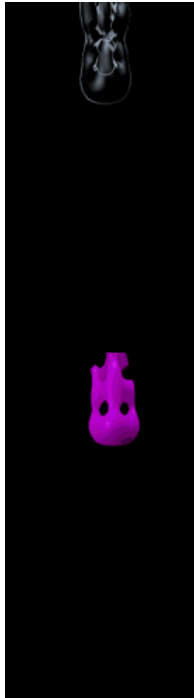


Quantum Mechanics_ *Viscosity*

Viscosity



A simulation of substances with different viscosities. The substance above has lower viscosity than the substance below

Common symbols

η, μ

SI unit

$\text{Pa} \cdot \text{s} = \text{kg}/(\text{s} \cdot \text{m})$

Derivations from $\mu = \underline{G} \cdot t$
other quantities

The **viscosity** of a fluid is a measure of its resistance to gradual deformation by shear stress or tensile stress. For liquids, it corresponds to the informal notion of "thickness". For example, honey has a much higher viscosity than water.^[1]

Viscosity is due to the friction between neighboring particles in a fluid that are moving at different velocities. When the fluid is forced through a tube, the fluid generally moves faster near the axis and very slowly near the walls; therefore, some stress (such as a Pressure difference between the two ends of the tube) is needed to overcome the friction between layers and keep the fluid moving. For the same velocity pattern, the

stress required is proportional to the fluid's viscosity. A liquid's viscosity depends on the size and shape of its particles and the attractions between the particles.^{[[citation needed](#)]}

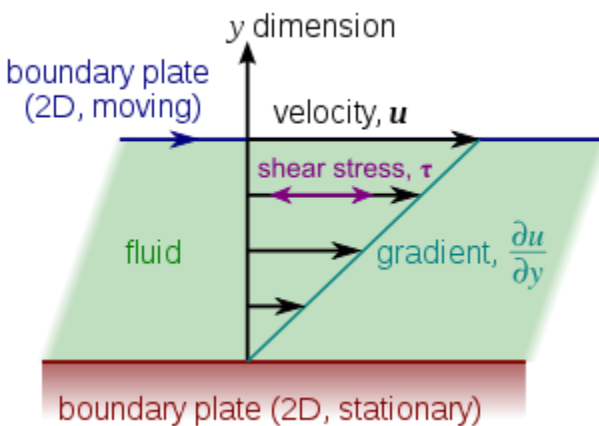
A fluid that has no resistance to shear stress is known as an **ideal fluid** or **inviscid fluid**. Zero viscosity is observed only at very low temperatures, in superfluids. Otherwise all fluids have positive viscosity. If the viscosity is very high, for instance in pitch, the fluid will appear to be a solid in the short term. A liquid whose viscosity is less than that of water is sometimes known as **amobile** liquid, while a substance with a viscosity substantially greater than water is called a **viscous** liquid.

Etymology

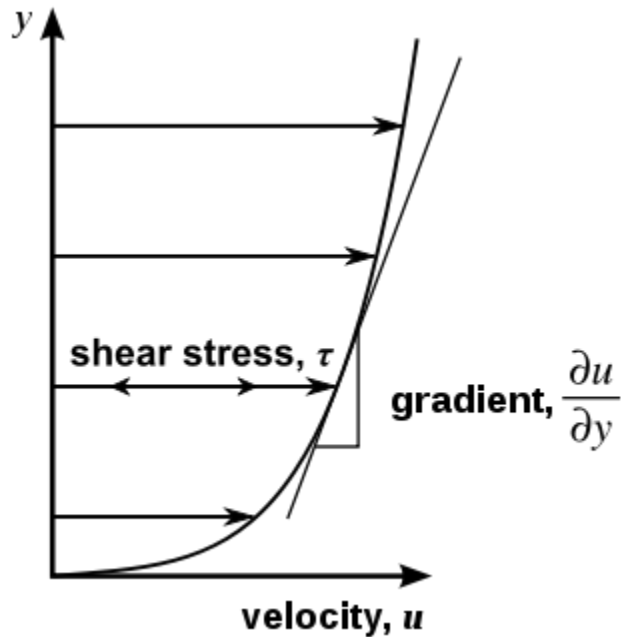
The word "viscosity" is derived from the Latin "viscum", meaning "anything sticky, birdlime made from mistletoe, mistletoe". A viscous glue called birdlime was made from mistletoe berries and was used for lime-twigs to catch birds.^[2]

Definition

Dynamic (shear) viscosity



Laminar shear of fluid between two plates. Friction between the fluid and the moving boundaries causes the fluid to shear. The force required for this action is a measure of the fluid's viscosity.



In a general parallel flow (such as could occur in a straight pipe), the shear stress is proportional to the gradient of the velocity.

The **dynamic (shear) viscosity** of a fluid expresses its resistance to shearing flows, where adjacent layers move parallel to each other with different speeds. It can be defined through the idealized situation known as a Couette flow, where a layer of fluid is trapped between two horizontal plates, one fixed and one moving horizontally at constant speed u . (The plates are assumed to be very large, so that one need not consider what happens near their edges.)

