

## **Analysis of Various Puncturing Patterns and Code Rates: Turbo Code**

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### **Abstract**

Puncturing is the process of deleting some parity bits from the codeword according to a puncturing matrix.

Puncturing is the tradeoff between rate and performance. Puncturing increases code rate without increasing complexity for code rate from  $1/3$  to  $1/2$  or more and decreases free distance of code. The redundant bits in coding decrease the bandwidth efficiency. The bandwidth efficiency decreases with increase in redundant bits in coding. the puncturing pattern adjust code rate To achieve system requirement, the puncturing pattern adjust code rate. The performance of punctured turbo code is worse than without puncturing turbo code but coding scheme improve bandwidth efficiency. The design principles of puncturing for turbo code are based on the minimization of puncturing of systematic bits and to provide puncturing of parity bits of the two encoders.

Performance level varies with different puncturing pattern. since different punctured location affect different performance at the receiver. The puncturing patterns are analyzed to compare their performances.. When the bit in codeword has high weight, avoid puncturing of systematic bit sequence since it has high weight but puncturing preference is given to bits having lower weight .In high rate system selection of good pattern plays very important role. Coding on different number of punctured bit has been suggested. Uniformly distributed puncturing pattern perform better than puncturing non-uniformly distributed pattern.

**Keywords:** : Turbo code, Punctured Turbo code, Channel coding, Performance analysis, Rate compatible systematic Turbo code (RCPTC).

## Introduction

To provide a reliable communication over a noisy channel a coding technique Known as Turbo code is used. Turbo-codes outperform all previous FEC scheme by at least 3 dB, thereby doubling the battery lifetime or saving 20% of the required spectrum. Thus it gives a breakthrough in coding technology. In most of current wireless applications its properties make turbo-codes attractive. Different puncturing patterns are suggested to increase the rate of Turbo code without increasing complexity from code rate from  $1/2$  to  $1/3$  or more and improve the performance of Turbo code. At lower error rates a symmetric interleaver performs better than non-symmetric. Symmetric interleaver performs both interleaving and de-interleaving. Puncturing allow different levels of protection. Puncturing decreases the performance of Turbo codes due to uneven protection of the parity bits caused by the random interleaver. Convolutional codes are constructed from a single lower rate binary convolutional code. The advantage of punctured codes for binary transmission is that the encoders and decoders for the entire class of codes constructed easily by modifying the single encoder and decoder for the rate  $1/2$  binary convolutional code from which the high rate punctured code was derived. The objectives of this paper are to focus on the problem of selection of good interleavers and puncturing patterns to improve turbo code performance [1] [2][3][10].

## Parallel Turbo code encoder

An encoder is used to add redundancy to a digital data stream, in the form of additional data bits, thereby provides an error correction capability at the receiver. This overall process is referred to as Forward Error Correction (FEC). Among the most popular FEC schemes are convolutional coding, block coding and trellis coding. It is important to note that usually the output bit rate of an encoder is not equal to the input bit rate. To distinguish between the two bit rates, the transmitter's input rate is referred to as the information data rate, while the transmitter output rate is referred to as the channel data rate. The response of the encoder to an arbitrary input sequence represented by state diagram.

A parallel concatenated coding scheme consists of two simple constituent systematic encoders linked by an interleaver the second encoder. The codeword of the parallel concatenated code consists of the input bits to the first encoder followed by the parity check bits of both encoders. This construction can be applied to any number of constituent codes. Parallel concatenated schemes utilizes two convolutional codes as constituent codes, in connection with an iterative decoding algorithm of complexity comparable to that of the constituent codes, to obtain coding gains very close to theoretical limits. They are known as, "turbo codes".

