Separation of Multicomponent Mixtures By Use of Conventional Distillation Column with Multiple Stages

Keywords: Convention Distillation Column, Theta and Zedadda method, triangular matrix,

A conventional distillation column is defined as one that has one feed and two product streams, the distillate D and the bottoms B. consider the case in which the following specifications are made for a column at steady state operation:

- Number of plates in each section of the column
- Quality, composition and the thermal condition of the feed
- Column pressure
- Type of overhead condenser (total or partial)
- Reflux ratio, L_o/D or V_1 or L_o
- One specification on the distillate such as the total flow rate D.

For this set of conditions the problem is to find the compositions of the top and bottom products. The set of equations required to represent such a system is as follows:

1. ***Material balance equations***

$V_{j+1}.y_{j+1,i} = L_j x_{j,i} + DX_{D,i}$ $(1 \le i \le C)$ and $(0 \le j \le f\text{-}2)$

$V_f.y_{f,i} + V_f.y_{f,i} = L_{f-1} x_{f-1,i} + DX_{D,i}$ $(1 \le i \le C)$

$V_{j+1}.y_{j+1,i} = L_j x_{j,i} - BX_{B,i}$ $(1 \le i \le C)$ and $(f \le j \le N)$

$FX_i = DX_{D,i} + BX_{B,i}$ $(1 \le i \le C)$

2. ***Equilibrium Relations***

$$\sum_{i=1}^{C} y_{j,i} = 1$$

$$\sum_{i=1}^{C} x_{j,i} = 1$$

Where i = 1 to C …29.1

and j = 0 to N + 1

3. ***Summation Equation***

$$y_{j,i} = k_{j,i}.x_{j,i}$$

4. Enthalpy balance equations

$V_{j+1}.H_{j+1} = L_j h_j + DH_D + Q_C$ $(0 \le j \le f\text{-}2)$

$V_f.H_f + V_f.H_f = L_{f-1}h_{f-1} + DH_D + Q_C$

$V_{j+1}.H_{j+1} = L_j h_j - Bh_B + Q_R$ $(f \le j \le N)$

$FH = B.h_B + DH_D + Q_C - Q_R$

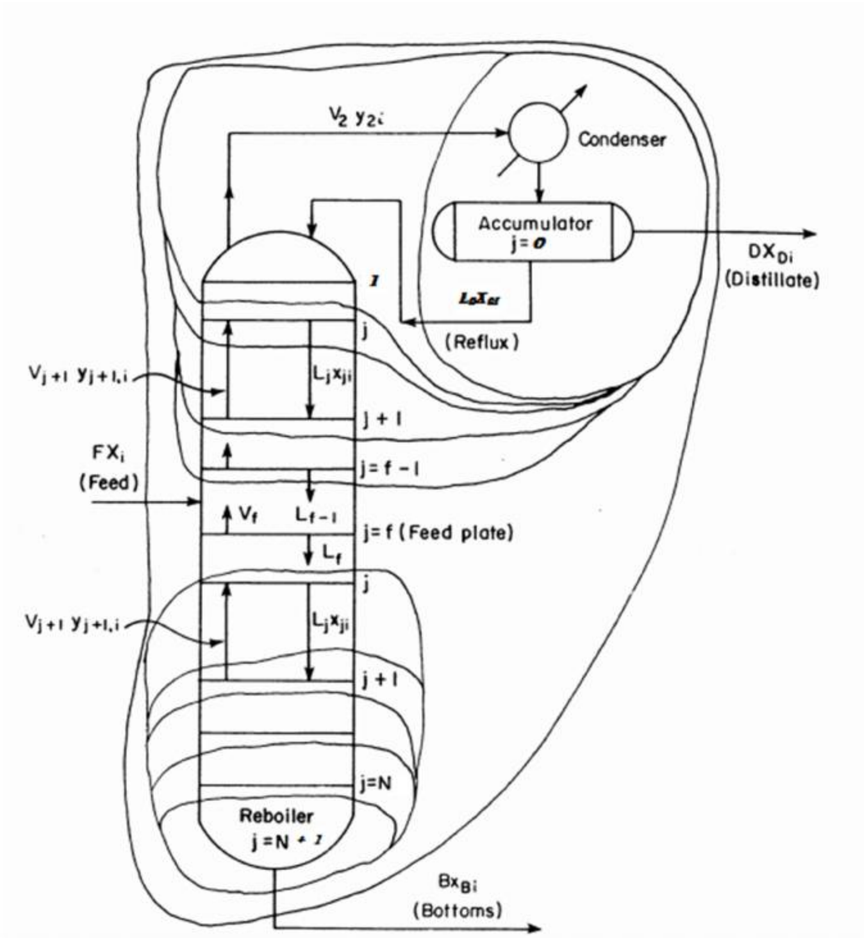

Fig. 29.1: Conventional Distillation Column

Inspection of this set of equations show that the equations are logical extension of those may be stated for a binary system.