If the speed of the top plate is small enough, the fluid particles will move parallel to it, and their speed will vary linearly from zero at the bottom to u at the top. Each layer of fluid will move faster than the one just below it, and friction between them will give rise to a force resisting their relative motion. In particular, the fluid will apply on the top plate a force in the direction opposite to its motion, and an equal but opposite one to the bottom plate. An external force is therefore required in order to keep the top plate moving at constant speed.

The magnitude F of this force is found to be proportional to the speed u and the area A of each plate, and inversely proportional to their separation y :

$$F = \mu A \frac{u}{y}.$$

The proportionality factor μ in this formula is the viscosity (specifically, the **dynamic viscosity**) of the fluid.

The ratio u/y is called the *rate of shear deformation* or *shear velocity*, and is the derivative of the fluid speed in the direction perpendicular to the plates. Isaac Newton expressed the viscous forces by the differential equation

$$\tau = \mu \frac{\partial u}{\partial y},$$

where $\tau = F/A$ and $\partial u/\partial y$ is the local shear velocity. This formula assumes that the flow is moving along parallel lines and the y axis, perpendicular to the flow, points in the direction of maximum shear velocity. This equation can be used where the velocity does not vary linearly with y , such as in fluid flowing through a pipe.

Use of the Greek letter mu (μ) for the dynamic stress viscosity is common among mechanical and chemical engineers, as well as physicists.[3][4][5] However, the Greek letter eta (η) is also used by chemists, physicists, and the IUPAC. [6]

Kinematic viscosity

The **kinematic viscosity** is the ratio of the dynamic viscosity μ to the density of the fluid ρ . It is usually denoted by the Greek letter nu (ν).

$$\nu = \frac{\mu}{\rho}$$

It is a convenient concept when analyzing the Reynolds number, that expresses the ratio of the inertial forces to the viscous forces:

$$Re = \frac{\rho u L}{\mu} = \frac{u L}{\nu},$$

where L is a typical length scale in the system.

Bulk viscosity

When a compressible fluid is compressed or expanded evenly, without shear, it may still exhibit a form of internal friction that resists its flow. These forces are related to the rate of compression or expansion by a factor σ , called the volume viscosity, **bulk viscosity** or **second viscosity**.

The bulk viscosity is important only when the fluid is being rapidly compressed or expanded, such as in sound and shock waves. Bulk viscosity explains the loss of energy in those waves, as described by Stokes' law of sound attenuation.

Viscosity tensor

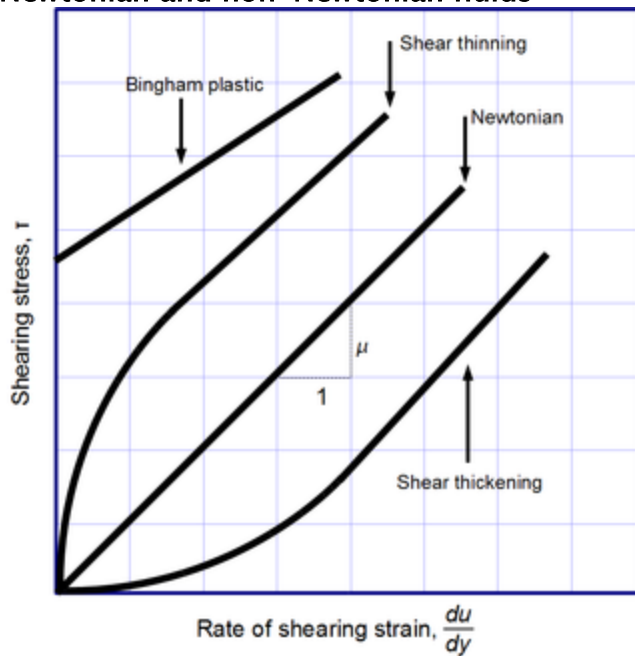
Main article: Viscous stress tensor

In general, the stresses within a flow can be attributed partly to the Deformation of the material from some rest state (elastic stress), and partly to the rate of change of the deformation over time (viscous stress). In a fluid, by definition, the elastic stress includes only the hydrostatic pressure.

In very general terms, the fluid's viscosity is the relation between the strain rate and the viscous stress. In the Newtonian fluid model, the relationship is by definition a linear map, described by a viscosity tensor that, multiplied by the strain rate tensor (which is the gradient of the flow's velocity), gives the viscous stress tensor.

The viscosity tensor has nine independent degrees of freedom in general. For isotropic Newtonian fluids, these can be reduced to two independent parameters. The most usual decomposition yields the stress viscosity μ and the bulk viscosity σ .

Newtonian and non-Newtonian fluids



Viscosity, the slope of each line, varies among materials

Newton's law of viscosity is a constitutive equation (like Hooke's law, Fick's law, Ohm's law): it is not a fundamental law of nature but an approximation that holds in some materials and fails in others.