Figure 1. Shows parallel Turbo code encoder in which information bits are directly fed to the first RSC Encoder<sub>1</sub>. One set of information bits are directly fed to the decoder, whereas information bits are fed to the RSC encoder<sub>1</sub> whose output is parity bit stream<sub>1</sub> i.e.P<sub>1</sub> but the information bits are fed to the RSC ENC<sub>2</sub> through the interleaver. It is the second parity stream bit i.e.P<sub>2</sub>. These two parity streams are then applied to the puncturing map for puncturing the redundant bits to remove unwanted

bits and to increase the code rate thereby reducing the bandwidth of transmitted signals.[1] [4][5][6]

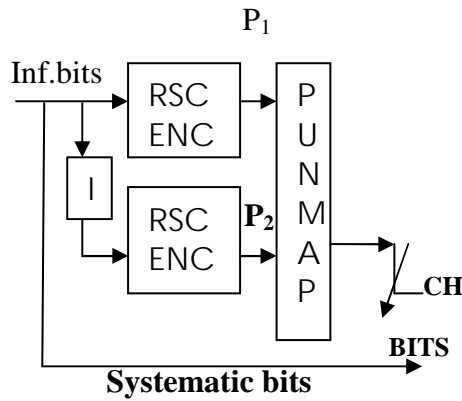


Figure 1: Parallel Turbo code encoder.

**Trellis Termination**

Trellis structure is used to reduce redundancy. Trellis representation is used to decode a linear block code. It is essential that the decoder knows both the starting state and finishing state. This gives rise to the need to terminate the trellis of an RSC code. Due to the recursive structure of the RSC encoder, it is not sufficient to set the last M information bits to zero i.e. where M is the number of memory elements in the encoder to drive the encoder to the all zero state. Their RSC encoder, shown in Figure 2. includes a switch with two positions A and B. The switch is in position A for the first N clock cycles, and in position B for additional M cycles to drive the encoder to the all-zero state. This method is equivalent to setting the last M information bits equal to the value of the feedback path so that the addition of the feedback and the input is 0 (mod 2). Shift Register

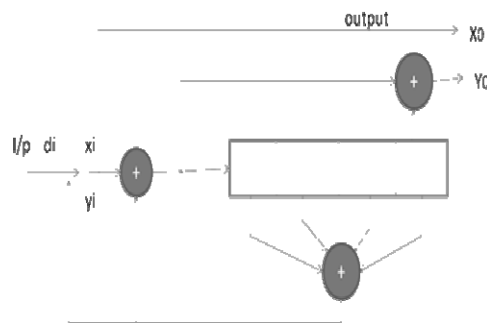


Figure 2: RSC Encoder with Trellis Termination.

In the Turbo Code encoder the trellis is terminated in this manner and the other trellis is left unterminated. This results in a  $(n(N+M);N)$  block code, where the rate of the code (excluding termination) is  $1/n$ . For large block sizes, the effect of trellis termination on the BER performance and the code rate is negligible but, for small block sizes, trellis termination becomes much more important [7] [8].

### Rate-compatible punctured convolutional codes

Codes are compatible; rate variation within a data frame is possible to achieve unequal error protection: this is required when different levels of error protection i.e. different code rates are needed for different parts or blocks of an information sequence. Unequal error protection methods applied to JPEG image transmission using turbo codes. The JPEG image is partitioned into two groups, i.e., DC components and AC components according to their respective sensitivity to channel noise. The highly sensitive DC components are better protected with a lower coding rate, while the less sensitive AC components use a higher coding rate. The DC and AC components are protected with different coding rates. In some applications like transmission of compressed digital signals needs to transmit some group of information bits with more redundancy than others requires unequal protection provided in the transmission of the information sequence, where most important bits are transmitted with more redundancy. For this it is desirable to use a single code that has variable redundancy obtained by puncturing low rate  $1/n$  convolutional code by different. The selected puncturing matrix satisfy a rate compatibility criterion, basic requirement is that lower rate i.e. higher redundancy codes transmit the same coded bits as all higher-rate codes plus additional bits. The resulting codes obtained, called Rate compatible punctured codes

#### Rate 4/11

P0= [11111111]  
P1= [11111110]  
P2= [11101110]

#### Rate 2/5

P0= [11111111]  
P1= [11111110]  
P2= [10101010]

#### Rate 4/9

P0= [11111111]  
P1= [11111110]  
P2= [10001000]

#### Rate 1/2

P0= [11111111]  
P1= [11111111]  
P2= [00000000]

#### Rate 4/7

P0= [11111111]  
P1= [11101110]  
P2= [00000000]

#### Rate 4/6

P0= [11111111]  
P1= [10101010]  
P2= [00000000]

#### Rate 4/5

P0= [11111111]  
P1= [10001000]  
P2= [00000000]

#### Rate 8/9

P0= [11111111]  
P1= [10000000]  
P2= [00000000]

**Figure 2:** RCPC codes constructed from rate  $1/3$ ,  $k=4$  code maximum free distance with  $P=8$ ,  $RC= P/(P+M)$ ,  $M = 1,2,4,6,8,10,12,14$ . [7][9].

