{dr} \cdot k_{pr}$ = Rotor winding factor

k_{dr} = Distribution factor of rotor winding

k_{pr} = Pitch factor of rotor winding

It may be noted for the wound rotor, the number of poles in the rotor winding must be the same as those in the stator winding, which has been explained in the previous lesson (#30). The squirrel cage rotor can be considered as equivalent to a wound one.

The frequency of the induced emf in the rotor winding with the rotor rotating at any speed, is proportional to the relative speed between the speed of the rotating magnetic field, and that of the rotor, and is given by

$$f_r = (p/2) \cdot (n_s - n_r) = (p/2) \cdot s \cdot n_s = s \cdot f$$

where,

$n_r = (1 - s) \cdot n_s$ = Speed of the rotor (rev/sec)

$s = (n_s - n_r) / n_s = 1 - (n_r / n_s)$ = slip in IM

The ratio of the induced emf's per phase, in the stator and rotor windings, with the rotor at standstill, is given by,

$$E_s / E_r \approx T_s / T_r = Z_s / Z_r$$

assuming that the winding factor in both windings are same. It may be noted that the induced voltage per phase in the winding is proportional to the effective turns per phase, of the winding ($= k_w \cdot T$). The formula is similar to that obtained in a transformer, where the induced voltage per turn ($E_t = E / T$) is the same in the two (primary/secondary) windings.

The induced emf in the rotor winding with the motor running at any speed is proportional to the rotor frequency ($f_r = s \cdot f$), and is given by

$$s \cdot E_r = 2.22 \cdot k_{wr} \cdot s \cdot f \cdot \phi \cdot Z_r$$

The currents in the balanced three-phase windings of the stator produce a resultant mmf, rotating at the synchronous speed (n_s) relative to stator conductors, which is stationary. Similarly, the currents in the balanced three-phase windings of the rotor (taken

as a wound one) produce another resultant mmf, rotating at the speed of $(n_s - n_r = s \cdot n_s)$ relative to rotor conductors in the same direction as that of the rotor, which rotates at a speed $(n_r = (1-s) \cdot n_s)$. Hence, the speed of the resultant rotor mmf relative to stator conductors is $(s \cdot n_s + n_r = n_s)$. The axes of the resultant rotor mmf rotate at the same speed as those of the resultant stator mmf, so that the mmfs are stationary relative to each other.

The equivalent circuit per phase, of the rotor

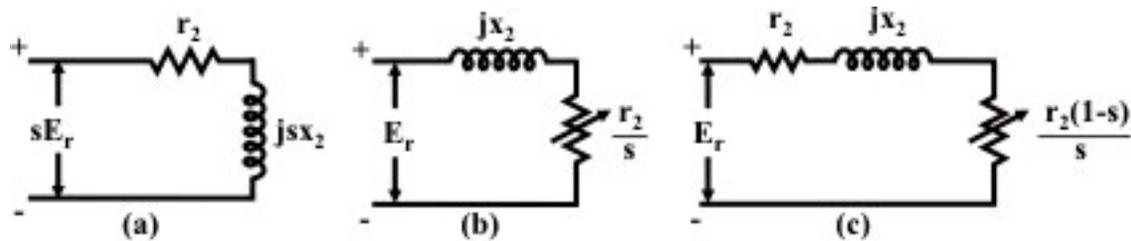The current per phase in the rotor winding (Fig. 31.1a) is given by

Fig. 31.1: Equivalent circuit of the rotor winding (IM)

$$I_2 = \frac{s \cdot E_r}{\sqrt{(r_2)^2 + (s \cdot x_2)^2}}$$

where,

r_2 = Resistance (Ω) per phase, of the rotor winding

$x_2 = 2 \cdot \pi \cdot f \cdot l_2$ = Leakage reactance (Ω) per phase, of the rotor winding at standstill ($s = 1.0$ or $\omega_r = 0.0$)

The leakage reactance per phase of the rotor winding at any other speed n_r, is proportional to the rotor frequency ($f_r = s \cdot f$), and is given by $x_{2s} = s \cdot x_2$. The term in the denominator of the expression is the magnitude of the rotor impedance per phase at any other speed, which is ($|z_{2s}| = \sqrt{(r_2)^2 + (s \cdot x_2)^2}$), with its phase angle as $\phi_{2s} = \tan^{-1}[(s \cdot x_2)/r_2]$. It may be noted that the subscript '2' is used here, instead of 'r' earlier. Similarly, the subscript '1' may be used, instead of 's', i.e. stator.

In the expression for the rotor current per phase, if both numerator and denominator is divided by the term s, I_2 is

$$I_2 = \frac{E_r}{\sqrt{(r_2/s)^2 + (x_2)^2}}$$

The circuit of the rotor (Fig. 31.1a) can now be drawn as shown in Fig. 31.1b, from the equation given earlier. It may be noted here the input voltage per phase to the circuit is constant, i.e. E_r and the leakage reactance per phase is also constant, x_2, both being same as at standstill, with the rotor frequency being same as line or input frequency. Also the phase angle of the impedance, whose magnitude is the term in the denominator, remains same as $\phi_{2s} = \tan^{-1}[x_2/(r_2/s)] = \tan^{-1}[(s \cdot x_2)/r_2]$, as obtained earlier.