A fluid that behaves according to Newton's law, with a viscosity μ that is independent of the stress, is said to be Newtonian. Gases, water and many common liquids can be considered Newtonian in ordinary conditions and contexts. There are many non-

Newtonian fluids that significantly deviate from that law in some way or other. For example:

- Shear thickening liquids, whose viscosity increases with the rate of shear strain.
- Shear thinning liquids, whose viscosity decreases with the rate of shear strain.
- Thixotropic liquids, that become less viscous over time when shaken, agitated, or otherwise stressed.
- Rheopectic liquids, that become more viscous over time when shaken, agitated, or otherwise stressed.
- Bingham plastics that behave as a solid at low stresses but flows as a viscous fluid at high stresses.

Shear thinning liquids are very commonly, but misleadingly, described as thixotropic. Even for a Newtonian fluid, the viscosity usually depends on its composition and temperature. For gases and other compressible fluids, it depends on temperature and varies very slowly with pressure.

The viscosity of some fluids may depend on other factors. A magnetorheological fluid, for example, becomes thicker when subjected to a magnetic field, possibly to the point of behaving like a solid.

Viscosity in solids

The viscous forces that arise during fluid flow must not be confused with the elastic forces that arise in a solid in response to shear, compression or extension stresses. While in the latter the stress is proportional to the *amount* of shear deformation, in a fluid it is proportional to the *rate* of deformation over time. (For this reason, Maxwell used the term **fugitive elasticity** for fluid viscosity.)

However, many liquids (including water) will briefly react like elastic solids when subjected to sudden stress. Conversely, many "solids" (even granite) will flow like liquids, albeit very slowly, even under arbitrarily small stress.[7] Such materials are therefore best described as possessing both elasticity (reaction to deformation) and viscosity (reaction to rate of deformation); that is, being viscoelastic.

Indeed, some authors have claimed that amorphous solids, such as glass and many polymers, are actually liquids with a very high viscosity (e.g.~greater than 10^{12} Pa·s). [8] However, other authors dispute this hypothesis, claiming instead that there is some threshold for the stress, below which most solids will not flow at all,[9] and that alleged instances of glass flow in window panes of old buildings are

due to the crude manufacturing process of older eras rather than to the viscosity of glass.[10]

Viscoelastic solids may exhibit both shear viscosity and bulk viscosity. The **extensional viscosity** is a linear combination of the shear and bulk viscosities that describes the reaction of a solid elastic material to elongation. It is widely used for characterizing polymers.

In geology, earth materials that exhibit viscous deformation at least three times greater than their elastic deformation are sometimes called rheids. [11]

Viscosity measurement

Main article: Viscometer

Viscosity is measured with various types of viscometers and rheometers. A rheometer is used for those fluids that cannot be defined by a single value of viscosity and therefore require more parameters to be set and measured than is the case for a viscometer. Close temperature control of the fluid is essential to acquire accurate measurements, particularly in materials like lubricants, whose viscosity can double with a change of only 5 °C.

For some fluids, viscosity is a constant over a wide range of shear rates (Newtonian fluids). The fluids without a constant viscosity (non-Newtonian fluids) cannot be described by a single number. Non-Newtonian fluids exhibit a variety of different correlations between shear stress and shear rate.

One of the most common instruments for measuring kinematic viscosity is the glass capillary viscometer.

In coating industries, viscosity may be measured with a cup in which the efflux time is measured. There are several sorts of cup– e.g. Zahn cup, Ford viscosity cup– with usage of each type varying mainly according to the industry. The efflux time can also be converted to kinematic viscosities (centistokes, cSt) through the conversion equations.[12]

Also used in coatings, a Stormer viscometer uses load-based rotation in order to determine viscosity. The viscosity is reported in Krebs units (KU), which are unique to Stormer viscometers.

Vibrating viscometers can also be used to measure viscosity. These models such as the *Dynatrol* use vibration rather than rotation to measure viscosity.

Extensional viscosity can be measured with various rheometers that apply extensional stress.

Volume viscosity can be measured with an acoustic rheometer.

Apparent viscosity is a calculation derived from tests performed on drilling fluid used in oil or gas well development. These calculations and tests help engineers develop and maintain the properties of the drilling fluid to the specifications required.

Units

Dynamic viscosity

The SI physical unit of dynamic viscosity is the pascal-second (Pa·s), (equivalent to (N·s)/m², or kg/(m·s)). If a fluid with a viscosity of one Pa·s is placed between two plates, and one plate is pushed sideways with a shear stress of one pascal, it moves a distance equal to the thickness of the layer between the plates in one second. Water at 20 °C has a viscosity of 0.001002 Pa·s, while a typical motor oil could have a viscosity of about 0.250 Pa·s.[13]

The cgs physical unit for dynamic viscosity is the poise[14] (P), named after Jean Léonard Marie Poiseuille. It is more commonly expressed, particularly in ASTM standards, as centipoise (cP). Water at 20 °C has a viscosity of 1.0020 cP.

$$1 \text{ P} = 0.1 \text{ Pa} \cdot \text{s},$$

$$1 \text{ cP} = 1 \text{ mPa} \cdot \text{s} = 0.001 \text{ Pa} \cdot \text{s} = 0.001 \text{ N} \cdot \text{s}/\text{m}^2.$$

Kinematic viscosity

The SI unit of kinematic viscosity is m²/s.

