Special Relativity

Option G – Relativity

A little historical background: So far when we have talked about things being relative we have been talking about Galilean relativity. Things like velocity addition, if a train is moving at 100 kph and you walk towards the front of the train at 5 kph, then an outside observer would measure your speed to be 105 kph. This kind of relativity confused you a bit way back when, but now it's simple and makes sense, er well, most of the time. There's a problem with it… it only works for low velocities. If that train was moving 250,000 kilometers per second then Galilean relativity breaks down.

After Maxwell put together his 4 equations to describe Electricity and Magnetism it was quickly realized that they did not obey Galilean relativity. If fact they predict that the speed of light as measured by an observer is the same whether that observer is moving towards or away from the light source! Either Maxwell was wrong, which he wasn't, or a new kind of relativity was needed.

In the 1880's two guys produced the most successful and famous failed physics experiment, even more so than Luca's clay ball lab. The guys, Michelson and Morley, set out to show that the speed of light was different for different relative motions. In their day many people believed in an "aether," this was believed to be the medium that E&M waves propagated in. They were going to prove it existed… They set up an inferometer that could rotate 90 degrees, remember the diffraction pattern formed is a function of the speed of light in the media. Then at different times of the day and different times of the year (and thus different relative speeds) they measured the speed of light. With EVERY experiment they got the same number. Oops, sort of… Their results like many "failed" experiments were ignored for a while. Today they are given credit for establishing that the speed of light is constant regardless of relative motion.

A guy by the name of Lorentz is given credit (though he wasn't the first to write down the equations) for coming up with a set of equations that formed a new sort of relativity for Maxwell's equations. He just made a math guess and it worked…

Einstein gets all the credit, but in reality he did only two things. Proved the assumptions made by Lorentz and suggested that these equations applied to all physical laws not just Maxwell's E&M equations. The genius was in suggesting that what we all believe intuitively is wrong, or not exactly as we think it is… From this one suggestion and the use of Lorentz equations the rest of special relativity is just an exercise is somewhat complex math. His ideas were so radical they weren't fully accepted for years and today you can still find a few whack-jobs that think he's wrong. And no doubt he is not 100% correct, but no one has come up with a better idea yet.

G.1.1 Explain what meant by a frame of reference.

G.2.1 Explain what is meant by an inertial reference frame

A frame of reference could be defined as a set of object or a set of coordinates that are not moving relative to an observer. In other words the stuff that is going at the same speed as you… For example when you are traveling down a highway in car, everything in the car is in your frame of reference, it doesn't appear to be moving. Whereas everything outside the car is moving relative to you and is in a different reference frame, an exception would be a car traveling with the same velocity as your car…

Special relativity is limited to reference frames that are not accelerating. A reference frame that is not accelerating is referred to as an inertial reference frame (think inertia). Think back to the idea of not being able to measure speed we can only measure acceleration… An inertial reference frame is one where you can not measure an acceleration. If the reference frame is accelerating we must employ General Relativity, Special Relativity is a special case of General Relativity, and we'll get there in due time… but lets blow one neuron at a time.

G.2.2 State the two postulates of the special theory of relativity.

1. The laws of physics are the same for all inertial observers.
2. All inertial observers will measure the same value for the free space velocity of light irrespective of their relative velocity.

Galilean relativity is explained by the first postulate, however it does not take in to account the second postulate. It can also be argued that the second postulate is contained in the first, if Maxwell's equations are considered, since they predict the speed of light is constant!

G.2.3 Discuss the concept of simultaneity.

In our every day experience if two events happen simultaneously for one observer we assume that the events happen simultaneously for any observer. However this is not quite right…

That the classic situation of two observers one on a train and the second standing on the ground watching the train pass. The train, along with observer Y, is traveling to the right with velocity v, while observer X is standing on the ground with zero velocity. Just as observer Y passes observer X, observer X (ground) sees two lighting bolts hit the front and rear of the train at the same time, i.e. the two events happened simultaneously for observer X. But observer Y on the train will not see the events simultaneously…