### Rate-Compatible Punctured Turbo Codes (RCPTC)

To achieve unequal error protection: parallel concatenated convolutional codes (PCCC) with two identical constituent encoders with a variable rate of  $1/2$  to  $1/3$  using the same mother encoder, the RCPTCs is introduced. The variation in rate is obtained by puncturing, with  $M$  puncturing matrices, a rate  $1/M$  turbo encoder, consisting of one rate  $1/2$  recursive systematic convolutional (RSC) encoder cascaded in parallel with  $M-2$  rate  $1$  RSCs. The puncturing scheme is periodic not limited to parity bits, so that both systematic and partially systematic RCPTCs obtained. In multidimensional turbo code where  $M - 1$  is the number of constituent encoders used in hybrid FEC/ARQ protocols, with a variable rate of  $1$  to  $1/M$ . Both systematic and partially systematic RCPTCs obtained since puncturing scheme is not limited to parity bits.

### Design Methodology

The specification of an RCPTC consists in finding suitable mother encoder(s), the interleaver(s), and the puncturing patterns to obtain the desired code rate range. PCCC with rate  $1/3$  selected as mother code. The design methodology includes the following three consecutive steps.

In first step is the selection of constituent encoders having good performance at low signal-to noise ratio with particular attention to decoding complexity. The final choice was the optimum 4-state recursive systematic encoder. The second step is the design of turbo-code interleaver which is based on the codeword weight distribution and on the achievable performance on the additive white Gaussian noise (AWGN) channel, using a maximum likelihood approach.

The third step is to select puncturing schemes based on both the weight distribution and the achievable performance on the AWGN channel. Both cases of systematic and partially systematic RCPTCs were utilized. In design methodology for rate  $R = k/(k + 1)$  ( $2 \leq k \leq 16$ ) punctured turbo codes were deriving high rate codes via puncturing a basic rate  $1/3$  PCCC. To obtain a code rate of  $k/(k + 1)$ , one parity bit only is transmitted every  $k$  information bits presented to the encoder input. The rates of the two constituent encoders after puncturing are assumed to be the same and the parity bits to be transmitted alternate between the two encoders. Therefore, for every  $2k$  input bits, only two parity bits are transmitted by the puncturing scheme, one from each of the two constituent encoders. The most important design parameters are, the generator polynomials, the interleaves  $I$ , the puncturing pattern  $P$ . Since weight-two input and its multiplicity  $N_2$  and weight-three inputs and their multiplicity  $N_3$ , dominate the performance. The design criterion is the maximization of the minimum weight turbo-codeword for weight-2 i.e.  $d_2$  and the maximization of the minimum weight turbo-codeword for weight-3 inputs i.e.  $d_3$  and the minimization of  $N_2$  and  $N_3$  over the above parameters. To obtain a chain of RCPTCs with rates  $V = \{1/3, 1/2, 2/3, 4/5, 8/9, 16/17\}$ , starting from a puncturing period of 32 bits which is halved when passing from one rate to the next lower rate. This technique applicable for only rates of the type  $k/(k + 1)$ ,  $k = 2i$ .

### RCPTC puncturing scheme for Turbo codes

The concept of rate-compatible punctured convolutional codes is obtained by adding a rate-compatibility restriction to the puncturing rule. This restriction requires that the codes are organized in hierarchy, where coded bits of higher rate are used by all lower rate codes i.e. the higher rate codes are embedded in to the lower rate codes of the family. After encoding, puncturing is the next process and basically it is used to increase the code rate. Puncturing is performed on the incoming encoded stream of bits. By this process we obtained high rate code from low rate code. The resulting code obtained by this scheme is known as Rate compatible punctured turbo Code (RCPTC) scheme.. For instance for puncturing period 8, rate 1/3, 8/23 and 8/22 puncturing patterns are as follows:-