The cgs physical unit for kinematic viscosity is the stokes (St), named after George Gabriel Stokes. It is sometimes expressed in terms of centistokes (cSt). In U.S. usage, stoke is sometimes used as the singular form.

$$1 \text{ St} = 1 \text{ cm}^2 \cdot \text{s}^{-1} = 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}.$$

$$1 \text{ cSt} = 1 \text{ mm}^2 \cdot \text{s}^{-1} = 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}.$$

Water at 20 °C has a kinematic viscosity of about 1 cSt.

The kinematic viscosity is sometimes referred to as **diffusivity of momentum**, because it is analogous to diffusivity of heat and diffusivity of mass. It is therefore used in dimensionless numbers which compare the ratio of the diffusivities.

Fluidity

The reciprocal of viscosity is fluidity, usually symbolized by $\phi = 1 / \mu$ or $F = 1 / \mu$, depending on the convention used, measured in reciprocal poise (cm·s·g⁻¹), sometimes called the *rhe*. Fluidity is seldom used in engineering practice.

The concept of fluidity can be used to determine the viscosity of an ideal solution. For two components a and b , the fluidity when a and b are mixed is

$$F \approx \chi_a F_a + \chi_b F_b,$$

which is only slightly simpler than the equivalent equation in terms of viscosity:

$$\mu \approx \frac{1}{\chi_a/\mu_a + \chi_b/\mu_b},$$

where χ_a and χ_b is the mole fraction of component a and b respectively, and μ_a and μ_b are the components' pure viscosities.

Non-standard units

The Reyn is a British unit of dynamic viscosity.

Viscosity index is a measure for the change of kinematic viscosity with temperature. It is used to characterise lubricating oil in the automotive industry.

At one time the petroleum industry relied on measuring kinematic viscosity by means of the Saybolt viscometer, and expressing kinematic viscosity in units of Saybolt Universal Seconds (SUS).[15] Other abbreviations such as SSU (Saybolt Seconds Universal) or SUV (Saybolt Universal Viscosity) are sometimes used. Kinematic viscosity in centistoke can be converted from SUS according to the arithmetic and the reference table provided in ASTM D 2161. [16]

Molecular origins



Pitch has a viscosity approximately 230 billion (2.3×10^{11}) times that of water. [17]

The viscosity of a system is determined by how molecules constituting the system interact. There are no simple but correct expressions for the viscosity of a fluid. The simplest exact expressions are the Green-Kubo relations for the linear shear viscosity or the Transient Time Correlation Function expressions derived by Evans and Morriss in 1985.[18] Although these expressions are each exact, in order to calculate the viscosity of a dense fluid using these relations currently requires the use of molecular dynamics computer simulations.

Gases

Viscosity in gases arises principally from the molecular diffusion that transports momentum between layers of flow. The kinetic theory of gases allows accurate prediction of the behavior of gaseous viscosity.

Within the regime where the theory is applicable:

- Viscosity is independent of pressure and
- Viscosity increases as temperature increases.[19]

James Clerk Maxwell published a famous paper in 1866 using the kinetic theory of gases to study gaseous viscosity.[20] To understand why the viscosity is independent of pressure, consider two adjacent boundary layers (A and B) moving with respect to each other. The internal friction (the viscosity) of the gas is determined by the probability a particle of layer A enters layer B with a corresponding transfer of momentum. Maxwell's calculations show that the viscosity coefficient is proportional to the density, the mean free path, and the mean velocity of the atoms. On the other hand, the *mean free path* is inversely proportional to the density. So an increase in density due to an increase in pressure doesn't result in any change in viscosity.

Relation to mean free path of diffusing particles

In relation to diffusion, the kinematic viscosity provides a better understanding of the behavior of mass transport of a dilute species. Viscosity is related to shear stress and the rate of shear in a fluid, which illustrates its dependence on the mean free path, λ , of the diffusing particles.

From Fluid mechanics, for a Newtonian fluid, the shear stress, τ , on a unit area moving parallel to itself, is found to be proportional to the rate of change of velocity with distance perpendicular to the unit area:

$$\tau = \mu \frac{du_x}{dy}$$

for a unit area parallel to the x-z plane, moving along the x axis. We will derive this formula and show how μ is related to λ .

Interpreting shear stress as the time rate of change of momentum, ρ , per unit area A (rate of momentum flux) of an arbitrary control surface gives

$$\tau = \frac{\dot{p}}{A} = \frac{\dot{m}\langle u_x \rangle}{A}$$

where $\langle u_x \rangle$ is the average velocity, along the x axis, of fluid molecules hitting the unit area, with respect to the unit area.

Further manipulation will show[21]

$$\dot{m} = \rho \bar{u} A$$

$$\langle u_x \rangle = \frac{1}{2} \lambda \frac{du_x}{dy}$$

, assuming that molecules hitting the unit area come from all distances between 0 and λ (equally distributed), and that their average velocities change linearly with distance (always true for small enough λ). From this follows:

$$\tau = \underbrace{\frac{1}{2} \rho \bar{u} \lambda}_{\mu} \cdot \frac{du_x}{dy} \Rightarrow \nu = \frac{\mu}{\rho} = \frac{1}{2} \bar{u} \lambda,$$

where

\dot{m} is the rate of fluid mass hitting the surface,

ρ is the density of the fluid,

\bar{u} is the average molecular speed ($\bar{u} = \sqrt{\langle u^2 \rangle}$),

μ is the dynamic viscosity.