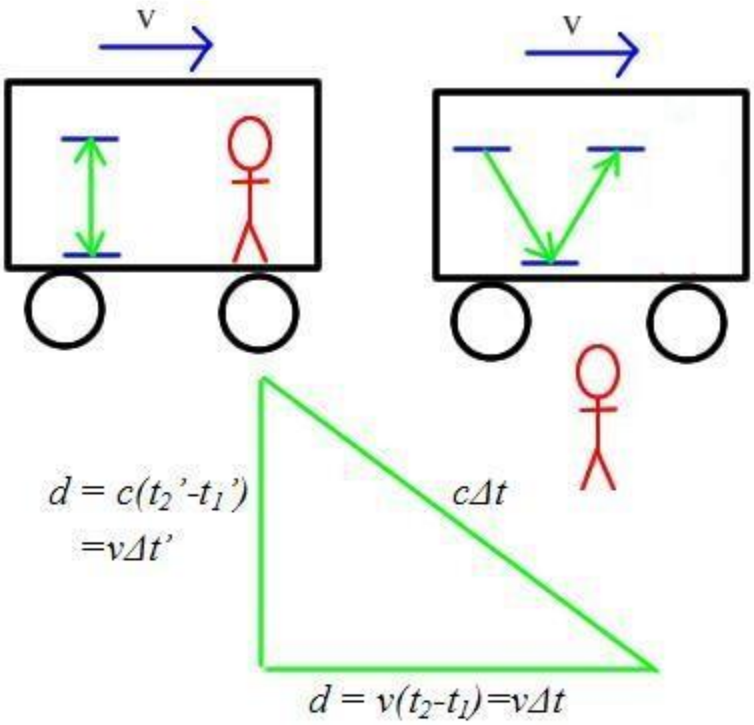
Since the speed of light is independent of the observers speed, by the time the light from each lightning bolt reaches observer Y the train will have moved forward. Thus the distance the light must travel is different for the bolt that hit the front and for the bolt that hit the back. If the distances are different, but the speeds are the same, the time taken must be different. In other words the observer Y will see the bolt in the front of the train first followed by the bolt in the back, i.e. the events for observer Y do not occur simultaneously! This is something new…

G.3.1 Explain the concept of a light clock.

A light clock is an imaginary device that bounces light back and forth between two parallel flat mirrors. The distance between the mirrors is fixed and since the speed of light is a constant the time for light to bounce back and forth is constant. We can therefore think it of as a type of clock.

G.3.2 Derive the time dilation formula

If we do another "thought experiment," this time with a light clock. If observer Y is on a train with a light clock traveling at a constant velocity, observer Y "sees" the light bounce back in forth in a vertical line. Whereas the observer on the ground sees the light bounce in a saw-tooth pattern as shown below. From this we can find a relationship between observer X's time and observer Y's time.



We can create a triangle of the motion. Where the hypotenuse is the distance traveled as seen by observer X, the light travels at speed c for time Δt. The vertical portion is the distance traveled as seen by observer Y. The horizontal portion is the horizontal distance traveled as seen by observer X. If we apply Pythagorean theorem:

$$(c\Delta t)^2 = (c\Delta t')^2 + (v\Delta t)^2 \tag{1}$$

$$c^2\Delta t^2 = c^2\Delta t'^2 + v^2\Delta t^2 \tag{2}$$

$$c^2\Delta t^2 - v^2\Delta t^2 = c^2\Delta t'^2 \tag{3}$$

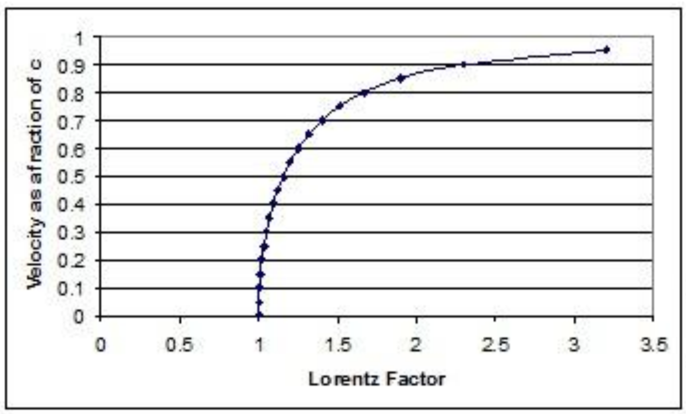
$$\Delta t^2 = \frac{c^2}{c^2 - v^2}\Delta t'^2 \tag{4}$$

$$\Delta t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\Delta t' \tag{5}$$

$$\Delta t = \lambda \Delta t' \tag{6}$$

Where λ is the Lorentz factor. Notice that λ is always larger than one. This means that observer X will see observer Y's clock move slowly. But the same can be said for Y looking at X's clock. Both observers see the other clock move more slowly. You can never see a clock move faster… always slower.

G.3.3 Draw and annotate a graph of how a Lorentz factor varies with relative velocity.



The graph to the right shows how the Lorentz factor changes as the velocity of the object increases as a fraction of the speed of light.

G.3.4 Define the term proper time.

Proper time is the time measured when the clock is at rest relative to the observer. In the example of the moving train and light clock, proper time is the time measured by the observer in the Train for the light to travel from the top of the clock to the bottom.

G.3.6 Describe the phenomenon of length contraction.

