Similitude for physical models

A physical model is a system whose operation can be used to predict the characteristics of a similar system, or prototype, usually more complex or built to a much larger scale.”A model can be either smaller or bigger than the real construction.It is believed that model is always smaller but that is not true always for example if we want to make a very small computer chip then to illustrate its function properly the model is made bigger as compared to the original chip.

Ratios of the forces of gravity, viscosity, and surface tension to the force of inertia are designated, Froude number, Reynolds number, and Weber number, respectively. Equating the Froude number of the model and the Froude number of the prototype ensures that the gravitational and inertial forces are in the same proportion. Similarly, equating the Reynolds numbers of the model and prototype ensures that the viscous and inertial forces are in the same proportion. Equating the Weber numbers ensures proportionality of surface tension and inertial forces.

$F=V/(Lg)^{1/2}$

where

F= Froude number (dimensionless)

V= velocity of fluid, ft/s (m/s)

L= linear dimension (characteristic, such as depth or diameter), ft (m)

g= acceleration due to gravity, 32.2 ft/s2 (9.81 m/s2)

For hydraulic structures, such as spillways and weirs, where there is a rapidly changing water-surface profile, the two predominant forces are inertia and gravity. Therefore, the Froude numbers of the model and prototype are equated:

$F_m=F_p$

$V_m/(L_m g)^{1/2}=V_p/(L_p g)^{1/2}$

Viscous forces are usually predominant when flow occurs in a closed system, such as pipe flow where there is no free surface. The following relations are obtained by equating Reynolds numbers of the model and prototype:

$$\frac{V_m L_m}{v_m} = \frac{V_p L_p}{v_p} \qquad V_r = \frac{v_r}{L_r}$$

The variable factors that fix the design of a true model when the Reynolds number governs are the length ratio and the viscosity ratio.

The *Weber number* is

$$\mathbf{W} = \frac{V^2 L\rho}{\sigma}$$

where ρ = density of fluid, lb·s^2/ft^4 (kg·s^2/m^4) (specific weight divided by g); and σ = surface tension of fluid, lb/ft^2 (kPa).

The Weber numbers of model and prototype are equated in certain types of wave studies.

For the flow of water in open channels and rivers where the friction slope is relatively flat, model designs are often based on the Manning equation. The relations between the model and prototype are determined as follows:

$$\frac{V_m}{V_p} = \frac{(1.486/n_m)R_m^{2/3}S_m^{1/2}}{(1.486/n_p)R_p^{2/3}S_p^{1/2}}$$

where n = Manning roughness coefficient ($T/L^{1/3}$, T representing time)

R = hydraulic radius (L)

S = loss of head due to friction per unit length of conduit (dimensionless)

= slope of energy gradient

For true models, $S_r = 1$, $R_r = L_r$. Hence,

$$V_r = \frac{L_r^{2/3}}{n_r}$$

In models of rivers and channels, it is necessary for the flow to be turbulent. The U.S. Waterways Experiment Station has determined that flow is turbulent if

$$\frac{VR}{\nu} \geq 4000$$

where V = mean velocity, ft/s (m/s)

R = hydraulic radius, ft (m)

ν = kinematic viscosity, ft^2/s (m^2/s)

If the model is to be a true model, it may have to be uneconomically large for the flow to be turbulent.