Effect of temperature on the viscosity of a gas

Sutherland's formula can be used to derive the dynamic viscosity of an ideal gas as a function of the temperature:[22]

$$\mu = \mu_0 \frac{T_0 + C}{T + C} \left(\frac{T}{T_0} \right)^{3/2}$$

This in turn is equal to

$$\lambda \frac{T^{3/2}}{T + C}, \quad \text{where} \quad \lambda = \frac{\mu_0(T_0 + C)}{T_0^{3/2}} \quad \text{is a constant for the gas.}$$

in Sutherland's formula:

- μ = dynamic viscosity (Pa · s or $\mu\text{Pa} \cdot \text{s}$) at input temperature T ,
- μ_0 = reference viscosity (in the same units as μ) at reference temperature T_0 ,

- T = input temperature (kelvin),
- T_0 = reference temperature (kelvin),
- C = Sutherland's constant for the gaseous material in question.

Valid for temperatures between $0 < T < 555$ K with an error due to pressure less than 10% below 3.45 MPa.

According to Sutherland's formula, if the absolute temperature is less than C , the relative change in viscosity for a small change in temperature is greater than the relative change in the absolute temperature, but it is smaller when T is above C . The kinematic viscosity though always increases faster than the temperature (that is, $d \log(\nu)/d \log(T)$ is greater than 1).

Sutherland's constant, reference values and λ values for some gases:

| Gas | C [K] | T_0 [K] | μ_0 [$\mu\text{Pa s}$] | λ [$\mu\text{Pa s K}^{-1/2}$] |
|------------------------|------------|--------------|---------------------------------|--|
| <u>air</u> | 120 | 291.15 | 18.27 | 1.512041288 |
| <u>nitrogen</u> | 111 | 300.55 | 17.81 | 1.406732195 |
| <u>oxygen</u> | 127 | 292.25 | 20.18 | 1.693411300 |
| <u>carbon dioxide</u> | 240 | 293.15 | 14.8 | 1.572085931 |
| <u>carbon monoxide</u> | 118 | 288.15 | 17.2 | 1.428193225 |
| <u>hydrogen</u> | 72 | 293.85 | 8.76 | 0.636236562 |
| <u>ammonia</u> | 370 | 293.15 | 9.82 | 1.297443379 |
| <u>sulfur dioxide</u> | 416 | 293.65 | 12.54 | 1.768466086 |
| <u>helium</u> | 79.4[23] | 273 | 19[24] | 1.484381490 |

Viscosity of a dilute gas

The Chapman-Enskog equation[25] may be used to estimate viscosity for a dilute gas. This equation is based on a semi-theoretical assumption by Chapman and Enskog. The equation requires three empirically determined parameters: the collision diameter (σ), the maximum energy of attraction divided by the Boltzmann constant (ϵ/κ) and the collision integral ($\omega(T^*)$).

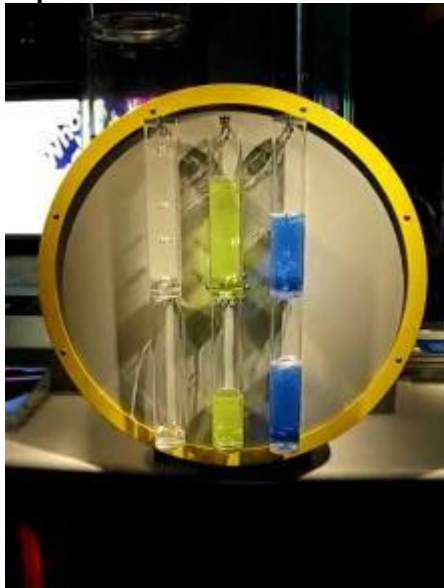
$$\mu_0 \times 10^6 = 2.6693 \frac{(MT)^{1/2}}{\sigma^2 \omega(T^*)},$$

with

- $T^* = \kappa T/\epsilon$ — reduced temperature (dimensionless),

- μ_0 = viscosity for dilute gas ($\mu\text{Pa}\cdot\text{s}$),
- M = molecular mass (g/mol),
- T = temperature (K),
- σ = the collision diameter (\AA),
- ϵ / κ = the maximum energy of attraction divided by the Boltzmann constant (K),
- ω_μ = the collision integral.

Liquids



In liquids, the additional forces between molecules become important. This leads to an additional contribution to the shear stress though the exact mechanics of this are still controversial.^[*citation needed*] Thus, in liquids:

- Viscosity is independent of pressure (except at very high pressure); and
- Viscosity tends to fall as temperature increases (for example, water viscosity goes from 1.79 cP to 0.28 cP in the temperature range from 0 °C to 100 °C); see [temperature dependence of liquid viscosity](#) for more details.

The dynamic viscosities of liquids are typically several orders of magnitude higher than dynamic viscosities of gases.

Viscosity of blends of liquids

The viscosity of the blend of two or more liquids can be estimated using the Refutas equation.^[26] The calculation is carried out in three steps.