When it is assumed that the vapour and liquid streams form ideal solutions, the enthalpy per mole of vapour and the enthalpy per mole of liquid leaving plate ‘j’ are given by following expressions.

$$H_j = \sum_{i=1}^{C} H_{j,i} y_{j,i} \qquad \text{(Vapour)} \qquad \ldots 29.2$$

$$h_j = \sum_{i=1}^{C} h_{j,i} x_{j,i} \qquad \text{(Liquid)} \qquad \ldots 29.3$$

Where, the enthalpy of each pure component 'i' in the vapour and liquid streams leaving plate 'j' are represented by $H_{j,i}$ & $h_{j,i}$ respectively. These enthalpies are evaluated at the temperature and pressure of j^{th} plate, H_D depends on condenser used. For a total condenser (D is withdrawn from the accumulation as a liquid at its bubble point temperature 'T'$_0$ at the column pressure, and $y_{1,i} = x_{Di} = X_{Di}$),

$$H_D = \sum_{i=1}^{C} h_{0,i} X_{Di} = \sum_{i=1}^{C} h_{0,i} x_{0i} = h_0 \qquad \ldots 29.4$$

For a partial condenser (D is withdrawn from the accumulation as a vapour at its dew point temperature 'T'$_0$ at the column pressure, and $y_{0,i} = X_{Di}$),

$$H_D = \sum_{i=1}^{C} H_{0,i} X_{Di} = \sum_{i=1}^{C} H_{0,i} y_{0i} = H_0 \qquad \ldots 29.5$$

The enthalpy per mole of bottom has double but equivalent representation, h_B and h_{N+1} i.e.

$$h_B = \sum_{i=1}^{C} h_{Bi} x_{Bi} = \sum_{i=1}^{C} h_{N+1,i} x_{N+1,i} = h_{N+1} \qquad \ldots 29.6$$

The symbols Q_C & Q_R represent the condenser & reboiler heat duties, respectively. The calculation procedure consists

- An iterative technique which employs the θ method of convergence.
- The tridiagonal formulation of the component material balances & equilibrium relationships.
- The K_b method for the determination of temperature.
- The constant composition method for the determination of the total flow rates.

Formulation of Tridiagonal Matrix:

In the case of the component material balances, a new set of variables, the component flow rates in the vapour and liquid phases are introduced, namely,

$$v_{j,i} = V_j y_{j,i} \quad \text{and} \quad l_{j,i} = L_j x_{j,i} \qquad \ldots 29.7$$

Also the flow rates of component 'i' in the distillate and the bottoms are represented by

$$d_i = DX_{Di} \text{ and } b_i = BX_{Bi} \quad \text{…29.8}$$

and the flow rates of component 'i' in the vapour and liquid parts of the feed are represented by

$$v_{f,i} = V_f y_{f,i} \text{ and } l_{f,i} = L_f X_{f,i} \quad \text{…29.9}$$

The equilibrium relationship $y_{j,i} = k_{j,i} x_{j,i}$ may be restated in an equivalent form in terms of the component flow rates $v_{j,i}$ & $l_{j,i}$ as follows. First, observe that the expression $y_{j,i} = k_{j,i} x_{j,i}$ may be restated in the form:

$$V_j y_{j,i} = \left(\frac{V_j k_{j,i}}{L_j} \right) L_j x_{j,i} \quad \text{…29.10}$$

$$v_{j,i} = S_{j,i} l_{j,i} \text{ & } l_{j,i} = A_{j,i} v_{j,i} \quad \text{…29.11}$$

Where, $A_{j,i} = \frac{1}{S_{j,i}} = \frac{L_j}{V_j k_{j,i}}$

Instead of enclosing the end of the column and respective plates in each section of the column as demonstrated by eq. (29.1), an equivalent set of component material balances is obtained by enclosing each plate (j = 0, 1, 2, ..., N, N+1) by a component material balance as demonstrated ion following figure. The corresponding set of material balances for each component 'i' are as follows:

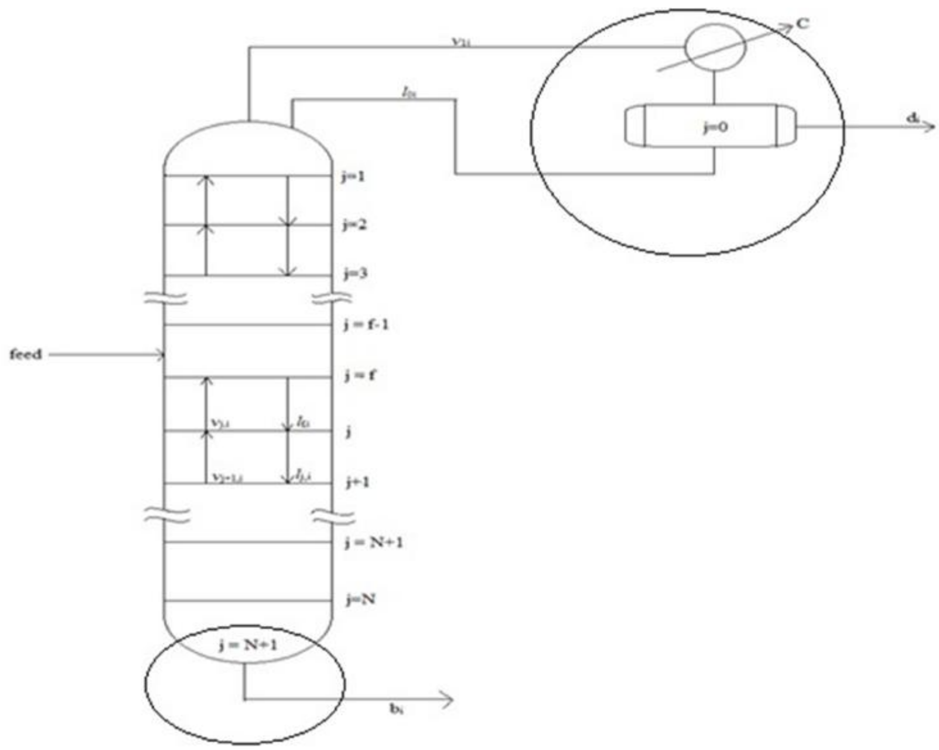

Fig. 29.2: Conventional Distillation Column

$$\left.\begin{array}{ll}
-l_{0,i}-d_i+v_{j,i}=0 & \\
-l_{j-1,i}-v_{j,i}-l_{j,i}+v_{j+1,i}=0 & (1\le j\le f-2) \\
l_{f-2,i}-v_{f-1,i}-l_{f-1,i}+v_{f,i}=-v_{f,i} & \\
l_{f-1,i}-v_{f,i}-l_{f,i}+v_{f+1,i}=-l_{f,i} & \\
l_{j-1,i}-v_{j,i}-l_{j,i}+v_{j+1,i}=0 & (f+1\le j\le N) \\
l_{N,i}-v_{N+1,i}-b_i=0 &
\end{array}\right\} \qquad \ldots 29.12$$

Except for the first expression of eq. (29.12), the $l_{j,i}$'s may be eliminated from eq. (29.12) by use of equilibrium relationship eq. (29.11).

For the case of total condenser, $l_{0,i}$ and d_i have the same composition and thus

$$l_{0,i}=\left(\frac{L_0}{D}\right)d_i \qquad \ldots 29.13$$

For a partial condenser, $y_{0,i}=X_{Di}$ and hence

$$D.X_{Di} = \left(\frac{Dk_{0,i}}{L_0}\right)L_0 x_{0,i} \qquad \ldots 29.14$$

$$l_{0,i} = A_{0,i} d_i \qquad \ldots 29.15$$

Where $A_{0,i} = \dfrac{L_0}{k_{0,i} D}$

The expression given by eq.(29.15) may be used to represent both a partial condenser and a total condenser, provided $A_{0,i}$ is set equal to L_0/D for a total condenser. Also, the form of $A_{N+1,i}$ differs slightly from that of $A_{j,i}$ because of the double representation of the reboiler by the subscripts $"N+1"$ and B, i.e.

