

## PRINCIPLES OF LEAST SQUARES

It is found from the probability equation that the most probable values of a series of errors arising from observations of equal weight are those for which the sum of the squares is a minimum. The fundamental law of least squares is derived from this. According to the principle of least squares, the most probable value of an observed quantity available from a given set of observations is the one for which the sum of the squares of the residual errors is a minimum. When a quantity is being deduced from a series of observations, the residual errors will be the difference between the adopted value and the several observed values,

Let  $V_1, V_2, V_3$  etc. be the observed values

$x$  = most probable value

### The laws of weights.

From the method of least squares the following laws of weights are established:

(i) The weight of the arithmetic mean of the measurements of unit weight is equal to the number of observations.

For example, let an angle  $A$  be measured six times, the following being the values:

$\angle A$	Weight	$\angle A$	Weight
$30^\circ 20' 8''$	1	$30^\circ 20' 10''$	1
$30^\circ 20' 10''$	1	$30^\circ 20' 9''$	1
$30^\circ 20' 7''$	1	$30^\circ 20' 10''$	1

$\therefore$  Arithmetic mean

$$= 30^\circ 20' + 1/6 (8'' + 10'' + 7'' + 10'' + 9'' + 10'')$$

$$= 30^\circ 20' 9''.$$

Weight of arithmetic mean = number of observations = 6.

(2) The weight of the weighted arithmetic mean is equal to the sum of the individual weights.

For example, let an angle A be measured six times, the following being the values :

$\angle A$	Weight	$\angle A$	Weight
30° 20' 8"	2	30° 20' 10"	3
30° 20' 10"	3	30° 20' 9"	4
30° 20' 6"	2	30° 20' 10"	2

$$\text{Sum of weights} = 2 + 3 + 2 + 3 + 4 + 2 = 16$$

$$\begin{aligned} \text{Arithmetic mean} &= 30^\circ 20' + \frac{1}{16} (8'' \times 2 + 10'' \times 3 + 7'' \times 2 + 10'' \times 3 + 9'' \times 4 + 10'' \times 2) \\ &= 30^\circ 20' 9''. \end{aligned}$$

$$\text{Weight of arithmetic mean} = 16.$$

(3) The weight of algebraic sum of two or more quantities is equal to the reciprocals of the individual weights.

For Example angle  $A = 30^\circ 20' 8''$ , Weight 2  
 $B = 15^\circ 20' 8''$ , Weight 3

Weight of  $A + B =$

(4) If a quantity of given weight is multiplied by a factor, the weight of the result is obtained by dividing its given weight by the square of the factor.

(5) If a quantity of given weight is divided by a factor, the weight of the result is obtained by multiplying its given weight by the square of the factor.

(6) If an equation is multiplied by its own weight, the weight of the resulting equation is equal to the reciprocal of the weight of the equation.

(7) The weight of the equation remains unchanged, if all the signs of the equation are changed or if the equation is added or subtracted from a constant.