The first step is to calculate the Viscosity Blending Number (VBN) (also called the Viscosity Blending Index) of each component of the blend:

$$(1) \quad \text{VBN} = 14.534 \times \ln [\ln(\nu + 0.8)] + 10.975$$

where ν is the kinematic viscosity in centistokes (cSt). It is important that the kinematic viscosity of each component of the blend be obtained at the same temperature.

The next step is to calculate the VBN of the blend, using this equation:

$$(2) \quad \text{VBN}_{\text{Blend}} = [x_A \times \text{VBN}_A] + [x_B \times \text{VBN}_B] + \dots + [x_N \times \text{VBN}_N]$$

where x_X is the mass fraction of each component of the blend.

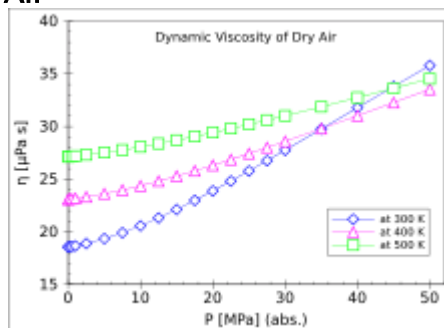
Once the viscosity blending number of a blend has been calculated using equation (2), the final step is to determine the kinematic viscosity of the blend by solving equation (1) for ν :

$$(3) \quad \nu = \exp \left(\exp \left(\frac{\text{VBN}_{\text{Blend}} - 10.975}{14.534} \right) \right) - 0.8,$$

where $\text{VBN}_{\text{Blend}}$ is the viscosity blending number of the blend.

Viscosity of selected substances

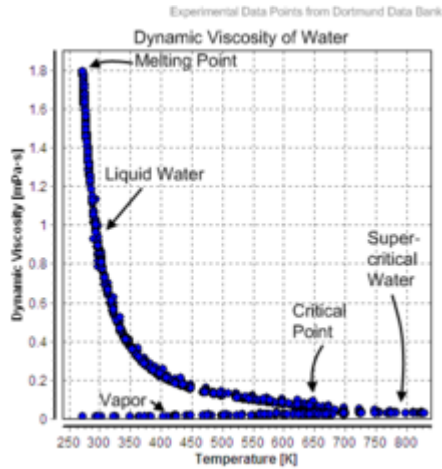
Air



Pressure dependence of the dynamic viscosity of dry air at the temperatures of 300, 400 and 500 K

The viscosity of air depends mostly on the temperature. At 15 °C, the viscosity of air is $1.81 \times 10^{-5} \text{ kg}/(\text{m}\cdot\text{s})$, 18.1 $\mu\text{Pa}\cdot\text{s}$ or $1.81 \times 10^{-5} \text{ Pa}\cdot\text{s}$. The kinematic viscosity at 15 °C is $1.48 \times 10^{-5} \text{ m}^2/\text{s}$ or 14.8 cSt. At 25 °C, the viscosity is 18.6 $\mu\text{Pa}\cdot\text{s}$ and the kinematic viscosity 15.7 cSt. One can get the viscosity of air as a function of temperature from the [Gas Viscosity Calculator](#)

Water



Dynamic viscosity of water

The dynamic viscosity of water is $8.90 \times 10^{-4} \text{ Pa} \cdot \text{s}$ or $8.90 \times 10^{-3} \text{ dyn} \cdot \text{s}/\text{cm}^2$ or 0.890 cP at about 25 °C.

Water has a viscosity of 0.0091 poise at 25 °C, or 1 centipoise at 20 °C.

As a function of temperature T (K): $(\text{Pa} \cdot \text{s}) = A \times 10^{B/(T-C)}$ where $A=2.414 \times 10^{-5} \text{ Pa} \cdot \text{s}$; $B = 247.8 \text{ K}$; and $C = 140 \text{ K}$. *citation needed*

Viscosity of liquid water at different temperatures up to the normal boiling point is listed below.

| Temperature [°C] | Viscosity [mPa · s] |
|------------------|---------------------|
| 10 | 1.308 |
| 20 | 1.002 |
| 30 | 0.7978 |
| 40 | 0.6531 |
| 50 | 0.5471 |
| 60 | 0.4658 |
| 70 | 0.4044 |
| 80 | 0.3550 |
| 90 | 0.3150 |
| 100 | 0.2822 |

Other substances



Example of the viscosity of milk and water. Liquids with higher viscosities make smaller splashes when poured at the same velocity.



honey being drizzled.



Peanut butter is a semi-solid and can therefore hold peaks.