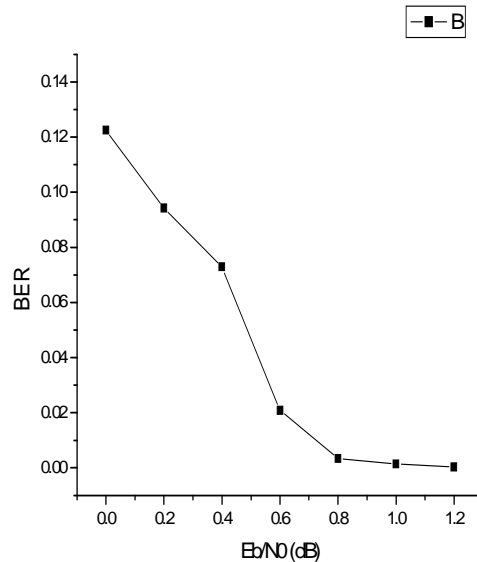
Rate 1/3	Rate 8/23	Rate 8/22
P0= [11111111]	P0= [11111111]	P0= [11111111]
P1= [11111111]	P1= [11111110]	P1= [11111110]
P2= [11111111]	P2= [11111111]	P2= [10111111]
		[7][9][10]

**Figure 3:** Rate compatible punctured turbo Code (RCPTC) scheme, puncturing patterns for puncturing period 8, rate 1/3, 8/23 and 8/22.

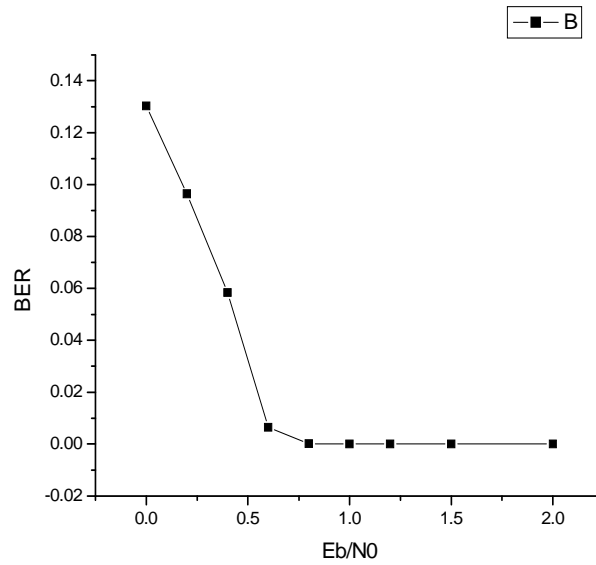
### Results and Comparisons Among Different Criteria

Consider the mother code having a puncturing period  $P$  and rate  $R = 1/M$ , determine the range of code rates  $R = P / (P + l)$  with  $l = 1, (M - 1) P$  (1) ranging between  $P / (P + 1)$ , where  $1/M$  described a family of rate compatible codes. The rate-compatible codes are obtained from the mother code with puncturing matrices  $\mathbf{a}(l) = a_{ij}(l)$ , where  $a_{ij} \in (0, 1)$  and 0 means puncturing. They are applied to find a family of systematic RCPTCs based on a rate 1/3 mother PCCC. This is obtained by concatenating an 8-state rate 1/2 and a rate 1 convolutional encoders. Optimal puncturing followed by periodic and non periodic, as well as homogeneous and non homogeneous patterns. The algorithm is used to find a well-performing puncturing pattern. The three criteria are used sequentially to design on Rate-Compatible Punctured Turbo Codes, by puncturing one bit at a time in the optimal position, subject to the constraint of rate compatibility. The sequential puncturing is performed starting from the lowest rate code i.e., the mother code and at the end the highest possible rate is achieved. The relative merits of the different design criteria are compare for this simulation of the performance of the resulting RCPTCs using a random interleaver (with size 1000, 4000 and, 8000) and 8 decoding iterations are carried out. The interleaver used in the simulation is selected randomly frame by frame: in this sense, it comes down to averaging simulation results over many randomly chosen interleavers. The uniform interleaver aspect is implementing for designing the puncturing pattern. Where the average distance spectrum of the code

