# Quantum Mechanics_polytropic process

A **polytropic process** is a thermodynamic process that obeys the relation:

$$pv^n = C$$

where p is the pressure, v is specific volume, n, the **polytropic index**, is any real number, and C is a constant. The polytropic process equation is particularly useful for characterizing expansion and compression processes which include heat transfer. This equation can accurately characterize a very wide range of thermodynamic processes, that range from $n<0$ to $n=\infty$ which covers, $n=0$ (Isobaric), $n=1$ (Isothermal), $n=\gamma$ (Isentropic), $n=\infty$ (Isochoric) processes and all values of n in between. Hence the equation is polytropic in the sense that it describes many lines or many processes. In addition to the behavior of gases, it can in some cases represent some liquids and solids. The one restriction is that the process should display an energy transfer ratio of $K=\delta Q/\delta W=$constant during that process. If it deviates from that restriction it suggests the exponent is not a constant. For a particular exponent, other points along the curve can be calculated:

$$P_1 v_1^n = P_2 v_2^n = ... = C$$

Derivation
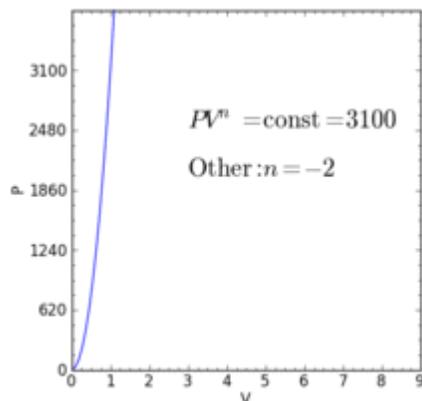
Polytropic processes behave differently with various polytropic indice. Polytropic process can generate other basic thermodynamic processes.

The following derivation is taken from Christians.[1] Consider a gas in a closed system undergoing an internally reversible process with negligible changes in kinetic and potential energy. The First Law of Thermodynamics is

$$\delta q - \delta w = du$$

Define the energy transfer ratio,
$$K = \frac{\delta q}{\delta w}.$$

For an internally reversible process the only type of work interaction is moving boundary work, given by *Pdv*. Also assume the gas is calorically perfect (constant specific heat) so *du* = *c$_v$dT*. The First Law can then be written
$$(K-1)Pdv = c_v dT \quad \text{(Eq. 1)}$$

Consider the Ideal Gas equation of state with the well-known compressibility factor, *Z*: *Pv* = *ZRT*. Assume the compressibility factor is constant for the process. Assume the gas constant is also fixed (i.e. no chemical reactions are occurring). The *PV = ZRT* equation of state can be differentiated to give
$$Pdv + vdP = ZRdT$$

Based on the well-known specific heat relationship arising from the definition of enthalpy, the term *ZR* can be replaced by *c$_p$* − *c$_v$*. With these observations First Law (Eq. 1) becomes
$$-\frac{vdP}{Pdv} = (1-\gamma)K + \gamma$$

where *γ* is the ratio of specific heats. This equation will be important for understanding the basis of the polytropic process equation. Now consider the polytropic process equation itself:
$$Pv^n = C$$

Taking the natural log of both sides (recognizing that the exponent *n* is constant for a polytropic process) gives
$$\ln P + n \ln v = C$$

which can be differentiated and re-arranged to give
$$n = -\frac{vdP}{Pdv}$$

By comparing this result to the result obtained from the First Law, it is concluded that **the polytropic exponent is constant (and therefore the process is polytropic) when the energy transfer ratio is constant for the process**. In fact the polytropic exponent can be expressed in terms of the energy transfer ratio:
$$n = (1-\gamma)K + \gamma.$$

This derivation can be expanded to include polytropic processes in open systems, including instances where the kinetic energy (i.e. Mach Number) is significant. It can also be expanded to include irreversible polytropic processes (see Ref [1]).

Applicability

The polytropic process equation is usually applicable for reversible or irreversible processes of ideal or near-ideal gases involving heat transfer and/or work interactions **when the energy transfer ratio (δq/δw) is constant for the process**. The equation may not be applicable for processes in an open system if the kinetic energy (i.e. Mach Number) is significant. The polytropic process equation may also be applicable in some cases to processes involving liquids, or even solids.

Polytropic Specific Heat Capacity

It is denoted by C_n and it is equal to $$C_n = C_V \frac{\gamma - n}{1 - n}$$

Relationship to ideal processes

For certain values of the polytropic index, the process will be synonymous with other common processes. Some examples of the effects of varying index values are given in the table.

Polytropic index	Relation	Effects
$n < 0$	—	Negative exponents reflect a process where the amount of heat being added is large compared to the amount of work being done (i.e. the energy transfer ratio > γ/(γ−1)). Negative exponents can also be meaningful in some special cases not dominated by thermal interactions, such as in the processes of certain plasmas in astrophysics.[2]
$n = 0$	$pV^0 = p$ (constant)	Equivalent to an isobaric process (constant Pressure)
$n = 1$	$pV = NRT$ (constant)	Equivalent to an isothermal process (constantTemperature)
$1 < n < \gamma$	—	A quasi-adiabatic process such as in an internal combustion engine during expansion, or in vapor compression refrigeration during compression. Also a "polytropic compression" process like gas through a

centrifugal compressor where heat loss from the compressor (into environment) is greater than the heat added to the gas through compression.

$\gamma = \dfrac{C_p}{C_V}$ is the isentropic exponent, yielding an isentropic process (adiabatic and reversible). It is also widely referred as adiabatic index, yielding an adiabatic process (no Heat transferred). However the term adiabatic does not adequately describe this process, since it only implies no heat transfer.[3] Only a *reversible* adiabatic

$n = \gamma$ — process is an isentropic process.

Normally polytropic index is greater than specific heat ratio (gamma) within a "polytropic compression" process like gas through a centrifugal compressor. The inefficiencies of centrifugal compression and heat added

$\gamma < n < \infty$ — to the gas outweigh the loss of heat into the environment.

$n = \infty$ — Equivalent to an isochoric process (constant Volume)

Variation of polytropic index n

When the index n is between any two of the former values (0, 1, γ, or ∞), it means that the polytropic curve[*clarification needed*] will be bounded by the curves of the two corresponding indices.

Note that $1 < \gamma < 2$, since $\gamma = \dfrac{C_p}{C_V} = \dfrac{C_V + R}{C_V} = 1 + \dfrac{R}{C_V} = \dfrac{C_p}{C_p - R}$.

Notation

In the case of an isentropic ideal gas, γ is the ratio of specific heats, known as the adiabatic index or as adiabatic exponent.

An isothermal ideal gas is also a polytropic gas. Here, the polytropic index is equal to one, and differs from the adiabatic index γ.

In order to discriminate between the two gammas, the polytropic gamma is sometimes capitalized, Γ.

To confuse matters further, some authors refer to Γ as the polytropic index, rather than n. Note that

$$n = \frac{1}{\Gamma - 1}.$$

Other

A solution to the <u>Lane-Emden equation</u> using a polytropic fluid is known as a<u>polytrope</u>.

References

1. ^ _a_ _b_ Christians, Joseph, "Approach for Teaching Polytropic Processes Based on the Energy Transfer Ratio,_International Journal of Mechanical Engineering Education_, Volume 40, Number 1 (January 2012), Manchester University Press

2. ^ G. P. Horedt <u>Polytropes: Applications In Astrophysics And Related Fields</u>, Springer, 10/08/2004, pp.24.

3. ^ GPSA book section 13