Some dynamic viscosities of Newtonian fluids are listed below:

| <u>Gas</u> | at 0 °C (273 K) | at 27 °C (300 K)[27] |
|-----------------------|-----------------|----------------------|
| <u>air</u> | 17.4 | 18.6 |
| <u>hydrogen</u> | 8.4 | 9.0 |
| <u>helium</u> | | 20.0 |
| <u>argon</u> | | 22.9 |
| <u>xenon</u> | 21.2 | 23.2 |
| <u>carbon dioxide</u> | | 15.0 |
| <u>methane</u> | | 11.2 |
| <u>ethane</u> | | 9.5 |

Viscosity of selected gases at 100 kPa, [$\mu\text{Pa} \cdot \text{s}$]

| <u>Fluid</u> | Viscosity [Pa · s] | Viscosity [cP] |
|---------------------------|------------------------|-------------------|
| <u>blood</u> (37 °C)[8] | $(3-4) \times 10^{-3}$ | 3-4 |
| <u>honey</u> | 2-10 | 2,000-10,000 |
| <u>molasses</u> | 5-10 | 5,000-10,000 |
| <u>molten glass</u> | 10-1,000 | 10,000-1,000,000 |
| <u>chocolate syrup</u> | 10-25 | 10,000-25,000 |
| <u>molten chocolate</u> * | 45-130[28] | 45,000-130,000 |
| <u>ketchup</u> * | 50-100 | 50,000-100,000 |
| <u>lard</u> | ≈ 100 | $\approx 100,000$ |
| <u>Peanut butter</u> * | ≈ 250 | $\approx 250,000$ |
| <u>shortening</u> * | ≈ 250 | $\approx 250,000$ |

Viscosity of Fluids with variable compositions

| Liquid : | Viscosity [Pa · s] | Viscosity [cP=mPa · s] |
|-------------------------------------|------------------------|---------------------------|
| <u>acetone</u> [27] | 3.06×10^{-4} | 0.306 |
| <u>benzene</u> [27] | 6.04×10^{-4} | 0.604 |
| <u>castor oil</u> [27] | 0.985 | 985 |
| <u>corn syrup</u> [27] | 1.3806 | 1380.6 |
| <u>ethanol</u> [27] | 1.074×10^{-3} | 1.074 |
| <u>ethylene glycol</u> | 1.61×10^{-2} | 16.1 |
| <u>glycerol</u> (at 20 °C)[24] | 1.2 | 1200 |
| <u>HFO-380</u> | 2.022 | 2022 |
| <u>mercury</u> [27] | 1.526×10^{-3} | 1.526 |
| <u>methanol</u> [27] | 5.44×10^{-4} | 0.544 |
| <u>motor oil</u> SAE 10 (20 °C)[19] | 0.065 | 65 |
| <u>motor oil</u> SAE 40 (20 °C)[19] | 0.319 | 319 |
| <u>nitrobenzene</u> [27] | 1.863×10^{-3} | 1.863 |
| <u>liquid nitrogen</u> @ 77K | 1.58×10^{-4} | 0.158 |
| <u>propanal</u> [27] | 1.945×10^{-3} | 1.945 |
| <u>olive oil</u> | .081 | 81 |
| <u>pitch</u> | 2.3×10^8 | 2.3×10^{11} |
| <u>sulfuric acid</u> [27] | 2.42×10^{-2} | 24.2 |
| <u>water</u> | 8.94×10^{-4} | 0.894 |

Viscosity of Liquids
(at 25 °C unless otherwise specified)

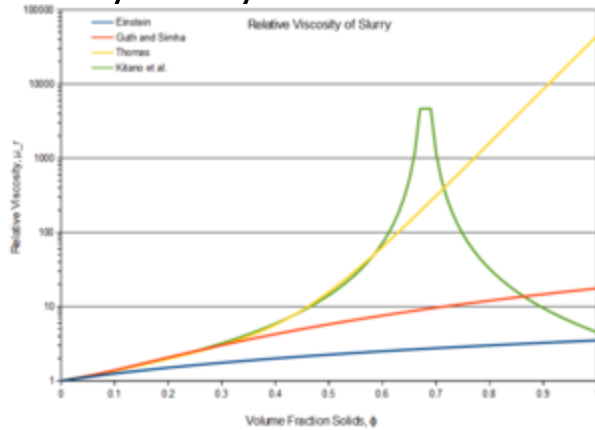
| Solid | Viscosity [Pa · s] | Temperature [K] |
|---------------------------|----------------------------|--------------------|
| <u>asthenosphere</u> [29] | 7.0×10^{19} | 900 °C |
| <u>upper mantle</u> [29] | $(0.7-1.0) \times 10^{21}$ | 1300-3000 °C |
| <u>lower mantle</u> | $(1.0-2.0) \times 10^{21}$ | 3000-4000 °C |

Viscosity of Solids

* These materials are highly Non-Newtonian.

Note: Higher viscosity means thicker substance

Viscosity of slurry



Plot of slurry relative viscosity μ_r as calculated by empirical correlations from Einstein,[30] Guth and Simha,[31] Thomas,[32]and Kitano *et al.*[33]

The term slurry describes mixtures of a liquid and solid particles that retain some fluidity. The viscosity of slurry can be described as relative to the viscosity of the liquid phase:

$$\mu_s = \mu_r \cdot \mu_l,$$

where μ_s and μ_l are respectively the dynamic viscosity of the slurry and liquid (Pa·s), and μ_r is the relative viscosity (dimensionless).

Depending on the size and concentration of the solid particles, several models exist that describe the relative viscosity as a function of volume fraction ϕ of solid particles.