The circuit of Fig. 31.1b can be redrawn as shown in Fig. 31.1c, with the resistance (r_2/s) now being shown as sum of two resistances, rotor resistance, r_2 and fictitious resistance, $((r_2/s) - r_2 = r_2(1-s)/s)$. The first part of the circuit consists of the rotor impedance at standstill, which is ($z_2 \angle \phi_2 = r_2 + j\,x_2$), whose magnitude and phase angle are $|z_2| = \sqrt{(r_2)^2 + (x_2)^2}$ and $\phi_2 = \tan^{-1}(x_2/r_2)$ respectively.

The relation between rotor input, rotor copper loss and rotor output

From the equivalent circuit (Fig. 31.1c) per phase of the rotor winding, the total input (W) to the rotor is $P_i = 3 \cdot E_r \cdot I_2 \cdot \cos\phi_{2s} = 3 \cdot (I_2)^2 \cdot (r_2/s)$. Please note that the phase angle, ϕ_{2s} is the angle between the induced voltage and current (both in the rotor). The total input to the rotor is the power transferred from the stator via air gap. The total copper loss in the rotor winding is $3 \cdot (I_2)^2 \cdot r_2$. The total output of the rotor is the total power loss in the fictitious resistance, $r_2(1-s)/s$ of the rotor circuit (Fig, 31.1c), and is given by $3 \cdot (I_2)^2 \cdot r_2 \cdot (1-s)/s$. The total rotor output is obtained also as the difference of the total rotor input and the total rotor copper loss. Alternatively,
Total input to the rotor = Total rotor copper loss + Total rotor output
It may be shown that
Total rotor copper loss = $s \times$ Total rotor input,
or, Total rotor input = $(1/s) \times$ Total rotor copper loss
In terms of total rotor output, the relationships are as follows,
Total rotor copper loss = $[s/(1-s)] \times$ Total rotor output,
or, Total rotor output = $\{(1-s)/s\} \times$ Total rotor copper loss
The relation between total rotor output and total rotor input is
Total rotor output = $(1-s) \times$ Total rotor input
or, Total rotor input = $[1/(1-s)] \times$ Total rotor output
In the above discussion, total rotor output is the output in electrical terms, also termed as gross one, which includes mechanical, or friction and windage losses.

The expression for gross power and torque developed, and the torque-speed or torque-slip characteristics of Induction motor along with other relevant points, will be presented in the next lesson.

Power flow diagram

In the induction motor, efficiency is
$$\text{Efficiency } (\eta) = \frac{Output}{Input} = \frac{Output}{Output + losses}$$
as Input = output + losses

The total losses are losses in both stator and rotor, and mechanical losses. The stator losses are iron or core loss and copper loss, i.e. loss in the stator winding resistance. The core loss is the sum of hysteresis and eddy current losses, with the frequency in the stator

being line frequency of 50 Hz. Both the above losses are related to frequency. It may be noted here that the flux in the air gap is rated, as the input or supply voltage fed to the stator winding is rated with the frequency is also rated. The copper loss is proportional to the square of the current in the stator winding. The loss in the rotor is only copper loss as shown earlier, with the iron loss being low due to low frequency in the rotor core. For example, the rotor frequency ($f_r = s f$) is only 2 Hz, if the slip is assumed as 4%, the speed being 1,440 rpm for a 4-pole motor. The line (input) frequency is 50 Hz. The iron loss is related to the frequency as stated earlier. The net output available at the shaft is the gross output minus the mechanical losses.

Before coming to the power flow diagram in an Induction motor, total input power fed to the motor via stator (winding) is $P_i = 3\,V\,I\cos\phi_i = 3\,V\,I\cos(V,I)$

where, V and I are the input voltage and current per phase, to the motor.

Total input power to the motor (stator) = Stator losses (iron & copper) + Total input power to the rotor (or Power transferred via air gap)
Total input power to the rotor = Rotor copper loss + Total output power (gross)
Total output power (gross) = Mechanical losses + Total output power (net) [or Power available at the shaft]
It may be noted that all the losses, excluding mechanical one, are total for all three phases, and as given earlier, iron losses in the rotor are very low..

In this lesson – the third one of this module, the equivalent circuit per phase of an IM has been derived, starting with the formula for the induced emfs per phase, in both stator and rotor windings. The various losses and where do they occur, are described next, followed by presentation of power flow diagram for IM. In the next lesson as given earlier, the expression for gross power or torque developed, and the torque-speed or torque-slip characteristics of an IM, along with other relevant points, will be presented in detail.