$$V_{N+1} y_{N+1,i} = \left(\frac{V_{N+1} k_{N+1,i}}{B}\right) B x_{Bi}$$

or, $b_i = A_{N+1,i} v_{N+1,i}$ …29.16

$$A_{N+1,i} = \frac{B}{k_{N+1,i} V_{N+1}}$$

Where the $l_{j,i}$'s and b_i are eliminated from eq. (29.12) by use of eqs. (29.11), (29.13), (29.15), and (29.16), the following results are obtained:

$$\left.\begin{array}{ll}
-\left(A_{0,i}+1\right)d_i + v_{j,i} = 0 & \\
A_{j-1,i} v_{j-1,i} - \left(A_{j,i}+1\right)v_{j,i} + v_{j+1,i} = 0 & \left(1 \le j \le f-2\right) \\
A_{f-2,i} v_{f-2,i} - \left(A_{f-1,i}+1\right)v_{f-1,i} + v_{f,i} = -v_{f,i} & \\
A_{f-1,i} v_{f-1,i} - \left(A_{f,i}+1\right)v_{f,i} + v_{f+1,i} = -l_{f,i} & \\
A_{j-1,i} v_{j-1,i} - \left(A_{j,i}+1\right)v_{j,i} + v_{j+1,i} = 0 & \left(f+1 \le j \le N\right) \\
A_{N,i} v_{N,i} - \left(A_{N+1,i}+1\right)v_{N+1,i} = 0 &
\end{array}\right\} \qquad \ldots 29.17$$

This set of equations may be written in matrix form:

$$A_i v_i = -L_i \qquad \ldots 29.18$$

Where,

$A_i =$

$$
\begin{pmatrix}
-P_{0,i} & 1 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \\
A_{0,i} & -P_{1,i} & 1 & 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & 1 & 0 & 0 & 0 \\
0 & \cdots & \cdots & A_{f-2,i} & \cdots & \cdots & -P_{f,i} & 1 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & A_{N-1,i} & -P_{N,i} & 1 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & A_{N,i} & -P_{N+1,i}
\end{pmatrix}
$$

$$v_i = [d_i v_{1,i} v_{2,i} \ldots\ldots v_{f-1,i} v_{f,i} \ldots\ldots v_{N,i} v_{N+1,i}]^T$$

$$L_i = [000 \ldots\ldots v_{f-1,i} v_{f,i} \ldots\ldots 00]^T$$

$$P_{ji} = A_{ji} + 1$$

On the basis of the assumed temperatures and total flow rates, the absorption factors $\{A_{ji}\}$ appearing in eq. (29.18) may be evaluated for component i on each plate j, since A_i in eq. (29.18) is of tridiagonal form, this matrix equation may be solved for the calculated values of the vapour rates for the component i [denoted by $(v_{ji})_{cal}$] by use of the Thomas algorithm. Consider the following set of linear equations in the variables x_0, x_1,........., x_N , x_{N+1} whose coefficients, form a tridiagonal matrix.

$$B_0 x_0 + C_0 x_1 = D_0$$

$$A_1 x_0 + B_1 x_1 + C_1 x_2 = D_1$$

$$A_2 x_1 + B_2 x_2 + C_1 x_2 = D_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$A_N x_{N-1} + B_N x_N + C_N x_{N+1} = D_N$$

$$A_{N+1} x_N + B_{N+1} x_{N+1} = D_{N+1}$$

The following recurrence formulas are applied in the order stated:

$$f_0 = \frac{C_0}{B_0} \; , \quad g_0 = \frac{D_0}{B_0}$$

$$f_K = \frac{C_k}{B_k - A_k f_{k-1}} \qquad (k = 1, 2, ..., N+1)$$

$$g_K = \frac{D_k - A_k g_{k-1}}{B_k - A_k f_{k-1}} \qquad (k = 1, 2, ..., N+1)$$

After the f 's and g 's have been computed, the values of $x_{N+1}, x_N, ... x_1, x_0$ are computed as follows:

$$v_{N+1} = g_{N+1}$$

$$x_k = g_k - f_k x_{k+1} \qquad (k = N, N-1, ..., 2, 1, 0)$$

After these recurrence formulas have been applied for each component i and the complete set of vapor rates $\{(v_{ji})_{ca}\}$ has been calculate, the corresponding set of liquid rates $\{(l_{ji})_{ca}\}$ is then calculated by use of Eq. (29.7). These sets of calculated flow rates are used in conjunction with $\bar{\ }$ method of convergence and the K_b method in the determination of an improved set of temperatures.

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