In the case of extremely low concentrations of fine particles, Einstein's equation[30] may be used:

$$\mu_r = 1 + 2.5 \cdot \phi$$

In the case of higher concentrations, a modified equation was proposed by Guth and Simha,[31] which takes into account interaction between the solid particles:

$$\mu_r = 1 + 2.5 \cdot \phi + 14.1 \cdot \phi^2$$

Further modification of this equation was proposed by Thomas[32] from the fitting of empirical data:

$$\mu_r = 1 + 2.5 \cdot \phi + 10.05 \cdot \phi^2 + A \cdot e^{B \cdot \phi},$$

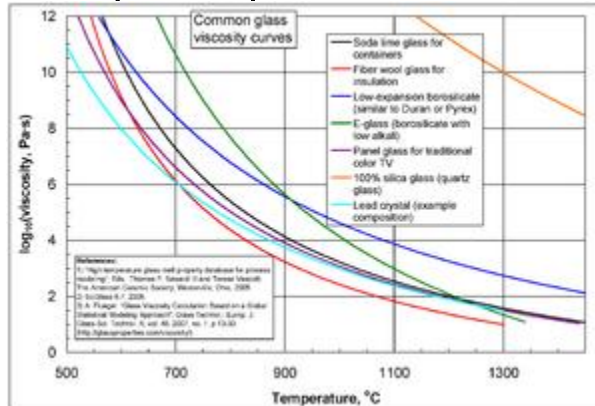
where $A = 0.00273$ and $B = 16.6$.

In the case of high shear stress (above 1 kPa), another empirical equation was proposed by Kitano *et al.* for polymer melts:[33]

$$\mu_r = \left(1 - \frac{\phi}{A}\right)^{-2},$$

where $A = 0.68$ for smooth spherical particles.

Viscosity of amorphous materials



Common glass viscosity curves.[34]

Viscous flow in amorphous materials (e.g. in glasses and melts)[35][36][37] is a thermally activated process:

$$\mu = A \cdot e^{Q/RT},$$

where Q is activation energy, T is temperature, R is the molar gas constant and A is approximately a constant.

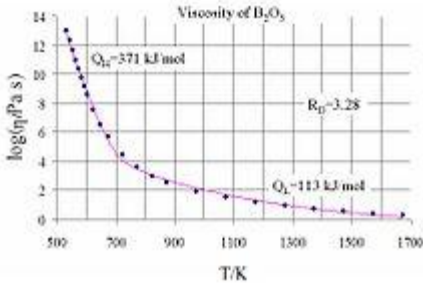
The viscous flow in amorphous materials is characterized by a deviation from the Arrhenius-type behavior: Q changes from a high value Q_H at low temperatures (in the glassy state) to a low value Q_L at high temperatures (in the liquid state). Depending on this change, amorphous materials are classified as either

- strong when: $Q_H - Q_L < Q_L$ or
- fragile when: $Q_H - Q_L \geq Q_L$.

The fragility of amorphous materials is numerically characterized by the Doremus' fragility ratio:

$$R_D = \frac{Q_H}{Q_L}$$

and strong material have $R_D < 2$ whereas fragile materials have $R_D \geq 2$.



Common log of viscosity vs temperature for B_2O_3 , showing two regimes

The viscosity of amorphous materials is quite exactly described by a two-exponential equation:

$$\mu = A_1 \cdot T \cdot \left[1 + A_2 \cdot e^{B/RT} \right] \cdot \left[1 + C \cdot e^{D/RT} \right],$$

with constants A_1 , A_2 , B , C and D related to thermodynamic parameters of joining bonds of an amorphous material.

Not very far from the glass transition temperature, T_g , this equation can be approximated by a Vogel-Fulcher-Tammann (VFT) equation.

If the temperature is significantly lower than the glass transition temperature, $T \ll T_g$, then the two-exponential equation simplifies to an Arrhenius type equation:

$$\mu = A_L T \cdot e^{Q_H/RT}$$

with:

$$Q_H = H_d + H_m,$$

where H_d is the enthalpy of formation of broken bonds (termed configurons) and H_m is the enthalpy of their motion. When the temperature is less than the glass transition temperature, $T < T_g$, the activation energy of viscosity is high because the amorphous materials are in the glassy state and most of their joining bonds are intact.

If the temperature is highly above the glass transition temperature, $T \gg T_g$, the two-exponential equation also simplifies to an Arrhenius type equation:

$$\mu = A_H T \cdot e^{Q_L/RT},$$

with:

$$Q_L = H_m.$$

When the temperature is higher than the glass transition temperature, $T > T_g$, the activation energy of viscosity is low because amorphous materials are melted and have most of their joining bonds broken, which facilitates flow.

Eddy viscosity

In the study of turbulence in Fluids, a common practical strategy for calculation is to ignore the small-scale *vortices* (or *eddies*) in the motion and to calculate a large-scale motion with an *eddy viscosity* that characterizes the transport and dissipation of energy in the smaller-scale flow (see *large eddy simulation*). Values of eddy viscosity used in modeling ocean circulation may be from 5×10^4 to $10^6 \text{ Pa} \cdot \text{s}$ depending upon the resolution of the numerical grid.

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