The phenomenon of length contraction is when a stationary observer sees a moving object contract in the direction of motion. The contraction is only in the direction of motion. A stationary observer will see a fast moving rocket ship very short but it will not look narrower, it will appear like a pancake.

$$l' = l\sqrt{1 - \frac{v^2}{c^2}} \tag{7}$$

$$\gamma l' =' l \tag{8}$$

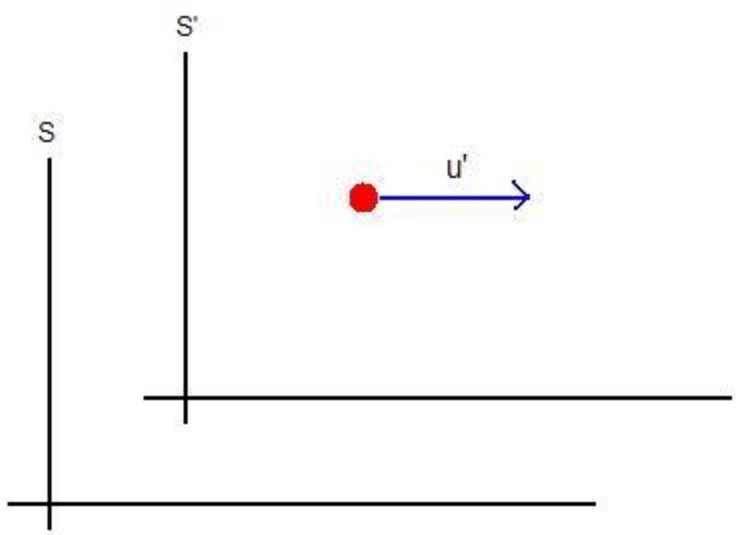
G.3.7 Define the term proper length

Much like proper time, proper length is the length measured by a stationary observer.

G.4.1 Describe how the concept of time dilation leads to the "twin paradox."

G.4.2 Solve one-dimensional problems involving the relativistic addition of velocities.

If a particle is moving with a velocity u' with respect to the frame S' and the frame S is moving with velocity v with respect to the frame S. What is the speed of the particle as measured from the frame S?



If we apply the Lorentz transformations we get the result:

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} \tag{9}$$

G.4.3 Define the term rest mass.

We use the term rest mass to describe the mass of an object when it is at rest relative to the observer. We give the rest mass the symbol m0. The mass m when moving at a speed v relative to the observer is given by:

(10)
$m = \gamma m0$

G.4.5 Explain in terms of the relativistic mass equation why no mass can ever attain or exceed the speed of light in a vacuum.

As a mass accelerates the mass increases, as the mass increases the force needed to accelerate the mass also increases. Therefore it gets harder and harder to accelerate the object. Looking at the equation for the mass of an object it can be seen that if the mass attained the speed of light it would have an infinite mass!

G.4.6 State that the equivalence of mass and energy is predicted by special relativity.

G.4.7 Distinguish between rest mass energy and total energy.

The equivalence between mass and energy was predicted by special relativity. The relationship is described mathematically by:

(11)
$E = mc$^2

Where m is the relativistic mass (as described above).

Rest mass energy is the energy of an object when it is measured by an observer in the same reference frame. Total energy takes into account both the rest mass energy and the kinetic energy of the object. Rest mass energy is the energy inherit in the object due solely to its mass. The rest mass is given by the equation:

(12)
$E0 = m0c$^2

If an object is at rest and has energy E0 and then accelerates to a given velocity (adding kinetic energy) its total energy is then given by E. The difference of E and E0 is the kinetic energy of the body:

(13)
$Ek = E - E0 = mc^2 - m0c^2$

G.5.1 Discuss muon decay as experimental evidence for time dilation and length contraction.

Some of the earliest confirmation of Special Relativity was the detection of Muons. Muons are created when sunlight strikes molecules in the upper atmosphere. Muons have a very short half life, they decay in a similar fashion to radioactive material. The half life of a Muon is only 2.2 microseconds, while at rest or in the same reference frame. Even at speeds approaching the speed of light (over 0.90c) the muons should decay before they reach the earths surface and so we should not be able to detect very many at the surface. However we can detect larger numbers of muons at the surface of the Earth. Because the muons are traveling at such high speeds we on earth see their clocks run slow and thus their half-life greatly increased. In the muon's reference frame it sees the distance to the earth's surface greatly reduced. So in both reference frames the muon has enough time to reach the surface of the earth…

G.5.2 Outline the set-up of the Michelson-Morley experiment.

G.5.3 Outline the results of the Michelson Morley experiment and its implication

Read this: http://www.physicsdaily.com/physics/Michelson-Morley_experiment

And this: http://www.physicsdaily.com/physics/Interferometer

G.6.1 Solve problems involving objects moving at relativistic speeds using Einstein's mass-energy equation.

Einstein's mass-energy equation can be derived starting with:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (14)$$

Square both sides and multiply by c4:

$$m^2c^4 - m^2v^2v^2 = m_0^2C^4 \tag{15}$$

$$E^2 = p^2c^2 + m_0^2C^4 \tag{16}$$

Source: http://ibphysicsstuff.wikidot.com/special-relativity