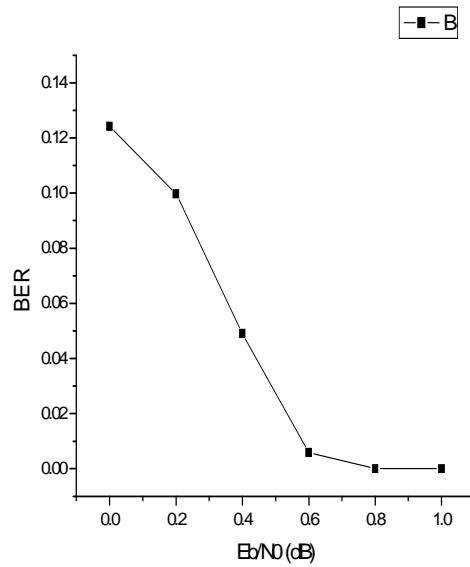
obtained applied to that pattern. Thus optimum pattern is not specific to particular interleaver hence the simulation is not performed using a specific interleaving scheme but a random interleaver, results approaches to a better closeness of simulation results to the union bound. Here we have compared three criteria applied to interleaver length, such as 1000, 4000 and 8000 has been considered. The computational complexity becomes highly complicated for the first two criterion when the interleaver size goes on increasing. The criterion introduced in this paper i.e. the third one also applied to longer interleaver lengths. Maximum-likelihood soft decoding applied at last to obtained the analytical upper bounds to the bit error probability. The coded bits are transmitted over an additive white Gaussian noise channel using an antipodal binary modulation to evaluate simulation and analytical results. A first set of simulation and bound results is shown in Figure 1, where we report the  $E_b/N_0$  required to obtain a bit error rate (BER) of  $10^{-5}$  versus the RCPTCs rate  $R_c$  ( $E_b/N_0$  being the ratio of the received energy per bit ( $E_b$ ) to the spectral noise density ( $N_0$ )). The interleaver size is set to  $N = 1000$  Systematic RCPTCs are considered, and only parity check bits are punctured. The homogeneous puncturing patterns are selected. The best performance is obtained by applying the optimization of the sequence  $(d_w, N_w)$  to the spectrum of the first component encoder, taken separately. The puncturing pattern for the whole turbo code scheme is obtained by applying the puncturing pattern obtained for the first constituent to the second-constituent check bits. The best puncturing patterns for  $N = 1000$  are reported for some rates, and are given, for each rate, in octal form for the first-constituent check bit positions going from 1 to  $N$ . Best code performance obtained at the optimal puncturing position as per criterion applied [10] [11][12].



**Figure 4:** Plot between  $E_b/N_0$  and BER for interleaver block=1000 where  $B(D) = 1+D^2+D^3$ ,  $A(D) = 1+D+D^3$  for 8- iterations.



**Figure 5:** Plot between  $E_b/N_0$  and BER for interleaver block=4000 where  $B(D) = 1+D^2+D^3$ ,  $A(D) = 1+D+D^3$  for 8- iterations.



**Figure 6:** Plot between  $E_b/N_0$  and BER for interleaver block=8000 where  $B(D) = 1+D^2+D^3$ ,  $A(D) = 1+D+D^3$  for 8- iterations.



### Simulation results

Different plots are drawn for a turbo code under basic conditions  $B(D) = 1 + D^2 + D^3$ ,  $A(D) = 1 + D + D^3$  for 8- iterations for various interleaver block sizes 1000,4000,8000. It was observed that as the size of the interleaver increases these graphs between  $E_b/N_0$  and BER achieve early error floor and reduces BER.

### Conclusion

In this paper it was concluded that rate-1/2 pseudo-randomly punctured PCCCs, which form a subset of rate-1/2 partially systematic PCCCs, not only approach the error floor region for same number of iterations but always yield a lower error floor than that of their rate-1/3 parent codes., Pseudo-random puncturing used to reduce the rate of a PCCC from 1/3 to 1/2 and at the same time achieve a coding gain at low bit error probabilities. RCPTC puncturing scheme for Turbo code is implement to arrange the code rates in a proper sequence. In other words, the higher rates are embedded in to the family of the lower rate codes. The two main applications of these techniques are its use in hybrid incremental ARQ/FEC schemes and to achieve unequal error protection of an information sequence. Rate compatible convolutional codes scheme codes are used in some applications like transmission of compressed digital signals needs to transmit some group of information bits with more redundancy than others; which requires unequal protection provided in the transmission of the information sequence, where most important bits are transmitted with more redundancy. Unequal error protection methods applied to JPEG image transmission using turbo codes.

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